

Fig. 2


Fig. 4

# SEMI-GRAPHIC CO-ORDINATE COMPUTATION 

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The semi-graphic method of determining the co-ordinates of a point is simple and often less laborious than others. A closer acquaintance with it would probably bring it into much more common use in Hydrographic Surveying.

It frequently occurs during the course of a survey that an additional main station or an important secondary station is found to be necessary. Much may depend on fixing this station with considerable precision, but, with the ordinary methods of adjustment, a large amount of re-computation might be necessary to connect is satisfactorily with the original triangulation. In cases of this description, and particularly when the additional station itself is inaccessible, the semi-graphic method is very useful as it enables the co-ordinates of the station to be determined with accuracy, after which the accepted angles connecting it can be calculated. The method is also useful when a considerable number of Ordnance Stations are available and, for the purposes of the survey, it is only necessary to fix a few additional minor points.

The method is as follows : The approximate co-ordinates of the unknown point must first be found either from one of the triangles connecting it or, when the scale is not too small, from a rough plot on the plotting sheet. A rectangular grid on some convenient scale - say 1 inch on the paper representing 2 feet on the ground - is then plotted on squared paper, the vertical sides of the grid being parallel to the co-ordinate meridian and co-ordinates being allotted to the sides which ensure that they will enclose the station. For instance, if the approximate co-ordinates of the station were found to be

$$
\begin{aligned}
& x=-4550 \text { feet } \\
& y=+2100
\end{aligned}
$$

a suitable grid would be one enclosing an area 10 ' by $10^{\prime}$ ( $5^{\prime \prime}$ by 5 " on the paper) with the vertical sides having co-ordinates of $x=-4545$ and - 4555 respectively and the horizontal sides having co-ordinates of $y=+2095$ and +2105 respectively. The problem is then to find the co-ordinate bearings of the rays from the known stations to the unknown one and to find at what points the rays first cut the sides of the grid. If the approximate co-ordinates of the unknown station are fairly correct, it is generally obvious which side of the grid, $i$, e. whether a vertical or a horizontal, any particular ray will cut, but, if an erroneous assumption in this respect should happen to be made, it is usually immaterial as the ray can still be plotted. Two examples of the method, which occurred in practice, follow.

## EXAMPLE I.

Figure $I$ shows the positions of the various stations. The points $E, F, G, H$ and $J$ were main stations, their co-ordinates had been calculated and the triangulation connecting them adjusted. It was necessary to fix the connecting station $d$ and connect it with the triangulation. The five main stations had observed rays to $d$ but $d$ itself was inaccessible.

The co-ordinate bearings connecting some of the main stations, if not already known, must be first calculated. In this case they were as follows:

| E |  |  | $4^{\circ 00} 54$ " |  |
| :---: | :---: | :---: | :---: | :---: |
| $J-H$ | N |  |  |  |
| $F-H$ | S |  | 51616 |  |
| G-J | S |  | I 54 |  |

The approximate co-ordinates of $d$ were found from the plotting sheet to be:

$$
\begin{aligned}
& x=+9080 \text { feet. } \\
& y=-1843 \text { Io }
\end{aligned}
$$



The sides selected for the grid had the following co-ordinates :

$$
\begin{array}{lr}
\text { N. side } y=-184305 \text { feet. } & \text { E. side } x=+9085 \text { feet. } \\
\text { S. side } y=-184315, & \text { W. side } x=+9075, "
\end{array}
$$

The first step is to find the co-ordinate bearing of the secondary station from each of the main observing stations by applying the observed angle to a known co-ordinate bearing connecting the main station with any other. The second step is the solution of the right-angled triangle of which the hypotenuse is formed by the observing station and the point of intersection of the ray on the near side of the grid, one side of the triangle being known from the difference of co-ordinates between the observing station and the same side of the grid. The working is shown on the attached sheet, the results giving the co-ordinates of the points at which the rays or bearings cut the near sides of the grid.

With this data the rays can be plotted, as shewn on Figure 2.
The position of $d$ can now be interpolated on the diagram. Strickly speaking, the position adopted should be such that its distance from each of the rays is proportional to the distance of $d$ from each of the observing stations. When more than three rays have been observed, this cannot usually be precisely done. In this case it is clear that the ray from $E$ should be neglected; either an error of observation or booking has occurred. The other four rays are, however, in good agreement and the position of $d$ (as shewn on the diagram) can be readily interpolated, its co-ordinates being :

$$
\begin{aligned}
& x=+ \text { 9080.1 feet. } \\
& y=-184310.9
\end{aligned}
$$

## EXAMPLE II.

In this case it was necessary to fix an inaccessible point $X$ which had been observed from three main stations, $A, B$, and $C$. The approximate co-ordinates of $X$ were found, by solving one of the triangles, to be:

$$
\begin{aligned}
& x=-148573 \text { feet. } \\
& y=+1085_{56}
\end{aligned}
$$

The sides selected for the grid had the following co-ordinates:

$$
\begin{array}{ll}
\text { N. side, } y=+108162 \text { feet. } & \text { E. side } x=-148567 \text { feet. } \\
\text { S. } \quad y=108152 & \text { W. } " x=-148577,
\end{array}
$$

Example I

| Observing Station ... H | $E$ | $J$ | $F$ | $G$ |
| :---: | :---: | :---: | :---: | :---: |
| Co-ord. Bg. H-E.... N $14^{\circ} \mathrm{OO}{ }^{\prime} 54{ }^{\prime \prime} \mathrm{W}$. |  | $J-H . N 53^{\circ} \mathrm{O} 7^{\prime} \mathrm{o} 8^{\prime \prime} \mathrm{W}$. | $F-H . . ~ S ~ 65^{\circ} 6^{\prime \prime} 16^{\prime \prime} \mathrm{W}$. | G-J... S $\mathrm{S}^{0} 54^{\prime} 45^{\prime \prime} \mathrm{E}$. |
| Obsd. Angle EHd... $12 \times 1659$ | HEd. 382905 | HJd $\quad 533942$ | HFd. 552225 | JGd.. 5712 Io |
| $\begin{aligned} & \text { Sum or Diff. }=\quad \text { Io } 7^{\circ} 16^{\prime} 05^{\prime \prime} W \\ & \text { Co-ord. Bg. } H-d \ldots \quad \text { S } 7^{\circ} 43^{\prime} 55^{\prime \prime} \mathrm{E} . \end{aligned}$ | E-d... S $5^{2}{ }^{\circ} 29^{\prime} 59^{\prime \prime} \mathrm{E}$. | $J-d . . \quad \mathrm{N} \quad 0^{\circ} 32^{\prime} 34 " \mathrm{E}$. | $F-d \ldots \quad \mathrm{~S} \quad 9^{0} 53{ }^{\prime} 5 \mathrm{I} \mathrm{W}$. | G-d... S $59^{\circ} 06^{\prime} 55^{\prime \prime} \mathrm{E}$. |
| $x$ co-ord. of H $\ldots \ldots \ldots . .5126 .6$ | $x$ of E... - 7129.3 | $y$ of $J \ldots . .190509 .5$ | $y$ of F... - 172393.3 | $x$ of $G \ldots+8808.9$ |
| $x$ of West side of grid.................... +9075.0 | ```x of W. sidc.... + 9075.0``` | $\begin{aligned} & y \text { of } \mathrm{S} . \\ & \text { side.... }-184315.0 \end{aligned}$ | $\begin{aligned} & y \text { of } N . \\ & \text { side.... - } 843305.0 \end{aligned}$ | $\begin{aligned} & x \text { of W. } \\ & \text { side.... }+9075.0 \end{aligned}$ |
| Diff. (Algbr.) .............. 14201.6 | 16204.3 | 6194.5 | 199x.7 | 266.1 |
| Log. Diff. ................. 4.1523373 | 4.2096302 | 3.7920063 | 4.0759737 | 2.4250449 |
| Co-ord. Bg. H-d cot.... 9.4925562 | E-d cot.. 9.8849848 | $J-d$ tan.. $7.976 \quad 5124$ | $F-d$ tan.. 9.2417531 | G-d cot.. 9.7767924 |
| Sum........................ 3.6448935 | 4.0946150 | 1.7685187 | 3.3177268 | 2.2018373 |
| Antilog (*) ................ - 4414.6 | - 12434.1 | $+\quad 58.7$ | - 2078.4 | 159.2 |
| $y$ co-ord. of H $\ldots \ldots \ldots . .$. | $y$ of E... - 171870.7 | $x$ of $J \ldots \ldots+902 \mathrm{I} 3$ | $x$ of $F \ldots+$ III59.8 | $y$ of $G \ldots-184148.7$ |
| Point of intersection, $y . \ldots . - \pm 84308.8$ | y......... - 184304.8 | $x \ldots \ldots \ldots \ldots$ | $x \ldots \ldots \ldots \ldots+9081.4$ | y.......... - 184307.9 |

$\left(^{*}\right)$ Note. - This antilog represents the distance on or at right angles to the meridian from the observing station to the point at which the observed ray cuts the near side of the grid.

The relative positions of the stations are shewn on Figure 3.


The co-ordinate bearings connecting the main stations were as follows:
$A-B \quad \mathrm{~N} 8 \mathrm{I}^{\circ} 2 \mathrm{I}$ '19" $\mathrm{E} . \quad B-C \quad \mathrm{~S} \quad 323052 \mathrm{E} . \quad C-B \quad \mathrm{~N} 323052 \quad \mathrm{~W}$.
The computation is then as shewn below.

| Observing Station ..... | A | $B$ | C |
| :---: | :---: | :---: | :---: |
| Co-ord. Bg. .... $A-B .$. <br> Obsd. Angle... BAX | $\begin{gathered} \text { N 8102 I'I9" } \mathrm{E} \\ \mathrm{I}_{4} 4_{5} . \end{gathered}$ | $\begin{array}{lc} B-C & \mathrm{~S}{32^{\circ} 30^{\prime} 52^{\prime \prime} \mathrm{E}}^{C B X} \\ 45494^{8} \end{array}$ | $\begin{array}{cc} C-B . . & \mathrm{N} 32^{\circ} 30^{\prime} 5^{\prime \prime} \mathrm{W} . \\ B C X & 102329 \end{array}$ |
| $\begin{aligned} & \text { Sum or Diff. }= \\ & \text { Co-ord. Bg..... } A-X . . \end{aligned}$ | N 83 10 10 E. | $B-X \quad \mathrm{~S} 782040 \mathrm{~W}$ | $C-X . . \mathrm{N} 220723 \mathrm{~W}$ |
| $x$ co-ord. of $A \ldots \ldots \ldots$. | - 186132.4 | $x$ of $B$ - 151962.9 | $y$ of $C \ldots+95405.5$ |
| $x$ of West side of grid.................... | - 148577.0 | $\begin{aligned} x & \text { of } W . \\ \text { side .. } & -148577.0 \end{aligned}$ | ```y of S side.... + 108152.0``` |
| Diff. (Algbr.) | $37555 \cdot 4$ | 3385.9 | 12746.5 |
| Log Diff. | 4.5746724 | 3.529674 I | 4.1053910 |
| Co-ord. Bg. A-X cot... | 9.0783977 | $B-X \cot 9.3144597$ | C-X tan. 9.6090891 |
| Sum | 3.653 0701 | 2.8441338 | 3.7144801 |
| Antilog | + 4498.5 | - 698.4 | - 5181.8 |
| $y$ co-ord. of $A \ldots \ldots$ | + 103661.0 | $y$ of $B .+108855.9$ | $x$ of C... -143389.3 |
| Point of intersection, y...................... | + 108159.5 | y....... + 108157.5 | $x \ldots \ldots \ldots .14857 \mathrm{I} . \mathrm{T}$ |

This computation supplies the data for plotting the rays from $A, B$ and $C$ as shewn on the Figure 4. A position is now assigned to $X$ such that the perpendiculars from it to each of the rays are proportional to the actual distances of $X$ from each of the respective stations. The distances $A-X, B-X$, and $C-X$ were $6.21,0.57$ and 2.25 miles respectively and the perpendiculars $a, b$ and $c$ on the diagram bear these proportions, the co-ordinates of $X$ being found to be:

$$
x=-148574.3 \text { feet. } \quad y=+108 \mathrm{I} 57.2
$$

