# THE ADJUSTMENT OF TRIANGULATION

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## INTRODUCTION.

Many difficulties have been experienced by those in charge of Hydrographic Surveys in adjusting triangulation. These difficulties not only lead to loss of accuracy, but result in many hours being wasted in the endeavour to reduce the network of triangulation to some state of consistency. In the absence of any defined system the methods used by Hydrographic Surveyors are many and far from accurate. The result is that many triangulations will not bear the test of a rigid examination, and it is surprising how clearly this is demonstrated when such a test is applied.

The system described in the following pages does not appear in standard works on Hydrographic Surveying. It is used, however, in adjusting minor triangulations of geodetic surveys.

In adjusting to self-consistency in work of high precision, the method of least squares (WRIGHT & HAYFORD : *Adjustment of Observations*, Ch. VII, London, 1906) is used in many government surveys. This method is laborious and need not be considered further here, as it is intended for use with observations of very high precision. The triangulations of Hydrographic Surveys may generally be considered to come in the category of Minor Triangulation, in which the accepted principle is that all observations must be adjusted to self-consistency, but that any distribution of error is tolerable provided that the maximum correction to any angle is not too great.

When adjustment is complete, it is known that any further discrepancies must be errors of computation since they cannot be errors of observation. This consideration is of great value since it makes it practically impossible for an error in computation to escape detection.

The fact that a figure has been adjusted to satisfy certain geometrical conditions is no guarantee that the angles have been correctly adjusted, *i. e.*, that their adjusted values are their true values, and it is important to realise this fact. We can say, however, that unless the figure is adjusted to geometrical consistency the adjustment must be incorrect.

This method of adjustment has the following advantages :

1. The surveyor is taught to group his triangles in polygons and quadrilaterals instead of confining his attention to single triangles.
2. Assistance is given to the surveyor in choosing the number and sites of his main stations.
3. The adjusted figure is symmetrically correct and geometrically consistent, that is to say, the values of the sides are proportional to the containing angles.
4. When once a triangle has been adjusted, no further correction is necessary to that triangle when it is used in conjunction with an adjoining figure.
5. The process of adjustment includes the provision of the *Log Sines* which are required for obtaining the values of the sides.
6. The method provides a quick means of checking the accuracy of a triangulation received in office.

This method of adjustment is not intended for use in beacon triangulation. In work of this character errors of observation are so large that no useful purpose is served by attempting to reduce them to a state of consistency in this way. In fact it is probable that it would only result in increasing the errors of the triangulation.

Examples are given in the following pages of the adjustment of a polygon and a quadrilateral. Large errors of observation have been supposed in order to demonstrate the method more clearly.

## ADJUSTMENT OF A POLYGON.

In Fig. 1 let the stations $B, C, D, E$ and $N$ form a polygon round the central station $A$. Let the component triangle $ABN$ and the small triangles round $S$ be previously adjusted. All the angles have been observed.

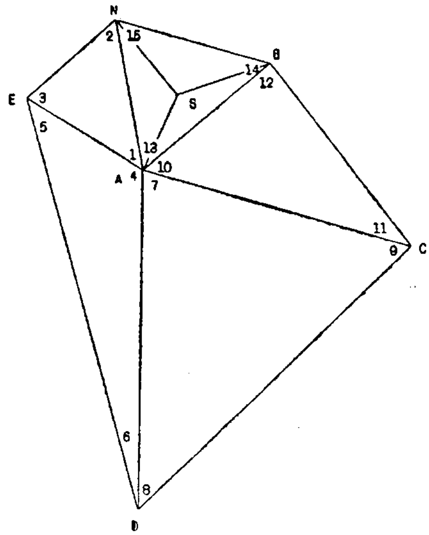



Fig. 1

It is required to adjust the observed angles in such a manner that the figure shall be geometrically consistent in every respect.

The procedure is as follows:

1. Prepare two forms, $A$ and $B$, as shown on pages 213 and 214. These can both be accommodated on one sheet of foolscap.

Then on form $A$:

2. Tabulate the observed angles of each triangle. It is convenient to keep the angles of each triangle in the same order; *e. g.*, start with the forward angle in each case and finish with the central angle. In the case of any part or parts of the polygon having been previously adjusted tabulate the adjusted angles. In this case triangle $ABN$ has already been adjusted; therefore tabulate its adjusted angles. No adjustment to these angles is to be applied in any of the subsequent working.

3. Tabulate the weighting values of the observations opposite each angle. In this case let the values be:

$$B = 2, \quad C = 3, \quad D = 1, \quad E = 2, \quad N = 1 \quad \text{and} \quad A = 1,$$

*i. e.*, the angles at $C$ have to be corrected three times as heavily as those at $D$ or $N$, and so on.

4. Sum the angles of each triangle, and apply the necessary corrections in accordance with the weighting values to make each corrected sum equal 180°.

5. Deduce the central angles at $A$ and tabulate them in the next column. Adjust these angles so that they sum to 360°. Angle 13 having been previously adjusted is not altered.

6. The adjustment of the central angles will disturb the previous adjustment; therefore readjust the exterior angles so that the triangles again sum to 180°. The adjustments in Sections 5 and 6 will always be comparatively small and may be distributed equally without taking into consideration the weighting values.

7. Tabulate the angles under the heading "Angles, 2nd Adjst.". No further adjustment of the central angles is made; so their *Log Sines* may now be looked out for future use.

On form $B$:

1. Tabulate all the exterior angles of the polygon, transferring them from the

column headed "Angles, 2nd Adjst.". Place all the forward angles, *i. e.*, Nos. 2, 5, 8, 11 and 14 in the upper half of the form, and all the back angles, *i. e.*, Nos. 3, 6, 9, 12 and 15 in the lower half.

2. Tabulate the *Log Sines* of the angles; and at the same time the "Diff. for 1 sec." which can be obtained from the foot of each column in SHORTREDE'S Tables. If an angle is over 90°, its "Diff. for 1 sec." must be prefixed by minus sign.

3. Sum the *Log Sines* of each set of angles. Call the sums $S1$ and $S2$; take the difference between $S1$ and $S2$ and call this $D$.

4. Sum algebraically all the "Diffs. for 1 sec." of both sets: call this $S(d)$. Note that the "Diffs. for 1 sec." of the angles already adjusted, *i. e.*, Nos. 14 and 15 are not to be tabulated.

5. Divide $D$ by $S(d)$ and call this $e$. The quantity $e$ is the number of seconds which must be added to each of the forward angles 2, 5, 8 and 11, and subtracted from each of the back angles 3, 6, 9 and 12. It will be seen that in this example the sum of the *Log Sines* of the forward angles is less than that of the back angles. $e$ is therefore additive to the forward angles.

6. Applying the quantity $e$ to each angle, tabulate the finally adjusted angles, and look out their *Log Sines* for future use.

The adjustment of the figure is now complete, *i. e.*, the angles have been adjusted so that the figure is geometrically consistent.

The proof of this adjustment will be found on page 214.

## ADJUSTMENT OF A POLYGON.

FORM *A*.

| Δ | *Angle* | *Wt. Value* | *Observed Angles* | 1st *Corrn.* | *Deduced Central Angles* | 2nd *Corrn.* | *Angles* (2nd *Adjst.*) | *Log Sines* |
|---|---|---|---|---|---|---|---|---|
| | 2 | 1 | 71°47'00" | + 0'30" | | — 7".1 | 71°47'22".9 | |
| *AEN* | 3 | 2 | 73 06 00 | + 1 00 | | — 7 .1 | 73 06 52 .9 | |
| | 1 | 1 | 35 05 00 | + 0 30 | 35°05'30".0 | + 14.2 | 35 05 44 .2 | 9.7596245 |
| | | | 179 58 00 | | | | 180 00 00 .0 | |
| | 5 | 2 | 35 42 00 | + 0 45 | | — 7.2 | 35 42 37 .8 | |
| *ADE* | 6 | 1 | 19 32 30 | + 0 22.5 | | — 7.1 | 19 32 45 .4 | |
| | 4 | 1 | 124 44 00 | + 0 22.5 | 124°44'22".5 | + 14.3 | 124 44 36 .8 | 9.9147191 |
| | | | 179 58 30 | | | | 180 00 00 .0 | |
| | 8 | 1 | 48 49 00 | — 0 18 | | — 7.1 | 48 48 34 .9 | |
| *ACD* | 9 | 3 | 58 28 30 | — 0 54 | | — 7.1 | 58 27 28 .9 | |
| | 7 | 1 | 72 44 00 | — 0 18 | 72°43'42".0 | + 14.2 | 72 43 56 .2 | 9.9799707 |
| | | | 180 01 30 | | | | 180 00 00. 0 | |
| | 11 | 3 | 36 00 00 | — 0 30 | | — 7.1 | 35 59 22 .9 | |
| *ABC* | 12 | 2 | 86 22 00 | — 0 20 | | — 7.2 | 86 21 32 .8 | |
| | 10 | 1 | 57 39 00 | — 0 10 | 57°38'50".0 | + 14.3 | 57 39 04 .3 | 9.9267572 |
| | | | 180 01 00 | | | | 180 00 00 .0 | |
| | 14 | — | 59 24 20 .5* | — | | — | Previously | |
| *ANB* | 15 | — | 50 49 01 .0* | — | | — | adjusted. | |
| | 13 | — | 69 46 38 .5* | — | 69°46'38".5 | — | | 9.9723679 |
| | | | 180 00 00 .0 | | 359°59'03".0 | | | |

(*) **Previously adjusted.**

FORM *B*.

| *Angle* | *Angles* (2nd *Adjst.*) | *Log Sines* | *Diff. for* 1". | *Angles* (*finally*) *adjusted*). | *Log Sines* |
|---|---|---|---|---|---|
| 2 | 71°47'22".9 | 9.9776851 | 6.9 | 71°47'23".4 | 9.9776855 |
| 5 | 35 42 37 .8 | 9.7661823 | 29.3 | 35 42 38 .3 | 9.7661838 |
| 8 | 48 48 34 .9 | 9.8765217 | 18.4 | 48 48 35 .4 | 9.8765226 |
| 11 | 35 59 22 .9* | 9.7691111 | 29.0 | 35 59 23 .4 | 9.7691126 |
| 14 | 59 24 20 .5 | 9.9348985 | — | 59 24 20 .5 | 9.9348985 |
| | *S* 1 = | 49.3243987 | | | 4030 |
| 3 | 73 06 52 .9 | 9.9808611 | 6.4 | 73 06 52 .4 | 9.9808609 |
| 6 | 19 32 45 .4 | 9.5244775 | 59.3 | 19 32 44 .9 | 9.5244745 |
| 9 | 58 27 28 .9 | 9.9305707 | 12.9 | 58 27 28 .4 | 9.9305700 |
| 12 | 86 21 32 .8 | 9.9991226 | 1.3 | 86 21 32 .3 | 9.9991225 |
| 15 | 50 49 01 .0* | 9.8893753 | — | 50 49 01 .0 | 9.8893753 |
| | *S* 2 = | 49.3244072 | *S* (*d*) 163.5 | | 4032 |
| | *S* 1 = | 49.3243987 | | | |
| | Diff. (*D*) = | 85 | | | |

$$e = \frac{85}{163.5} = 0''.5.$$

(*) Previously adjusted.

PROOF OF THE ADJUSTMENT OF THE POLYGON.

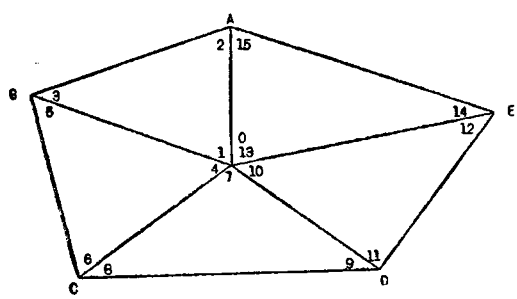



Fig. 2

In Fig. 2, *ABCDE* is a polygon in which the length of one side, say *AB*, is known and the angles 1 to 15 have been observed. To make this figure geometrically consistent the following equations must be satisfied:

(1) The sum of the angles of any triangle must equal 180°.
(2) The sum of the central angles must equal 360°.
(3) The product of the sines of angles 2, 5, 8, 11 and 14 must equal the product of the sines of angles 3, 6, 9, 12 and 15.

The necessity of satisfying equations (1) and (2) is obvious from first geometrical principles, and it might at first sight be supposed that if these two equations are satisfied the figure will be geometrically consistent. The need of a third equation can be demonstrated as follows:

Suppose it is required to find the length of the side *CD*, the side *AB* being known and equations (1) and (2) having been satisfied. This may be done in two ways, *viz*:

(*a*) through triangles *ABO*, *BCO* and *CDO*,
or (*b*) through triangles *ABO*, *AEO*, *EDO* and *CDO*.

Unless the third equation has been satisfied, the results will almost certainly differ because they manifestly depend on the values of the exterior angles of the polygon; and these can be varied at will provided that the sum of the angles of any triangle still equals 180°. For instance, a certain amount can be subtracted from angle 5 provided that an equal amount is added to angle 6. This would not disturb equations (1) and (2) but will evidently alter the length *CD* when calculated through the triangle *BCO*.

If the figure was geometrically consistent, it should be possible to calculate *CD* by any route and obtain the same result.

It thus appears that a further equation is required to complete the adjustment of the figure.

Suppose as before that it is required to calculate the length of the side *CD* from the known length *AB*. This may be done by two routes, the formulae being:

$$(1)\ CD = \frac{OC.\ \sin 7}{\sin 9} = \frac{OB.\ \sin 5.\ \sin 7.}{\sin 6.\ \sin 9.} = \frac{AB.\ \sin 2.\ \sin 5.\ \sin 7.}{\sin 6.\ \sin 9.\ \sin 1.}$$

$$\text{or } (2)\ CD = \frac{OD.\ \sin 7.}{\sin 8.} = \frac{OE.\ \sin 7.\ \sin 12.}{\sin 8.\ \sin 11.}$$

$$= \frac{OA.\ \sin 7.\ \sin 12.\ \sin 15.}{\sin 8.\ \sin 11.\ \sin 14.} = \frac{AB.\ \sin 3.\ \sin 7.\ \sin 12.\ \sin 15.}{\sin 1.\ \sin 8.\ \sin 11.\ \sin 14.}$$

Therefore, if the same result for the value of *CD* is to be obtained by either formulae, the figure must be adjusted so that the following equation is true:

$$\frac{AB.\ \sin 2.\ \sin 5.\ \sin 7.}{\sin 6.\ \sin 9.\ \sin 1.} = \frac{AB.\ \sin 3.\ \sin 7.\ \sin 12.\ \sin 15.}{\sin 1.\ \sin 8.\ \sin 11.\ \sin 14.}$$

Cancelling and cross-multiplying, we have:

$$(3)\quad \sin 2.\ \sin 5.\ \sin 8.\ \sin 11.\ \sin 14 = \sin 3.\ \sin 6.\ \sin 9.\ \sin 12.\ \sin 15.$$

Similarly, for calculating any other side from any known side, it is clearly necessary for this equation to be satisfied if identical results are to be obtained whatever the route followed.

In practice equation (3) will be used in the form:

Sum *log sines* 2, 5, 8, 11, 14 = Sum *log sines* 3, 6, 9, 12, 15.

## ADJUSTMENT OF A POLYGON WITHOUT CENTRAL STATION.

The foregoing method of adjusting a polygon can be used even if no central station exists.

In order to do this it is necessary to select one station to take the place of the central station and number the various angles accordingly. This can best be done by, as it were, turning the figure inside out, and drawing a second diagram with the selected station moved into the centre of the figure. Figures 3 and 4 will explain what is intended.

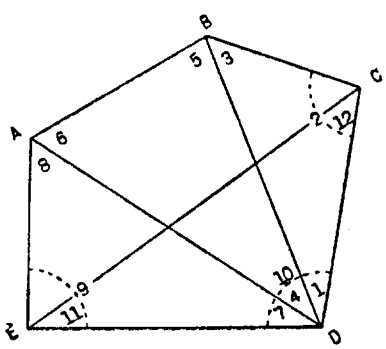



Fig. 3

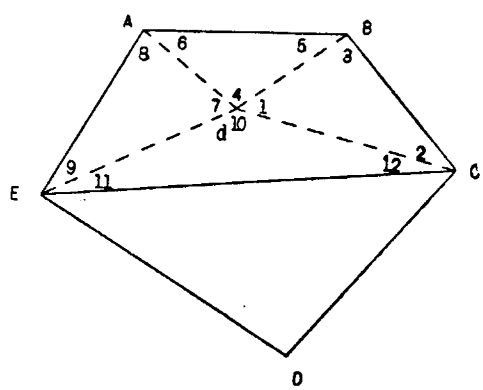



Fig. 4

Figure 3 represents a polygon without central station, in which station *D* has been selected to take the place of the central station. *D* is transferred to *d* as in Figure 4, and the angles are then numbered in the quadrilateral *ABCE* with central station *d*, in accordance with the system already described.

The numbered angles are then identified and marked on Figure 3, and the adjustment of the angles can be made on Form *A* as shown, the only difference being that, instead of making the central angles come to 360°, it is necessary to make the whole angle at *D* equal to its components.

FORM *A*.

| Δ | *Angle* | *Wt. Value* | *Observed Angles.* | 1st *Corrn.* | *Deduced Central Angles* | 2nd *Corrn.* | *Angles* (2nd *Adjst.*) | *Log Sines* |
|---|---|---|---|---|---|---|---|---|
| | 2 | 1 | 114°24'00" | — 0'30" | | — 0'23" | 114°23'07" | |
| *BCD* | 3 | 1 | 37 10 00 | — 0 30 | | — 0 22 | 37 09 08 | |
| | 1 | 2 | 28 28 00 | — 1 00 | 28°27'00" | + 0 45 | 28 27 45 | 9.6781390 |
| | | | 180 02 00 | | | | 180 00 00 | |
| | 5 | 1 | 58°01'00" | — 0'15" | | — 0'22" | 58°00'23" | |
| *ABD* | 6 | 1 | 73 07 00 | — 0 15 | | — 0 23 | 73 06 22 | |
| | 4 | 2 | 48 53 00 | — 0 30 | 48°52'30" | + 0 45 | 48 53 15 | 9.8770371 |
| | | | 180 01 00 | | | | 180 00 00 | |
| | 8 | 1 | 25°13'00" | — 1'00" | | — 0'22" | 25°11'38" | |
| *ADE* | 9 | 1 | 132 50 00 | — 1 00 | | — 0 23 | 132 48 37 | |
| | 7 | 2 | 22 01 00 | — 2 00 | 21°59'00" | + 0 45 | 21 59 45 | 9.5734972 |
| | | | 180 04 00 | | | | 180 00 00 | |
| | 11 | 1 | 46°23'00" | — 0'15" | | + 0'23" | 46°23'08" | |
| *CDE* | 12 | 1 | 34 16 00 | — 0 15 | | + 0 22 | 34 16 07 | |
| | 10 | 2 | 99 22 00 | — 0 30 | 99°21'30" | — 0 45 | 99 20 45 | 9.9941966 |
| | | | 180 01 00 | | 99°18'30" | | 180 00 00 | |

The final balancing is then carried out on form *B* in identically the same manner as before.

The station to be used as "centre" should be that which gives the best conditioned triangles irrespective of the reliability of the angles. In most cases this would be one of the stations opposite to the largest angle of the polygon.

The proof of this method is generally similar to the proof of the adjustment of the polygon already given.

Suppose that it is required to calculate the length of the side *CD* from the known length of *AB*. This may be done by two routes, the formulae being :

$$(1)\ CD = \frac{BD\ sin3.}{sin2.} = \frac{AB\ sin6.\ sin3.}{sin4.\ sin2.}$$

$$\text{or}\ (2)\ CD = \frac{ED\ sin11.}{sin12.} = \frac{AD.\ sin8.\ sin11.}{sin9.\ sin12.} = \frac{AB.\ sin5.\ sin8.\ sin11.}{sin4.\ sin9.\ sin12.}$$

Therefore if the same result for the value of *CD* is to be obtained by either formula, the figure must be adjusted so that the following equation is true :

$$\frac{AB\ sin6.\ sin3.}{sin4.\ sin2.} = \frac{AB\ sin5.\ sin8.\ sin11.}{sin4.\ sin9.\ sin12.}$$

Cancelling and cross-multiplying we have :

$$sin2.\ sin5.\ sin8.\ sin11. = sin3.\ sin6.\ sin9.\ sin12.$$

## ADJUSTMENT OF A QUADRILATERAL.

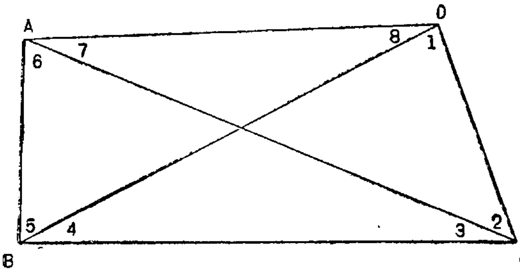



Fig. 5

In Fig. 5 let the stations $A, B, C$ and $D$ form a quadrilateral. It is required to adjust the observed angles in such a manner that the figure shall be geometrically consistent in every respect. All the angles have been observed.

If the weighting values of the observations are not equal, a preliminary adjustment of the angles should be made by considering the various triangles separately and adjusting their angles in accordance with the weighting values. The further adjustment required to make the figure consistent can then be equally divided as shown below.

Assume in this case that the observations have equal weighting values. The procedure is as follows:

1. Prepare a form as shown on page 218.
2. Tabulate the eight observed angles, putting the odd-numbered angles in the upper half of the form and the even-numbered angles in the lower half.
3. Sum the observed angles. The excess or deficiency from $360^{\circ}$ is the amount required for quadrilateral correction. This divided equally between the eight angles gives the First Correction.
4. Applying this correction, tabulate the angles in the column headed "Angles,1stAdjst.".
5. It is now necessary to make the angles $(1+2)$ equal to angles $(5+6)$, and angles $(3+4)$ equal to angles $(7+8)$. This is done as shown in the lower part of the form, by applying to each angle $1/4$ of the difference between each of the pairs. This gives the Second Correction.
6. Applying these corrections, tabulate the angles in the column headed "Angles, 2nd Adjust.".
7. Tabulate the *Log Sines* of the angles, at the same time noting the "Diffs. for 1 sec.", in the next column. These differences are given at the foot of each column in SHORTREDE's Tables. If an angle is over $90^{\circ}$, its "Diff. for 1 sec." must be prefixed by a minus sign.
8. Sum the *Log Sines* of each set of angles and take the difference between the sums. Call this $D$.
9. Sum algebraically the "Diffs. for 1 sec." for all the angles. Call this quantity $S\,(d)$.
10. Divide $D$ by $S\,(d)$, and the resulting quantity $e$ will be the number of seconds to be added to each of the odd-numbered angles and subtracted from each of the even-numbered angles. Note that in this example the sum of the *log sines* of the odd-numbered angles is less than that of the even-numbered angles, $e$ is therefore additive to the odd-numbered angles.
11. Applying the correction $e$, tabulate the finally adjusted angles and look out their *log sines* for future use.

The adjustment of the figure is now complete, *i. e.* the angles have been adjusted so that the figure is geometrically consistent.

The proof of the adjustment will be found on page 219.

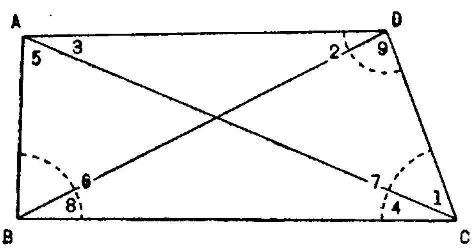



Fig. 6

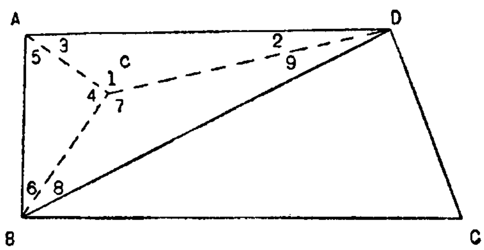



Fig. 7

## ADJUSTMENT OF A QUADRILATERAL.

| *Angle* | *Observed Angles.* | 1st *Corrn.* | *Angles* (1st *Adjst.*) | 2nd *Corrn.* | *Angles* (2nd *Adjst.*) | *Log Sines* | *Diff. for* 1" | *Angles* (*finally adjusted*). | *Log Sines.* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 67°29'00" | —0'15" | 67°28'45" | —0'30" | 67°28'15" | 9.9655237 | 8.7 | 67°28'23".1 | 9.9655308 |
| 3 | 27 13 45 | —0 15 | 27 13 30 | +0 15 | 27 13 45 | 9.6604392 | 40.9 | 27 13 53 .1 | 9.6604723 |
| 5 | 54 01 00 | —0 15 | 54 00 45 | +0 30 | 54 01 15 | 9.9080723 | 15.3 | 54 01 23 .1 | 9.9080848 |
| 7 | 32 04 15 | —0 15 | 32 04 00 | —0 15 | 32 03 45 | 9.7249669 | 33.6 | 32 03 53 .1 | 9.7249941 |
| | | | | | | *S*1 = 39.2590021 | | | 0820 |
| 2 | 52°51'00" | —0'15" | 52°50'45" | —0'30" | 52°50'15" | 9.9014177 | 16.0 | 52°50'06".9 | 9.9014048 |
| 4 | 32 27 45 | —0 15 | 32 27 30 | +0 15 | 32 27 45 | 9.7297700 | 33.1 | 32 27 36 .9 | 9.7297432 |
| 6 | 66 17 00 | —0 15 | 66 16 45 | +0 30 | 66 17 15 | 9.9616939 | 9.2 | 66 17 06 .9 | 9.9616864 |
| 8 | 27 38 15 | —0 15 | 27 38 00 | —0 15 | 27 37 45 | 9.6662812 | 40.2 | 27 37 36 .9 | 9.6662486 |
| | 360 02 00 | | 360 00 00 | | 360 00 00 | *S*2 = 39.2591628 | 197.0 | | 0830 |
| | | | | | | *S*1 = 39.2590021 | | | |
| | | | | | | *D* (Diff.) = 1607 | | | |

Angles (1st Adjst). 1+2 = 120°19'30"
" " 5+6 = 120 17 30

Diff. = 2 00
1/4 Diff. = 0 30

Angles (1st Adjst.) 3+4 = 59°41'00"
" " 7+8 = 59 42 00

Diff. = 1 00
1/4 Diff. = 0 15

$$e = \frac{1607}{197.0} = 8''.1$$

NOTE.

If one triangle of a quadrilateral has already been balanced, it will be best to adjust the figure by treating it as a polygon without central station, and using the method outlined on page 215.

*For instance.*— If in Fig. 5, the triangle *BDC* is already balanced, let *c* take the place of the central station, and number the angles by drawing a second diagram with *c* inside the triangle *ABD*.

Figures 6 and 7 will explain the procedure.

The adjustment of angles can be made on Form *A* as before, making the whole angle at *c* equal to its components, and the final balancing can then be carried out on Form *B* as before.

PROOF OF THE ADJUSTMENT OF A QUADRILATERAL.

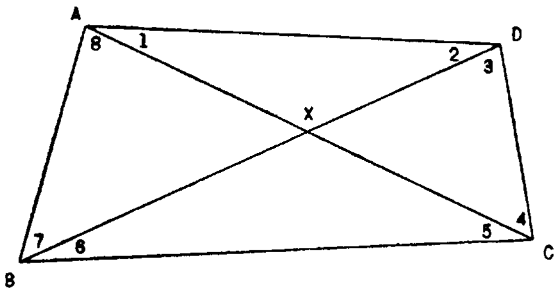



Fig. 8

In Fig. 8 *ABCD* is a quadrilateral, the diagonals of which intersect at *X*. To make this figure geometrically consistent the following equations must be satisfied :

(1) Angles 1 to 8 must sum to 360°.

(2) The sum of the angles of each triangle must be 180°.

(3) The product of the sines of angles 1, 3, 5 and 7 must equal the product of the sines of angles 2, 4, 6 and 8.

The necessity of satisfying equations (1) and (2) is obvious from first geometrical principles. A third equation, however, is required to make the figure consistent. This may be demonstrated as follows :

Suppose it is required to calculate the length of the side *CD*, the side *AB* being known and equations (1) and (2) having been satisfied. This may be done in two ways, *viz.* :

through triangles *ABC* and *BCD*,
or through triangles *ABD* and *ADC*.

The results will not agree (except accidentally) because equations (1) and (2) have not definitely fixed the values of the angles. For instance, it is possible to decrease angle 2 by any definite amount provided that we increase angle 1 by the same amount. The triangles *ABD* and *ADC* will still close to 180°, but the value of *CD* calculated through these triangles will be altered. When adjusted to geometrical consistency, it must be possible to calculate the length of any side from any known side and obtain identical results whatever the route employed. A third equation is therefore necessary to complete the adjustment of the figure.

Suppose, as before, that the side *AB* is known and that it is required to find the side *CD*. This may be done in two ways, the formulae being :

$$(1)\ CD = \frac{AD.\ \sin 1.}{\sin 4.} = \frac{AB.\ \sin 1.\ \sin 7.}{\sin 2.\ \sin 4.}$$

$$\text{or }(2)\ CD = \frac{BC.\ \sin 6.}{\sin 3.} = \frac{AB.\ \sin 6.\ \sin 8.}{\sin 3.\ \sin 5.}$$

Therefore, if we are to have the same result for *CD* by either formulae, the angles must be so adjusted that the following equation is true :

$$\frac{AB.\ \sin 1.\ \sin 7.}{\sin 2.\ \sin 4.} = \frac{AB.\ \sin 6.\ \sin 8.}{\sin 3.\ \sin 5.}$$

Cancelling and cross-multiplying, we have:

$$(3) \quad \sin 1.\ \sin 3.\ \sin 5.\ \sin 7. = \sin 2.\ \sin 4.\ \sin 6.\ \sin 8.$$

Equation (3) will generally be used in the form:

Sum *log sines* 1, 3, 5, 7 = Sum *log sines* 2, 4, 6, 8.

The means employed to satisfy equation (2) require a short explanation.

When two straight lines intersect, the opposite and alternate angles are equal, *i. e.*, in this case angle $AXD$ = angle $BXC$ and angle $AXB$ = angle $DXC$. Consequently, in a consistent figure angles $(1+2)$ = angles $(5+6)$ and angles $(3+4)$ = angles $(7+8)$. The second step in the adjustment is to satisfy these equations by adjusting the pairs of angles, and an inspection of the figure will show that it has the effect of closing triangles $ABD$, $ABC$, $ADC$ and $BCD$ so that equation (2) is satisfied and equation (1) is not disturbed.

# THE ADJUSTMENT OF A QUADRILATERAL WHEN ONE TRIANGLE HAS ALREADY BEEN ADJUSTED.

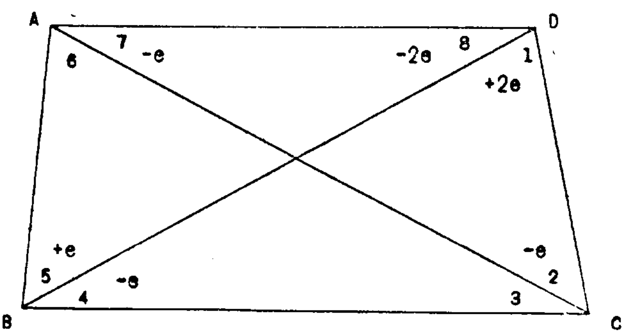
(*Addendum to* Publication H. D. 295 *of the Hydrographic Department, British Admiralty,* Febr. 1931.)

1. The method of closing and balancing a quadrilateral given on page 217 in the notes on "*The Adjustment of Triangulation*" (H. D. 295) published in September, 1929, cannot be used in the case where one triangle has already been adjusted, and whose angles cannot therefore receive any further correction.

A method of solving this problem is indicated on page 219 in the pamphlet referred to above, and, whilst this is probably the quickest method to employ, it confines the actual balancing correction to four angles of the figure.

The balancing correction can actually be spread over six angles by the employment of the method now described, but the amount of work involved is somewhat greather.

2. Let $A\ B\ C\ D$ be a quadrilateral of which the triangle $A\ B\ C$ has already been adjusted. The problem is to close and balance the figure without altering any of the angles 6, $(4 + 5)$, or 3.




3. The first step is to apply the corrections already found for the angles of the adjusted triangle $A\ B\ C$ to these angles where they appear in the other triangles of the figure.

The corrections to 3 and 6 must be applied to the double angles $(2 + 3)$ and $(6 + 7)$ respectively; whilst half the correction to the double angle $(4 + 5)$ must be applied to each of the single angles 4 and 5.

This comprises the first correction, and has the effect of making all the corner or double angles equal to their components.

4. To close the figure, next tabulate and sum the *whole* angles at the corners of the figure, and divide the excess or deficiency from 360° equally between the angles at

*A, D* and *C*, which have not been previously adjusted, 1/3rd to both 2 and 7, and 1/6th to both 1 and 8, in order to preserve the equality of the whole angles with their two components.

This comprises the second correction, and it automatically closes the triangle opposite to the previously adjusted triangle. (The external angles being 360°, and triangle *A B C* being 180°, it follows that triangle *A D C* is 180°).

5. The remaining triangular error in the other two triangles must be equal and opposite, and should be divided equally between the two *single* angles of the triangles, thus closing them without disturbing the exterior angles of the quadrilateral. This is the third correction, and the figure is now closed and ready for balancing.

6. The balancing is carried out in the usual way by making the sum of the *log sines* of the angles 1, 3, 5 and 7 equal to the sum of the *log sines* of the angles 2, 4, 6 and 8, but, owing to the fact that two of these angles (3 and 6) cannot be altered, the application of a constant balancing correction to the remaining angles is inadmissible, as it would disturb the closing of two of the triangles (*A B D* and *B D C*).

This difficulty is surmounted by putting a double correction into the two angles opposite to the previously adjusted triangle (1 and 8), which has the result of leaving the closing of the figure undisturbed by the balancing correction.

In finding the balancing correction *e*, the figures for angles 1 and 8 in the column headed "Diff. for 1" must be multiplied by two, and the correction to these angles must be 2*e* instead of *e*, which is applied to the angles 2, 4, 5 and 7.

7. The attached example will make the method of closing and balancing clear, and shows a convenient form for making the adjustment.

EXAMPLE :

CLOSING.

| *Triangle.* | | *Observed Angles.* | 1st *Corr.* | *Angles* 1st *Adjst.* | *Central Angles.* | 2nd *Corr.* | *Angles* 2nd *Adjst.* | 3rd *Corr.* | *Angles* 3rd *Adjstm.* |
|---|---|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | | ° ′ ″ | ° ′ ″ | | ° ′ ″ | | ° ′ ″ |
| *A* | 6 | 66 17 00 | + 15* | 66 17 15 | | | | | |
| *B* | (4 + 5) | 86 28 45 | + 30* | 86 29 15 | 86 29 15 | Prev. adjust. | Previous. adjusted. | Prev. adjust. | Previously adjusted. |
| *C* | 3 | 27 13 45 | — 15* | 27 13 30 | | | | | |
| | | 179 59 30 | | 180 00 00 | | | | | |
| *A* | (6 + 7) | 98 21 15 | + 15 | 98 21 30 | 98 21 30 | — 50 | 98 20 40 | | 98 20 40 |
| *B* | 5 | 54 01 00 | + 15 | 54 01 15 | | | 54 01 15 | + 7.5 | 54 01 22.5 |
| *D* | 8 | 27 38 15 | | 27 38 15 | | — 25 | 27 37 50 | + 7.5 | 27 37 57.5 |
| | | 180 00 30 | | 180 01 00 | | | 179 59 45 | | 180 00 00.0 |
| *A* | 7 | 32 04 15 | | 32 04 15 | | — 50 | 32 03 25 | | 32 03 25 |
| *C* | 2 | 52 51 00 | | 52 51 00 | | — 50 | 52 50 10 | | 52 50 10 |
| *D* | (1 + 8) | 95 07 15 | | 95 07 15 | 95 07 15 | — 50 | 95 06 25 | | 95 06 25 |
| | | 180 02 30 | | 180 02 30 | | | 180 00 00 | | 180 00 00 |
| *B* | 4 | 32 27 45 | + 15 | 32 28 00 | | | 32 28 00 | — 7.5 | 32 27 52.5 |
| *C* | (2 + 3) | 80 04 45 | — 15 | 80 04 30 | 80 04 30 | — 50 | 80 03 40 | | 80 03 40 |
| *D* | 1 | 67 29 00 | | 67 29 00 | | — 25 | 67 28 35 | — 7.5 | 67 28 27.5 |
| | | 180 01 30 | | 180 01 30 | 360 02 30 | | 180 00 15 | | 180 00 00.0 |

(*) From previous adjustment.

BALANCING.

| Nº of Angle. | Angles 3rd Adjst. | Log Sines. | Diff. for 1" | Angles finally adjusted. | Log Sines. |
|---|---|---|---|---|---|
| | ° ′ ″ | | | ° ′ ″ | |
| 1 | 67 28 27.5 | 9.9655347 | 17.4 (× 2) | 67 29 01.5 ** | 9.9655643 |
| 3 | 27 13 30 * | 9.6603778 | — | 27 13 30 | 9.6603778 |
| 5 | 54 01 22.5 | 9.9080838 | 15.3 | 54 01 39.5 | 9.9081097 |
| 7 | 32 03 25 | 9.7248997 | 33.6 | 32 03 42 | 9.7249568 |
| | | $S1$ = 39.2588960 | | | 39.2590086 |
| 2 | 52 50 10 | 9.9014097 | 16.0 | 52 49 53 | 9.9013826 |
| 4 | 32 27 52.5 | 9.7297948 | 33.1 | 32 27 35.5 | 9.7297386 |
| 6 | 66 17 15 * | 9.9616939 | — | 66 17 15 | 9.9616939 |
| 8 | 27 37 57.5 | 9.6663315 | 80.4 (× 2) | 27 37 23.5 ** | 9.6661947 |
| | | $S2$ = 39.2592299 | 195.8 | | 39.2590098 |
| | | $S1$ = .2588960 | | | |
| | | $D$ (Diff.) = 3339 | | | |

$$e = \frac{3339}{195.8} = 17''.0$$

(*) Previously adjusted. **Final adjustment to these angles is $2e$.