METHODS FOR PLOTTING AND GRADUATING SURVEYS

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1. The simple principles of the use of co-ordinates, and the method of converting rectangular co-ordinates to geographical co-ordinates or latitude and longitude, are well known, but in order to avoid confusion the following definitions are quoted as they will be referred to frequently in this paper :-

Point of Origin is a reference point on the earth's surface from which co-ordinate measurements are taken.

Co-ordinate Bearing is the relative bearing between two co-ordinated positions. Since the co-ordinate meridians are parallel lines the direct and reverse co-ordinate bearing between two points is the same.

Main Angle. The difference between two co-ordinate bearings at a station is the angle between the two objects at that station.

Co-ordinate Distance is the distance between two points as calculated from their co-ordinates.

X co-ordinate is the measurement east or west from the Origin along a line at right angles to the meridian of the Origin.

Y co-ordinate is the measurement north or south from the Origin along the meridian through the Origin.

True Meridian. In dealing with co-ordinates, the co-ordinate meridian through the Origin is the only True Meridian in the system.

2. Before considering the method to be used in plotting from co-ordinate data it is obvious that the co-ordinates of all the points to be plotted must be determined by previous calculation. Similarly the latitude and longitude of the Origin must be determined before the survey can be graduated.

3. *EXAMPLE OF PLOTTING :*

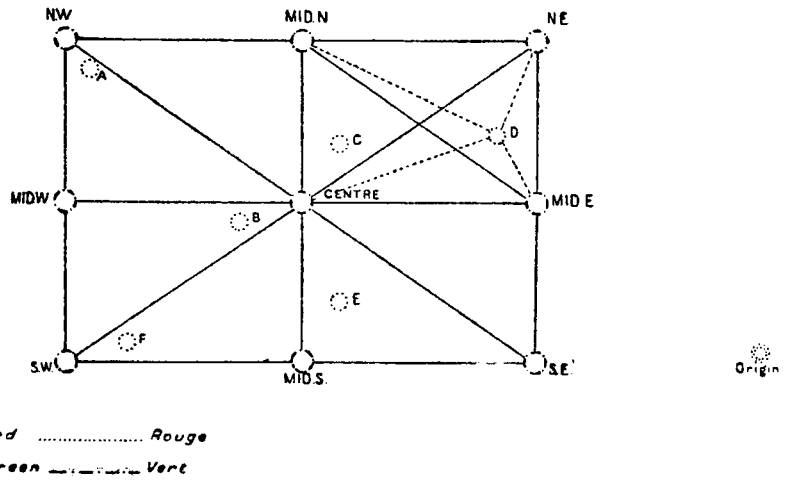

Fig. 1.

a) In Fig. 1, *A, B, C, D, E, F* are six points in a survey which are required to be plotted on a scale previously decided. The rectangular co-ordinates of these six points have been determined from the Point of Origin *O* to the eastward of the survey.

b) First it is necessary to determine the limits of a rectangle which will contain all the six points and to plot these points from suitable positions in the rectangle. The approximate position of the points will be known or can be taken from a chart or rough plot, for it is necessary to know the approximate positions in order to select the best plotting points.

I) The point *A* is the northern point of the survey, so that if we add a short distance, say 500 feet, to the *Y* co-ordinate of *A* we will obtain the *Y* co-ordinate of the containing rectangle.

Similarly the point *A* is the western point on the sheet and 500 feet added to the *X* co-ordinate of *A* will give the *X* co-ordinate of the sheet.

The *Y* co-ordinate of the south limit of the rectangle will be obtained in a similar manner from the co-ordinates of the point *F*, and the *X* co-ordinates of the eastern limit of the rectangle will be obtained from the co-ordinates of the point *D*. Thus we have a rectangle of which the co-ordinates of the four corners are known on the Point of Origin *O*. These corners will be referred to as the *NE, NW, SW* and *SE* corners of the rectangle.

II) Calculate the co-ordinate bearing of the diagonals of the rectangle from the differences of their co-ordinates, and at the same time determine the co-ordinates of the middle points of the sides which should be marked *Mid N, Mid S, Mid E* and *Mid W*. The intersection of the diagonals is the middle point of the rectangle, its co-ordinates should be determined and it should be marked *Centre*. The figure is now ready for calculating the angles to the stations from the plotting points *NE, NW, SE, SW, Mid N, Mid S, Mid E, Mid W* and the *Centre*.

4. *TO CALCULATE THE ANGLES TO THE POINTS FROM THE PLOTTING POINTS :*

a) It is obvious that all the points could be plotted by distances from the plotting points, but this practice is undesirable if the sheet be a large one, as the scale of a large sheet of paper is continually varying.

Protraction by small angles is therefore the most desirable method to use and is quite rapid once the calculations are made, as no large angles are involved and short radii can be employed. Distances should only be used when such are short.

b) As an example it is desired to calculate the angles to plot the point *D*.

By inspection of the rough plot we see that angles from the plotting points *NE, Mid E*, and *Centre* will give a good ***cut*** and a further check, if necessary, can be calculated from *Mid N*.

NOTE. — In this case the co-ordinate bearing *Mid N* to *Mid E* must be calculated to avoid the use of a large angle at *Mid N*. Calculate the co-ordinate bearings of the point *D* from each of the selected plotting points. Thus if the bearing *Centre - D* is *N* 65.00.00 *E*. and the bearing of the diagonal is *N* 55.00.00 *E*., the point *D* lies 10.00.00 to the right of the diagonal.

Similarly from *Mid E*., the bearing being *N*. 17.05.15 *W*., it is described as lying 17.05.15 to the left of the vertical. And so on for the remaining points.

c) Enter these angles in the Plotting Book in the following manner :

From	*To*	*Zero*	*Angle*	*R or L*	*Chord*	*Radius*
Mid. E	*D.*	Vert.	17.05.15	L.	As in ordinary plotting.	
NE	*D.*	Diag.	20.10.20	L.		
Centre	*D.*	Diag.	10.00.00	R.		

A space should be left in the plotting book between each station to afford room, should a further "shot" be necessary.

The calculations should be worked on a page opposite to the tabulated angles in the plotting book, as this is a convenience should a mistake in calculation be made.

It will be seen from this method that if errors occur and a point will not intersect, these errors must be due to errors of calculating the co-ordinate bearings or protraction. THEY CANNOT BE ERRORS OF OBSERVATION.

5. *PLOTTING:*

a) The requisite data for plotting the sheet is now available. Having selected the scale, the lengths in inches of the sides of the rectangle and the diagonals will have been calculated previously.

To commence plotting, three beam compasses are required; these should be set:-

1) to the length of the vertical;
2) to the length of the horizontal;
3) to the length of the diagonal.

The following procedure for erecting the figure has been found to be quick and accurate. Speed as in all plotting in the case of the containing rectangle is essential to minimise alteration of scale.

I) Use a needle for plotting.

II) Rule a diagonal line across the paper and prick off the length of the diagonal along this line. This will give the *NE* and *SW* corners of the rectangle.

III) Sweep the distances of the verticals and horizontals from those two positions, which will give the *NW* and *SE* corners of the rectangle.

IV) Check the length of the *NW* and *SE* diagonal which should give the same measurement on the paper as the first diagonal protracted.

V) Join up the sheet corners taking care to draw the lines from edge to edge of the paper.

VI) Divide the sides to obtain the *Mid Points* and join up opposite points.

VII) The diagonals and mid point lines should intersect at the centre point.

VIII) Circle all points in green and label them.

NOTE. — If the sheet is very large it can be further subdivided into rectangles as necessary.

b) The sheet is now ready for plotting the survey points from the plotting points. This operation can be carried out at leisure as only small angles and radii are required.

When plotting it is an advantage to place a thin piece of celluloid under the sheet between it and the table, as this will obviate the development of large holes under the plotting points when the sheet has been worked a considerable time.

6. *GRADUATION:*

a) *Calculation.*

In Fig. 2, the *green* rectangle in the figure has been described as previously explained and the co-ordinates of the four corners determined. It is required to graduate the rectangle in geographical co-ordinates based on the geographical position of the Point of Origin.

Considering the *SW* and *NE* corners of the rectangle:

At the *SW* corner the True Meridian through this position must lie at an angle to the co-ordinate meridian which is equal to the total convergency betwen the *SW* corner and the Point of Origin. Similarly the True Meridian through the *NE* corner lies at an angle equal to the convergency between the *NE* corner and the Point of Origin.

Since in the figure the *NE* corner is further to the eastward of the Point of Origin than the *SW* corner, it follows that the angle between the meridians at the *NE* corner is greater than that at the *SW* corner, and the difference between these two angles is the total convergency between the *NE* and *SW* corners.

The procedure in graduation is as follows:-

I) Given the co-ordinates of the *SW* corner, calculate the True Bearing and Distance between this point and the Origin.

II) With this true bearing and distance calculate the latitude and longitude of the *SW* corner, and the convergency between *SW* corner and the Origin.

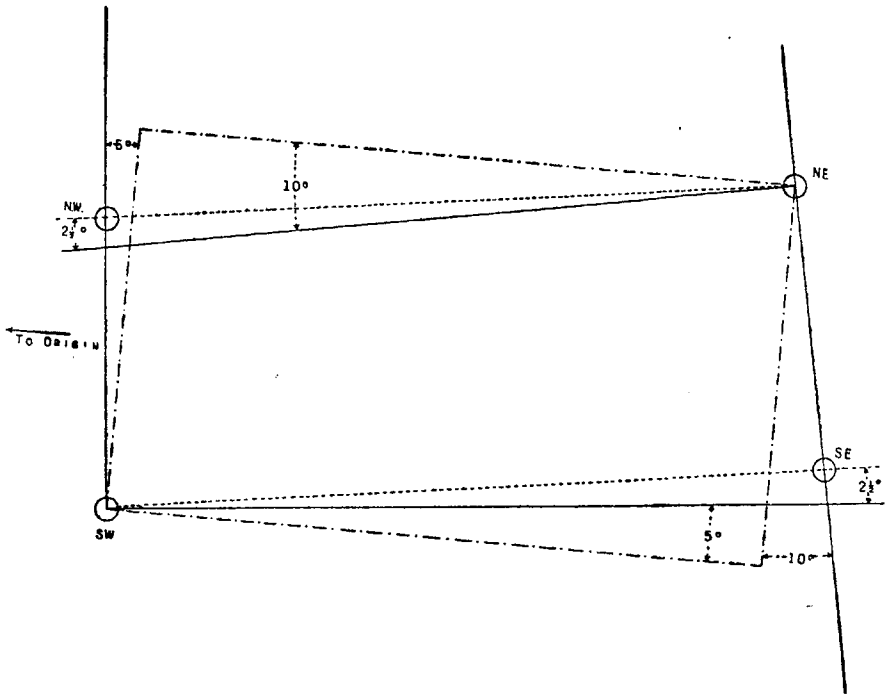

Fig. 2.

III) Similarly, calculate the latitude and longitude of the *NE* corner and the convergency.

IV) Obtain the difference of latitude between the *NE* and *SW* corners and the Middle latitude.

NOTE. — In graduating a sheet where the point of origin falls within the containing rectangle, care must be taken that the convergency is laid off in the correct direction when drawing the true Meridians at the *NE* and *SW* corners. In such cases the True Meridians must be laid off from the Co-ordinate Meridians in a direction towards the Point of Origin.

b) *Plotting.*

I) At *SW* corner lay off by a small chord the convergency between the Origin and *SW* as determined in (II) above. Care should be taken that the chord is protracted on the correct side of the co-ordinate meridian. This will give the True Meridian through the *SW* corner.

II) At *SW* corner lay off the same angle in a horizontal direction so that the line is drawn a normal to the True Meridian through *SW* corner in direction *D* in the figure.

III) At *SW* corner with the horizontal line of the rectangle as zero lay off an angle equal to the sum of the convergency between *SW* corner and Origin and half the convergency between *NE* and *SW* corners in direction *C* in figure.

IV) At *NE* corner lay off True Meridian as described in (I), using the convergency between the Origin and *NE* corner.

V) At *NE* corner lay off the normal to this meridian in direction *D* as described in (II).

VI) At *NE* corner with the horizontal line of the rectangle as zero lay off an angle equal to the *difference* between the total convergency at *NE* corner with the Origin and half the convergency between *NE* and *SW* corners in the direction *E*.

The intersection of the True Meridian through *SW* corner and the line drawn from *NE* corner in the direction *E* will give the position of the *NW* corner of the graduation. Similarly the intersection of the True Meridian through the *NE* corner and the line drawn from *SW* corner in direction *C* will give the position of the *SE* corner. A check on the differences of latitude between *NW* and *SW* corners and *NE* and *SE* corners can now be obtained using the difference of latitude according to the scale. The graduation for longitude can now be carried out in the usual manner (*see* Table for projection of Gnomonic Charts) since the normals to the meridians are drawn and half the convergency between *NE* and *SW* corners has been protracted in the direction of the pole.

7. *PLOTTING BY GEOGRAPHICAL CO-ORDINATES* (*See Fig.* 3):

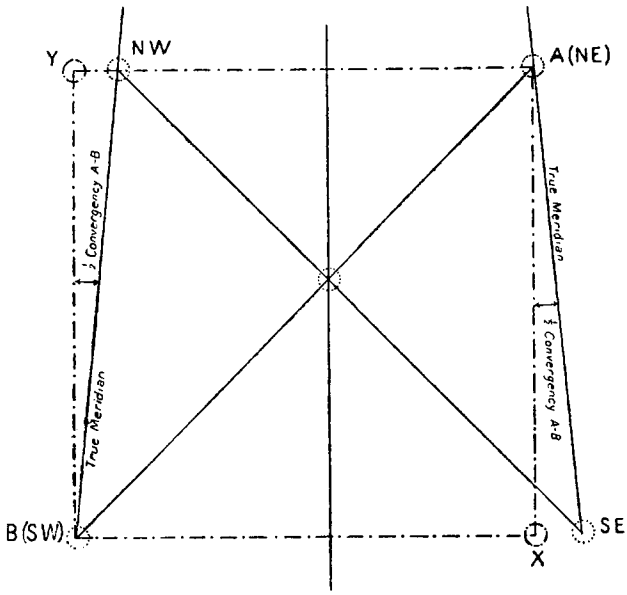

Fig. 3.

a) The positions of a series of trigonometrical stations forming the whole or part of a triangulation are sometimes given in latitude and longitude which are described as geographical co-ordinates. They differ from those in the system of rectangular co-ordinates, previously described, in the following principal respects:-

I) The meridians converge towards the poles and are therefore not parallel.
II) The normals to the meridians are small circles and not great circles.

b) In cases where a number of trigonometrical stations are given in geographical co-ordinates, it is sometimes more convenient to plot the survey in latitude and longitude. This is carried out by plotting first the sheet graduation and then plotting the points either from the graduated corners of the sheet or from other convenient points in the graduation.

The following method for carrying out the plotting accurately is suggested. The construction of the initial rectangle *AXBY* is recommended to give rigidity to the graduation by eliminating the practice of protracting large angles to form the meridians from the points *A* and *B*.

c) *To Plot the Graduation.*

I) Calculate the bearing, distance and convergency between *A* and *B*, the *NE* and *SW* corners of the sheet.
II) Determine the departure and diff. lat. between the two points.
III) Convert all distances into measurements according to scale selected.
IV) Construct the rectangle *AXBY* as previously described.
V) The True Meridians through *A* and *B* can now be drawn by applying the chorded angle of half the convergency between the points, laid off in the

correct direction from the lines *AX* and *BY*. As this angle is so small and the rectangle *AXBY* extremely rigid, the accuracy of the meridians can be relied upon to a far greater degree than would have been the case had the meridians been laid off by large chords from the diagonal.

VI) The detailed construction of the graduation is similar to that previously described under "Graduation of a Sheet plotted by Rectangular Co-ordinates."

VII) Join the *NW* and *SE* corners and draw the central meridian. The intersection of these lines with the original diagonal is the tangential point of the sheet and will check the accuracy of the work.

d) *To Plot the Points.*

I) Calculate the scale lengths of the distances between the *NW* and *NE* corners of the sheet and the *SW* and *SE* corners. These distances will differ slightly owing to the convergency.

II) The points can now be plotted according to scale either using diff. latitude and diff. longitude, or by true bearing and distance from the four corners of the graduation, or from the point of intersection of the diagonals, or from the central meridian using the diagonals or meridian as zero lines. It is important to remember that as the parallels are not straight lines they must not be used for protraction.