THE 45° PRISMATIC ASTROLABE

(Extract from a paper by CAPTAIN T. Y. BAKER,
read at the Royal Geographical Society, London, 19th January 1931.)

Any determination of latitude and longitude by astronomical observation is a determination of the direction which the vertical at the place of observation holds relative to the fixed stars at a certain time. The observation is usually made by actual measurement of certain zenith distances, or, alternatively, by noting when certain stars reach a specified zenith distance. The theodolite is a representation of the former case, the astrolabe, or the sextant, of the latter.

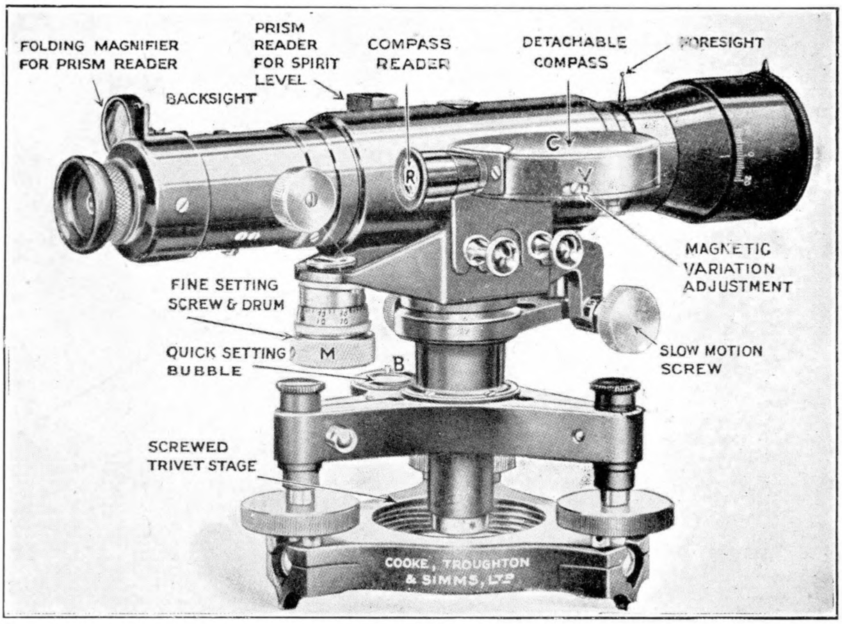
In looking at the problem of accurate determination of position from this standpoint it is clear that the two main sources of error lie (*a*) in the accuracy of the alleged vertical from which zenith distances are measured, and (*b*) in the measurement of the angles themselves, this latter case including that of noting the times when the various stars reach a certain specified zenith distance.

In talking of determination of position with high precision one is contemplating something that is true within one second of arc, and it is interesting to look at the meaning of one second of arc from the practical aspect both as regards levelling a theodolite and in the reliability of the surface of a pool of mercury. The theodolite is usually mounted on some sort of tripod, and no matter how carefully the instrument be made and levelled the vertical axis will only remain vertical throughout a series of observations provided the three feet of the tripod remain fixed. If one sinks ever so slightly in the ground the vertical axis will be displaced, and the amount of sinking required to produce a 1" tilt is round about 1/5000th of an inch, roughly a fifth of the thickness of a cigarette paper. The matter is really worse than this, for in order to get inside one second on the final result one cannot tolerate a possible systematic error that is anywhere near the same amount. Wind pressure on the side of the instrument is another possible cause of displacement of the vertical axis. Even when the ground is hard and wind is negligible the levelling to inside a second is a tedious and uncertain business.

The mercury pool as a means of registering the vertical seems to have much more to recommend it. The most satisfactory method is to use an amalgamated copper plate upon which a pool about a millimetre in depth will rest under capillary action without running off. The meniscus at the edge is of course curved, but the effect does not extend to more than a centimetre inwards, and as long as the place is large enough to allow this margin all round there is no difficulty in getting an optically flat surface that will give accurate imagery of the star observed. Some time ago some tests were carried out at the Admiralty Laboratory to determine how far such a plate could be tilted before the upper surface of the mercury was appreciably out of the horizontal. An optical flat was fixed tightly above the mercury and interference fringes observed by light reflected at the two surfaces. It was just possible to notice a contraction of the fringes as the tilt approached one degree, the corresponding tilt of the mercury surface being under a quarter of a second of arc. A mercury pool has the defect that it must be protected against the wind and that when disturbed it requires time to settle down. This time is quite small provided the pool is not more than a millimetre deep and any vibration set up by movement of the apparatus can usually be relied on to damp out in a couple of seconds or so.

All things considered, I think no one can doubt that of the two methods of defining the vertical that of the mercury pool is a long way the more satisfactory for really accurate work. If the copper plate is level within a few minutes there is no risk of any error in the vertical defined by the normal to the mercury.

In the measurement of the stars' zenith distance, the most satisfactory way seems to be to use a fixed angle and determine the times at which the various stars reach that altitude or zenith distance. It is not essential that the angle should be known with absolute accuracy since the method of working out the observations will look after that point. But it is in the highest degree essential that the angle shall remain constant to a fraction of a second for the whole series of stars observed. In using the method of equal altitudes with a theodolite one must set the telescope in the vertical circle and use that setting for all the stars observed. With an astrolabe one's standard angle is that between two polished faces of a prism. Every one will agree that the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COOKE, TROUGHTON
& SIMMS, LTD

FIG. 1

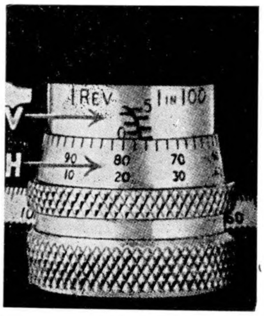
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FIG. 5.

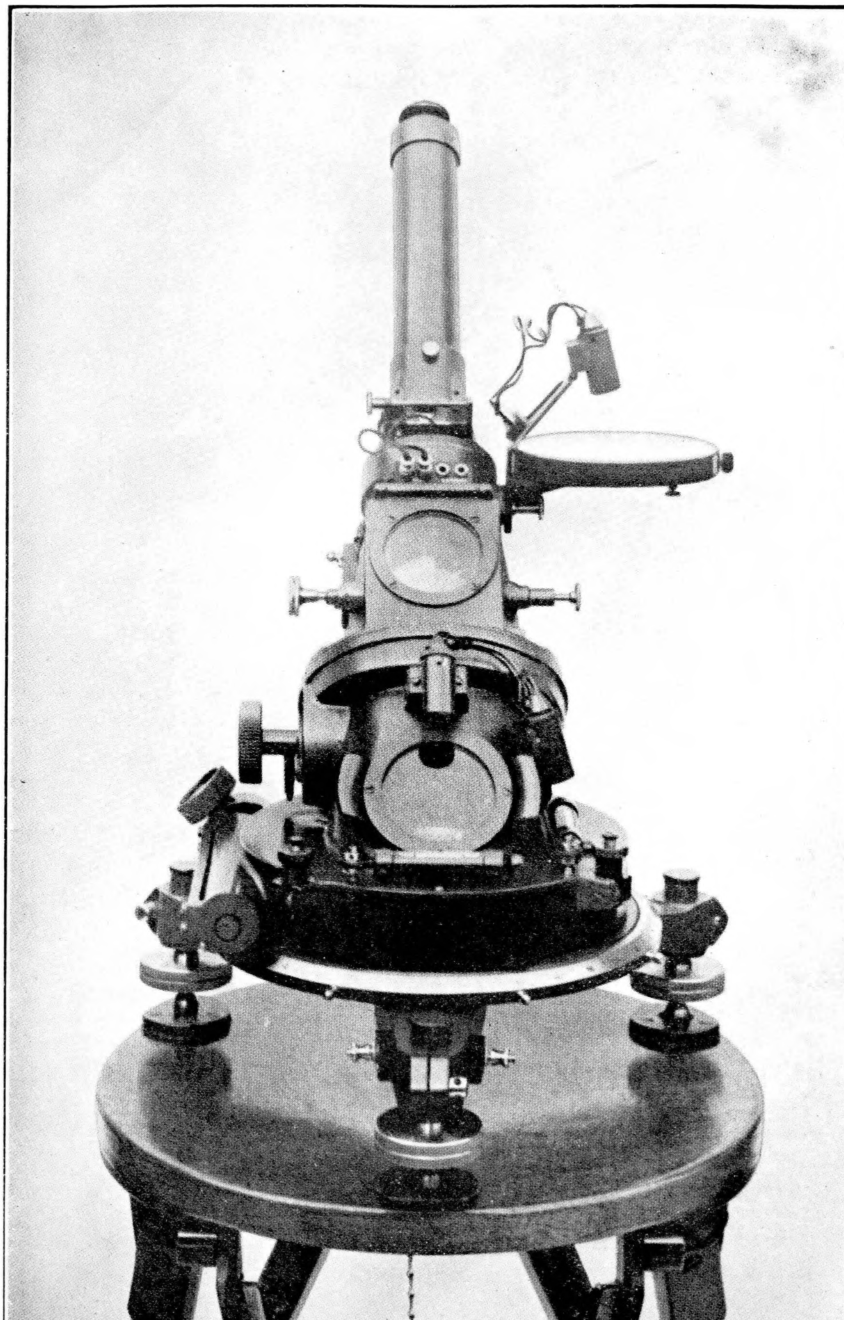

Fig. 1

45° Prismatic Astrolabe — Astrolabe à prisme de 45°

prism method is the more reliable. The chief danger lies in each case in a change of temperature, causing a local distortion, and it can hardly be questioned that the built-up mechanism of the telescope and vertical circle is much more vulnerable in this respect than a glass prism. The prism has a definite advantage that if unequal heating does distort it, the defect will first become visible as a loss of definition of the star image. There is however no evidence so far during the period that these astrolabes have been in use that temperature changes ever affect the prism angle to amounts that matter.

The prismatic astrolabe as an instrument embodying the two main principles of definition of the vertical by means of a mercury surface and the use of a fixed measuring angle contained in a reflecting prism originated apparently with CLAUDE and DRIENCOURT some five and twenty years ago (See *Hydrographic Review*, Vol. VII, N° 1, May 1930, pages 196-199). In this instrument (see Fig. 1) the prism is a 60° one, and each beam of light (*i.e.* the direct one and that reflected from the mercury surface) is reflected once in the prism. This astrolabe has been extensively used and some very accurate work has been done with it.

A modified form, making the prism rotable about a horizontal axis so that each edge of the prism could be used in turn, making one angle 60°, one less than 60° by a few minutes and the other more than 60°, an instrument was obtained that enabled three observations to be made on each star instead of one only. This astrolabe was described in one of the early numbers of the *Journal of Scientific Instruments*. Later on Captain BAKER experimented with a design embodying the same two basic principles, but entirely different in optical design from what had been attempted before.

The 60° astrolabe can be used only with stars that reach an altitude of 60° above the horizon, while the new one observed stars at an altitude of 45°. A little consideration shows that more stars become available during a night's work with a 45° astrolabe than with the other instrument. It is difficult to give an exact estimate, but very roughly the number of available stars is doubled.

The prismatic arrangement that enables stars to be observed at an altitude of 45° is shown in Fig. 2. A pentagonal or PRANDL prism *ABCDE* is held in the position shown. The faces *AE* and *CD* are normally silvered, and light entering the face *BC* is reflected first at the face *AE* and then at the face *CD* and passes out of the prism across the face *AB* and thence into the telescope. The deflection of the beam, 90°, is twice the prism angle between the faces *AE* and *CD* and is thus constant even if the prism is slightly tilted.

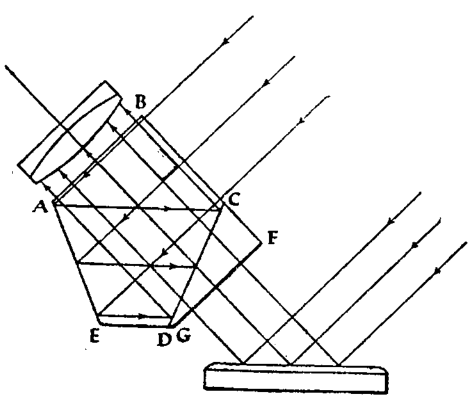

Fig. 1

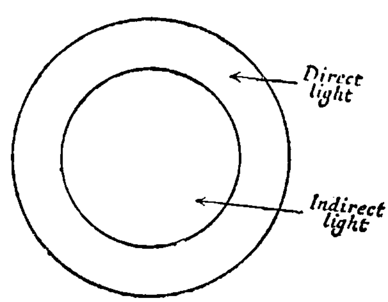

Fig. 2

In the astrolabe the face *CD* has a central part of the silvering cut away and a prism *CFGD* is cemented to the pentagonal. The angle of this prism is made so that *GF* and *AB* are substantially parallel and light from the star reflected in the mercury traverses the prism through the central hole in the cemented interface *CD*. Thus there are two beams of light entering the telescope and their cross-sections are shown in Fig. 3. The principal ray is in each case along the telescope axis. On the contrary in the 60° astrolabe the principal ray on the two bundles reflected in the telescope by the upper and the lower slanted faces of the prism are not parallel to the telescope axis and, when focussing, difference of holding accommodation may cause slight discrepancies in contact of two star images. It is obvious that the accommodation error of setting does not arise in the 45° astrolabe design.

The prism reflects the direct beam twice and the mercury reflects the indirect beam once, so that as the star moves in altitude the two star images move in opposite directions in the field, one up and the other down. The observation is made by noting when the two are in contact. The main function of the 36 × magnification telescope is merely to note more exactly when contact occurs. It is a magnifier and quite distinct in its function from the theodolite telescope where the measurement is made of the instants when the star crosses the spider-lines of the telescope. To get accuracy with the theodolite the angle between the line joining the cross-line to the nodal point of the objective must not depart by a fraction of a second from its position relative to the vertical axis. The astrolabe telescope could be tilted several minutes without introducing a tenth of a second inaccuracy in the latitude or longitude.

Contact between two bright stars is not an observation for which the eye has a very high degree of acuity, and in order to reduce the errors of observation the same method is adopted as in the 60° astrolabe. A weak prism is fitted over half the aperture of the direct beam so that one half of the direct beam, the outer zone of Fig. 3, is deflected sideways by about 1' of arc, and a second direct image of the star is formed to the side of, but on the same horizontal level as, the other. These two star images subtend 35' to 40' at the eye. The observation is made by allowing the mercury image (single star) to pass between the direct images (duplicate star) and note the time when all three star images are in line. This can be done with a much narrower spread of observation errors than occurs when the direct star is single.

The star images resulting from cutting up the telescope aperture in this manner are affected by diffraction. The indirect star image is formed by a central circular part of the objective and the diffraction pattern will be circular, the image being slightly larger than would be the case if the whole aperture of the objective could be used. The diffraction patterns of the duplicate star images are extended outwards horizontally. The appearance of the field of view when the star is in "contact" is shown in a very exaggerated manner in Fig. 4. In the 60° astrolabe one image was formed by a semicircle of the objective and the duplicate images each by a quadrant. The corresponding diffraction patterns were as shown in Fig. 5. This gave such an unsymmetrical arrangement that it became necessary to cut down the apertures still further into three circular ones. It made the three diffraction patterns more symmetrical, but at the expense of loss of light. The diffraction patterns of Fig. 4 do not prevent a pretty accurate determination of the instant of contact.

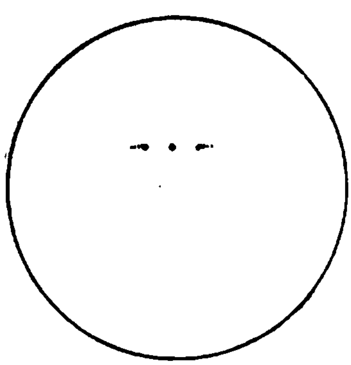

Fig. 4

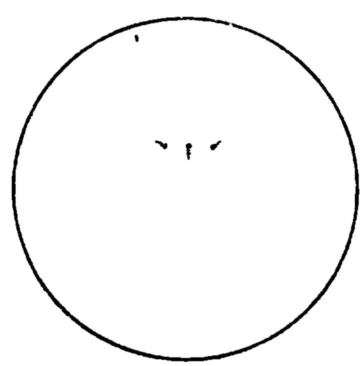

Fig. 5

In the form sketched in Fig. 2 the 45° astrolabe could yield only one observation on each star. The modified form of the 60° astrolabe where three shots were possible with each star was found to be so advantageous in reducing the number of stars to be observed that a series of weak refracting prisms has been introduced in the 45° instrument.

Suppose a telescope T is directed on a very distant bright point which is focussed at the centre of the field. The point is seen at A (Fig. 6). Now place a prism in front of the telescope with its thin edge uppermost. The image is deflected to B. If the prism is turned with its thin edge downwards the image is deflected to C. In any other position of the prism the star image is somewhere in the circumference of the circle shown in pecked line. The angular radius of the circle is the deflection angle of the prism, an invariable angle provided the prism is reasonably square (*i.e.* within a few degrees) with the beam which passes through it.

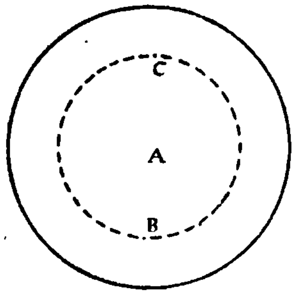

Fig. 6

Suppose now three prisms are arranged in cells so that the light of the indirect beam passes through all three. Each prism can be rotated in its cell and is located by means of a pawl in either of two positions, *e. g.* B or C, edge upwards or edge downwards. The prisms are of deflecting angles α, $2\,\alpha$, and $4\,\alpha$, and if they are all arranged edge upwards the total deviation of the light is

$$\alpha + 2\,\alpha + 4\,\alpha = 7\,\alpha$$

Now rotate the weakest prism in its cell by a half turn so that it is now turned edge downwards while the other two are still edge upwards. The deflection is now

$$-\alpha + 2\,\alpha + 4\,\alpha = 5\,\alpha$$

It is evident that the following eight arrangements are possible :

1. $\alpha + 2\,\alpha + 4\,\alpha = 7\,\alpha$
2. $-\alpha + 2\,\alpha + 4\,\alpha = 5\,\alpha$
3. $\alpha - 2\,\alpha + 4\,\alpha = 3\,\alpha$
4. $-\alpha - 2\,\alpha + 4\,\alpha = \alpha$
5. $\alpha + 2\,\alpha - 4\,\alpha = -\alpha$
6. $-\alpha + 2\,\alpha - 4\,\alpha = -3\,\alpha$
7. $\alpha - 2\,\alpha - 4\,\alpha = -5\,\alpha$
8. $-\alpha - 2\,\alpha - 4\,\alpha = -7\,\alpha$

It is to be noticed with this series that the amounts of deflection go in steps of $2\,\alpha$. Also the weak prism requires a half turn after each shot; the intermediate a half turn after each second shot, and the strongest a half turn after the fourth shot.

This device is used in the 45° astrolabe and Messrs. COOKE, TROUGHTON and SIMMS, who made the model illustrated, devised a most ingenious bevel gearing by means of which the necessary sequence of prism positions is ensured by giving a handle one complete turn after each observation. The handle has to be turned in one sense for a rising star and in the opposite for a setting one.

It has been found that if α is about 3' there is sufficient time between shots to turn the handle, give the mercury time to settle, tilt the prism and correct the instrument in azimuth and still leave a margin to get ready to make the next observation.

For really accurate work it is important to keep the instrument approximately trained in azimuth. Movement in azimuth results in the images tracking across the field of view not in a true straight line but in very shallow curves indicated in an exaggerated manner in Fig. 8. They are truly in contact only at the centre of the field. If the observation is taken at the edge of the field a second order error in altitude obviously results, and second order errors in precise work must be guarded against. Accordingly, a vertical graticule line is fitted in the telescope and contacts should be made near this line though not necessarily exactly on it.

From the above description it will be seen that the design calls for no high mechanical accuracy anywhere. The vertical looks after itself under the action of gravity. The instrument is levelled for convenience, but only to a few minutes. The measured angle is the constant angle of the prism combined with refractive effects of the three refracting prisms. These need be only approximately square with the beam and need only turn through an approximate 180° for each successive shot. If the half turn is 2° out the measured step in altitude is $\alpha \cos 88°$, an error of only 0.1" if α is 3'. This in itself is not a true error provided the prism located *always* 2° out. It is then a *constant* error in the measured altitude, and any constant error automatically eliminates itself in the computation of the positions from the observed times. An error in location does however displace the star image sideways so that it has to be corrected by a tilt of the prism block. For this reason a touch of the tilting head and a touch of the azimuth head are usually required just before each shot.

The cover contains the prism box in which is fitted a framework carrying the PRANDL prism. This framework is pivoted at the bottom to allow the small rocking motion that has been previously described, this angular motion being round the telescope's axis. The telescope screws into the cover and has a magnification of 36 × with a field of rather more than 1°. Inside the cover is the series of three rotating prisms which lie in the path of the beam between the mercury pool and the telescope objective. The actuating handle also works a counter and shows which of the eight prism combinations is in use. The series is 1, 2, 3.......8 or 8, 7, 6........1, according whether the star is rising or setting. Glass windows admit the two beams of light and the whole of the inside is thus effectively protected against wind that would otherwise ruffle the mercury surface. The windows are circular in shape and pinned so that they cannot rotate, and it is unnecessary to go to a high order of parallelism since their prismatic effect is all merged into the constant prism error. Attempts are however made to ensure that the prism effect of the windows lies in the vertical plane.

If the calculation is done carefully it should be possible for a good observer to determine the position well inside a second of arc by observations on twelve stars only. The length of time required to get twelve stars varies of course with the latitude and the time of the year, but since stars of magnitude 6 are easily observable there should in most cases be no difficulty in getting through the observations in a couple of hours or so.

Early in September 1930 a trial of the accuracy of the 45° astrolabe was made at the Royal Observatory, Greenwich, by two officers of the Hydrographic Service. Observations were obtained on four nights, twelve stars being observed by each officer on three nights and eight stars on the other.

Combining the observations of the three nights, giving half weight of the first, when the observations were regarded as experimental, the error in the geographical position accepted from them is

Error in latitude — 0.06". Error in longitude — 0.15".

The 45° astrolabe as made for the Hydrographic Service is rather heavy for transport. Messrs. COOKE, TROUGHTON and SIMMS now have taken in hand the design of a smaller model with a prism of 35 m/m aperture instead of 50 m/m. This entails a small reduction in magnification and consequently a slightly increased spread in errors of observation, but it should still be possible to get a fix within one second of arc as the result of one night's work. The chief difference between the two astrolabes would lie in the brightness of the stars that could be observed. With the larger model it is possible to make use of stars as faint as magnitude 7. With the smaller one the surveyor would be limited to stars not fainter than magnitude 5.