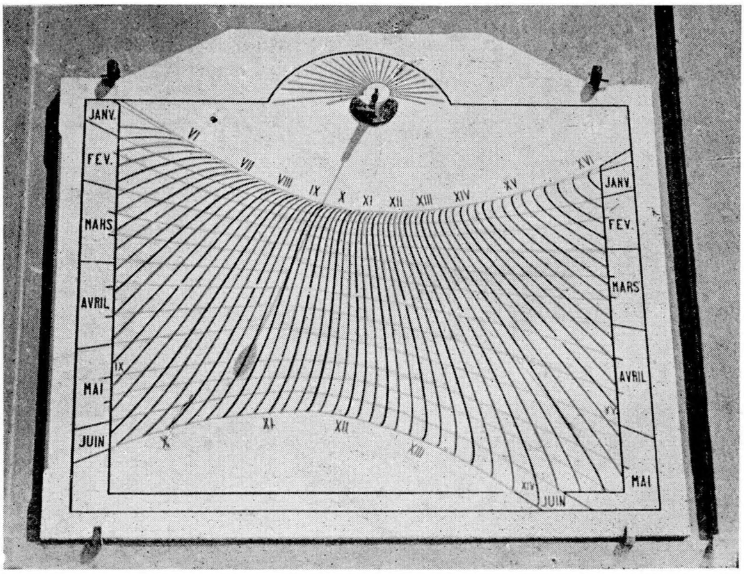


The Sun-Dial of the I.H.B. — Le cadran solaire du B.H.I.

THE SUN-DIAL
OF THE INTERNATIONAL HYDROGRAPHIC BUREAU

The new building erected by the Princely Government on the Quai de Plaisance of the Port of Monaco, and set apart for the use of the International Hydrographic Bureau, has a façade bathed in sunlight at all seasons and, as it lies in a practically East-West direction, particularly well suited to bear a sun-dial.

Although, in these days, this instrument has fallen somewhat into disuetude, it seemed to the author to be suitable to make a drawing of a dial which might decorate the façade.

Further, he attempted to modernise the drawing of the dial so that it would not show local apparent time, but the legal time in use in the Principality, automatically changing to summer-time on the proper dates, in such a manner that the time indicated by the sun-dial would agree with that shown by the town clocks.

This agreement is, naturally, realised only to the extent that engraving on marble will allow; the differences from clock time amount to about one minute and hardly exceed two minutes at the oblique extremes of the dial.

The drawing of the dial was presented to H. S. H. Prince LOUIS II of Monaco who kindly provided the funds necessary for its construction and installation.

In order to preserve a record of the way in which the dial was drawn up and so as to permit either its readings to be checked or changes in its adjustment to be made in the future, a short description of the summary method employed for making the drawing of the dial is given below (1).

For a gnomon *PS* (fig. 1), lying in the plane of the meridian and inclined to the horizon *h* of the place at an angle φ equal to the latitude of that place, it is easy to draw an equatorial dial on the plane *SHX*, perpendicular at *S* to the gnomon, by drawing a circle about the point *S* and dividing its circumference, beginning at the radius *SH*, into equal parts, representing hours (one hour being equivalent to 15° of arc) from 0 to XXIV hours.

By producing the radii corresponding to these divisions to meet the intersection *HX*, and by joining the points *A*, thus obtained, to the trace *P'* of the gnomon on the horizontal plane, a horizontal sundial is obtained.

If a vertical plane be erected, passing through *HX*, and if the same points be joined to the trace *P* of the gnomon in this plane, a vertical sundial lying in the East-West plane is obtained.

It is seldom that a wall on which it is desired to put a sundial is found to lie exactly East and West. It generally lies at some angle $XHX' = \alpha$ to this direction. The dial for such wall would be obtained by joining point *P* to the intersections *b* of the hour lines originating from *P'* with the horizontal trace of the wall *X'*.

These geometrical constructions are easily made on the drawing by turning down the various planes, but the angles involved, which must be determined, are often small and it is difficult to measure them or to redraw them exactly with a protractor. The

(1) For greater detail, those interested in the question should consult the "*Traité sur les Cadrans Solaires*", by G. BIGOURDAN, Director of the Bureau International de l'Heure, published in the *Annuaires du Bureau des Longitudes* of 1918 to 1920 ; and a descriptive work entitled "*Calculations for Sundials*", written about 1913 by Colonel F. P. WASHINGTON, of the British Ordnance Survey, and also the "*Book on Sundials*" by Mrs. GATTY, which, from a literary point of view, gives some thousand mottos pertaining to sundials.

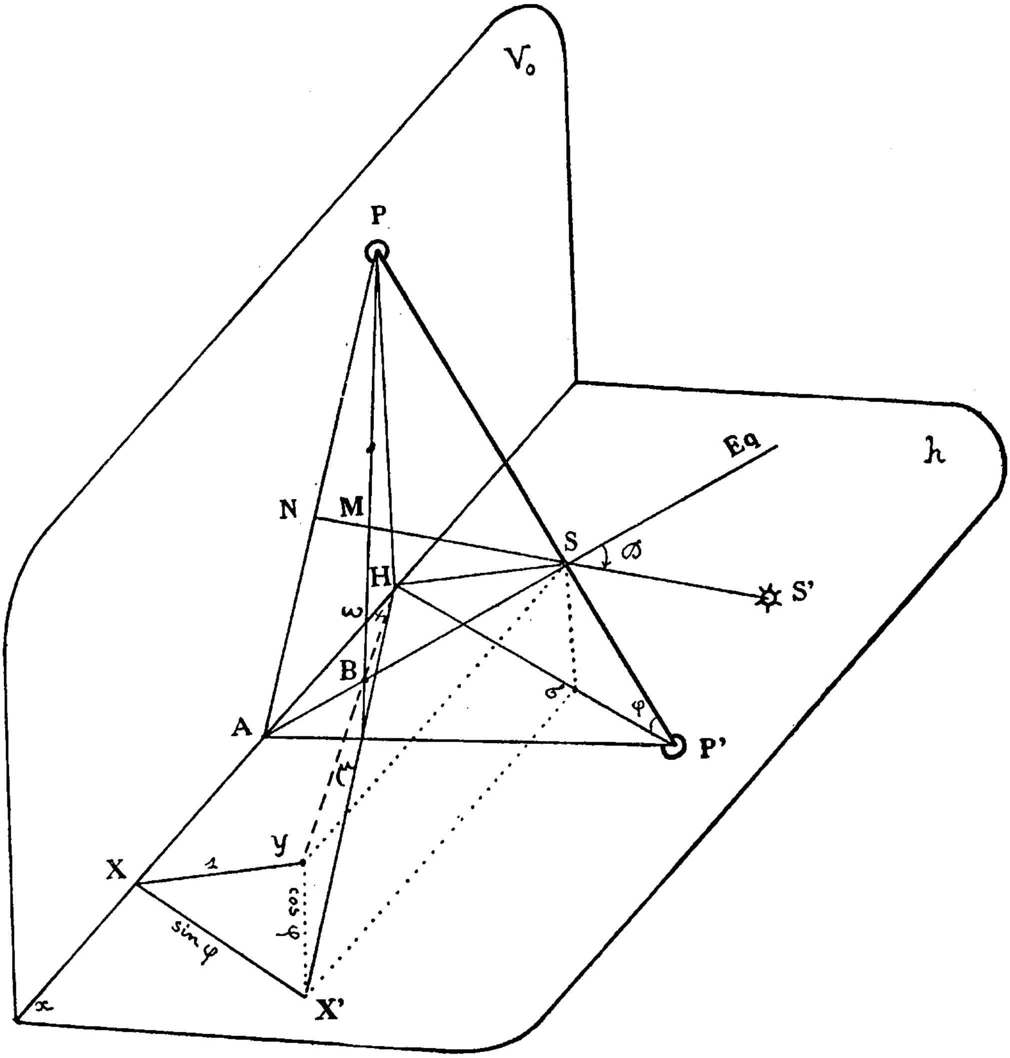


Fig. 1

lines of the dial-plate gain in accuracy if the calculation of the various lengths be made instead of that of the angles; the value of the former can be obtained by the usual methods of calculation with all the accuracy desirable.

It is this method which was employed for the construction of the lines of the dial-plate.

The bearing of the wall of the façade was determined by observations taken with a sextant at the moment of the rising of the sun above the sea horizon. The round value of 8° was adopted for the calculation, as it is not a difficult matter to displace the plane of the dial several minutes of arc at the moment of attaching it to the wall.

Further, instead of intersections *b*, intersections *B* were calculated, of the hourly radius system issuing from *P* with the trace *HY* of the wall lying in the equatorial plane *HSA*. This takes advantage of the relative symmetry of the dial with reference to the perpendicular, *SQ*, to *HB* (fig. 2).

The scale of construction of the model was left open by taking *SH* as the unit of length.

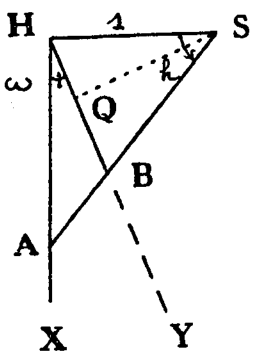
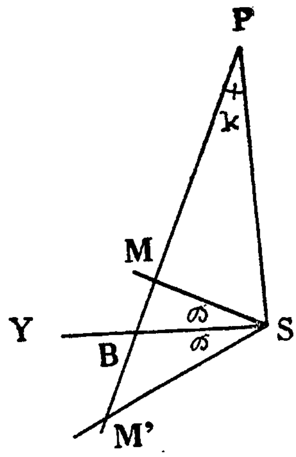


Fig. 2

Fig. 3



The diurnal umbral trajectories of the extremity S of the style which formed the gnomon on the dial-plate, which are generally hyperbolae, were determined at various points, by calculating the vectors BM and BM' (fig. 3) corresponding to the declination ϕ of the sun at different periods of the year. On days of zero declination the locus of the points M is the line HY (fig. 4), *i. e.*, the intersection of the wall with the equatorial plane passing through S, which line lies at an angle μ to the horizontal HX' and at an angle ω to the East-West line HX.

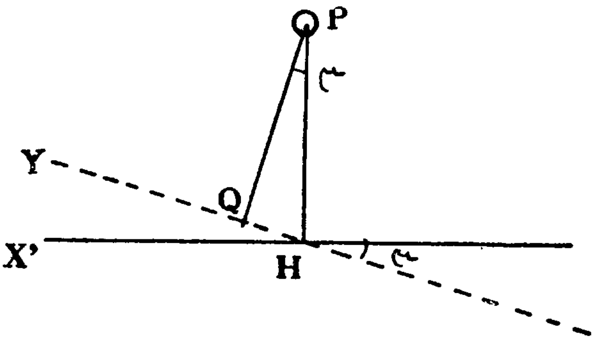


Fig. 4

The formulas used for these calculations are the following ; they are derived directly from the figures 1, 2, 3 and 4 :

$$SH = 1$$

$$X \tan \alpha = \sin \varphi \qquad \cos \varphi = HX \tan \mu \qquad PS = \tan \varphi$$

$$X \tan \omega = 1 \qquad \sin \varphi = HX \sin \alpha \qquad PH = \sec \varphi$$

$$\tan \omega = \tan \alpha \operatorname{cosec} \varphi \qquad \tan \mu = \sin \alpha \cot \varphi \qquad PQ = \sec \varphi \cos \mu$$

$$HQ = \sec \varphi \sin \mu$$

$$HQ = \sin \omega$$

$$\frac{HB}{\sin h} = \frac{SB}{\cos \omega} = \frac{1}{\cos (h\text{-}\omega)} \qquad SQ = \cos \omega$$

$$QB = \cos \omega \tan (h\text{-}\omega)$$

$$HB = \sin h \sec (h\text{-}\omega)$$

$$SB = \cos \omega \sec (h\text{-}\omega)$$

$$\tan k = SB \cot \varphi$$

$$\frac{BM}{\sin \phi} = \frac{SB}{\cos (\phi\text{-}k)} \qquad BM = SB \sin \phi \sec (\phi\text{-}k)$$

$$\frac{BM'}{\sin \phi} = \frac{SB}{\cos (\phi + k)} \qquad BM' = SB \sin \phi \sec (\phi + k)$$

Geographical position adopted for the International Hydrographic Bureau $\begin{cases} \varphi = 43^\circ 44.2' \text{ North} \\ \text{Long.} = 29^m 42^s \text{ East of Greenwich} \end{cases}$

$\alpha = 8^\circ \quad \mu = 8^\circ\ 15.4' \quad \omega = 11^\circ 29.5' = 45^m 58^s = 46^m.0$

Time of point Q $\begin{cases} \text{apparent local time} = \text{XIh } 14\text{m} \\ \text{apparent Greenwich time} = \text{Xh } 45\text{m} \end{cases}$

From these formulas the following values of the linear elements were deduced :

$PS = 0.956\ 835$
$PH = 1.384\ 04$
$PQ = 1.369\ 70$
$HQ = 0.203\ 385$
$SQ = 0.979\ 96$

The calculation is arranged in a tabular form which allows rapid evaluations of :-

1° the values of the vectors QB and SB for each ten minutes of time ;

2° the values of the vectors BM and BM' calculated in the triangles PSA for each twenty minutes of time, for the declination of the sun on the 1st, 10th and 20th days of each month.

These calculations give the exact values of more than 300 M points uniformly distributed over the dial-plate, thus permitting the lines of the dial-plate to be accurately drawn in functions of apparent Greenwich time, the time of point Q being Xh 45^m.

By joining all the M points calculated for the same declination of the sun, the traces of the diurnal umbral trajectories or diurnal hyperbolic arcs are obtained.

It should be noted that the arcs for the 20th of each month correspond approximately to the arcs of the signs of the zodiac.

In order to change from apparent time to mean time, a slight plus or minus correction of a few minutes must be made, viz :- the "Equation of Time", which varies, according to the period of the year, between + 14 and − 16 minutes.

This correction is usually indicated on a sundial by a curve, to the scale of the dial-plate, passing from side to side of the meridian according to the direction in which the correction is to be applied. This curve, which is in the form of a figure of eight, is called "mean noon line", as it takes the place, in actual fact, of the meridian for marking noon every day.

This curve was drawn on the dial-plate itself, and not only has it been substituted for the meridian, but also for all hour and other lines radiating from the point P, the root of the style, on all parts of the dial-plate where the shadow falls. The progressive deformation of the figure-of-eight curve which is noticeable on the dial is due to variations in the scale of minutes, which is not uniform and which varies all over the dial plate.

Thus it is possible directly to read Greenwich mean time, *i. e.*, the legal time in the Principality, at any time of the year, from the spot where the shadow of the extremity of the gnomon lies.

However, for several years, it has been customary to put the clocks on by one hour during summer or, more accurately in this region, from the early part of April to the early part of October. These periods correspond respectively to solar declinations of about + 5° in April and − 5° in October. In order to adapt the dial to this new arrangement, that part where summer time must be read is drawn in red, *i. e.*, the hours are read off the red lines, in the lower part of the dial. At other seasons, the normal time is read off the black lines in the upper part of the dial.

In order to avoid overlapping of the seasonal parts of the figure-of-eight, these two parts have been placed on separate dial-plates, one for the period from 21st December to to 21st June and the other for the period from 21st June to 21st December.

To avoid mistakes in reading, the style will be transferred from one dial-plate to the other on these dates. The accompanying photographs show the appearance of the dials.

Once the dial-plate is drawn the placing of the style is, in itself, a somewhat delicate operation ; but even here calculation allows an easier method of metric measurement to be substituted for the difficult determination by angles and bearing.

Point P, the root of the style, being already determined, it is but necessary to obtain the co-ordinates on the dial of the extremity S of the style and its perpendicular distance from the plane of the dial.

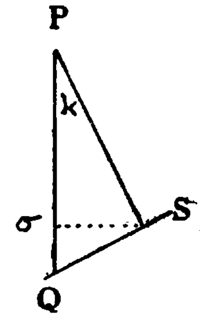
Since in the triangle *PSQ* (fig. 5) :

$$P\sigma = PS \cos k$$
$$Q\sigma = QS \sin k$$
$$S\sigma = PS \sin k = QS \cos k$$
$$QS = PS \tan k$$

hence :

$$k = 45^\circ 41'$$
$$P\sigma = 0.668\ 4$$
$$Q\sigma = 0.701\ 3$$
$$S\sigma = 0.684\ 6$$

Fig. 5



From these values, the position of the point σ being marked once for all on the marble plate (which was done), the extremity of the style can be regulated, whenever it is changed, by means of a triangular set square the edge of which represents the distance σ *S*.

The scale of the dial-plate partly depends on the thickness of the style. This must be sufficiently strong not to bend easily, but, at the same time, the shadow cast must not be too wide in comparison with the smallest divisions on the dial, in order to maintain its accuracy. In any case, the size of the dial is governed by the wall-space available for the sundial and it is this space which determines the scale of the sundial.

The scale of the Monaco sundials is determined by *SH* being equal to 300 millimetres.

H. B.