A SOURCE OF ERROR IN THE SOLAR COMPASS

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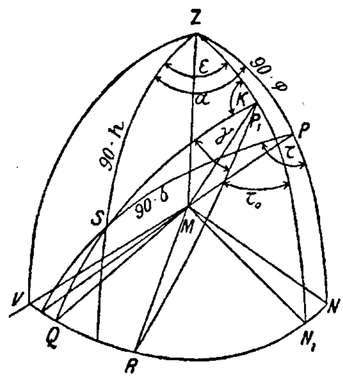
The *Boykow solar compass* has been successfully employed on polar flights as a direction indicator. Recently, this instrument has been recommended for use in flights in lower latitudes. For this purpose the axis of rotation of the clock mechanism which, in the type of instrument designed for polar flights, was oriented perpendicular to the local horizon, may be adjusted parallel to the axis of the earth. The use of this apparatus may then be conceived as follows :-

In order to be able to hold to the loxodromic course from the take-off to the goal of the flight with the aid of the solar compass, the setting at the point of departure should be made in such a manner that when the airplane is on the correct course, the axis of rotation of the clock mechanism may be oriented parallel to the axis of the earth. Since the clock mechanism completes one full revolution about its axis of rotation in exactly 24 hours, the influence of the diurnal movement of the sun is compensated in such a manner that at the point of departure one may be assured of maintaining position as well in a pre-determined plane parallel to the earth's axis (constant hour angle) as in the pre-determined azimuthal direction, while at the same time the fixed course is continuously indicated.

However, up to now no account has be taken of the fact that as a result of the changes in position of the airplane, the apparent movement of the sun differs with the changing positions of the airplane from that at the point of departure, for which in reality the apparatus is adjusted to function as a direction indicator. The significance of this will be illustrated by a few examples later to make this clear.

Let us assume that the airplane is to make a flight with the solar compass along the parallel of latitude 50° in an easterly direction. In order that the axis of rotation of the clock mechanism may assume the direction of the axis of the earth on the designated course, one should elevate it 50° above the horizontal bearing on the port beam. The figure then shows the errors which may be expected as a result of a change in the position of the airplane. Let Z be the zenith at the position occupied by the airplane, MV the horizontal line of direction ahead, MN_1 to port, MP_1 the direction of the axis of rotation of the clock mechanism : therefore the angle $\widehat{N_1MP_1} = 50^\circ$. The plane P_1SQM which is maintained by turning the airplane towards the sun S, passes through the axis MP_1. Under the action of the clock mechanism, the plane P_1RM is subject to an angular displacement with respect to the plane P_1SQM equal to $\gamma - 15^\circ u$ where u represents the hours in time which have elapsed since the start of the flight. If the airplane had remained stationary at the point of departure, this plane P_1RM would represent a fixed plane at that place, independent of the movement of the sun and having the fixed azimuthal bearing MR, which was the bearing of the sun at the time of the start. If therefore the azimuth of the sun was equal to a_0 at the instant of take-off and at the point of departure, then according to the sun compass the direction MN_1 where angle $\widehat{RMN_1} = a_0$ will also be taken as the North direction in every position which the airplane occupies later.

However, the later position of the airplane lies in reality to the eastward of the point of departure and the instantaneous azimuth of the sun (computed from North to East) at that point differs from the sun's azimuth at the point of departure at the same instant. The true direction of North MN therefore deviates to the left of the direction indicated by the compass, and in following this instrument we should be diverted too far to the Southward (On a westerly flight the opposite would occur). The actual elevation of the pole P is therefore less than 50°. The error in course which results, or $f = \widehat{N_1MN}$, is calculated from $f = a - \varepsilon$, in which $a = \widehat{PZS}$ is the true azimuth of the sun at the point of departure and $\varepsilon = \widehat{P_1ZS}$ is the erroneous azimuth given by the sun compass.



If the position of the airplane and the azimuth of the sun at that point are known at the time, then the following elements in the figure are known :-

Arc	PZ	$= 90^\circ - \varphi =$	polar distance of the airplane,
»	ZS	$= 90^\circ - h =$	zenith distance of the sun at the position of the airplane,
»	PS	$= 90^\circ - \delta =$	polar distance of the sun,
Angle	SZP	$= a =$	azimuth of the sun at position of airplane,
»	NPS	$= \tau =$	hour angle of the sun at position of plane (reckoned from North).

The angle ε is calculated from the triangle ZSP_1, in which the side $ZP_1 = 40^\circ$ according to the setting of the axis of rotation of the clock mechanism in the plane, the side $ZS = 90^\circ - h$, and the angle $\varkappa$ is found from the equation

$$\varkappa = 180^\circ - \gamma - \tau_0$$

In this, $\gamma = 15^\circ\ u$ in which u equals the elapsed time of flight in hours, and τ_0 is the hour angle of the sun at the point of departure at the instant of the take-off. The calculation of the resultant error in course and position is carried through approximately in the following example :- Let $\delta = 20^\circ$ N. be the declination of the sun, the take-off occurs at 0430 local time at the meridian of the point of departure. The approximation will be made by assuming that the course remains unaltered for one hour at a time and is then changed in one jump to correspond to the position arrived at. Let the distance covered over the ground equal 100 nautical miles per hour, and the effect of the wind be excluded.

With an error in course f with regard to the desired course east, we shall have each hour :-

Change in latitude $\Delta\varphi = - 100'\ sin\ f$.

Change in longitude $\Delta\lambda = 100'\ sec\ \varphi\ cos\ f$ or, in time, $= 6\ 2/3$ min. $sec.\ \varphi\ cos\ f$.

If we call the hour angle of the sun at the position of the airplane τ and the longitude difference between position of plane and point of departure λ, then at the start

$$\lambda = 0\ ;\ \tau = 4.5\ \text{h} = 67^\circ.5$$

which at the same time represents the constant angle τ_0. The corresponding initial azimuth of the sun is : $a_0 = 60^\circ 18'$.

For the further course of the flight we may compute

$\varkappa = 180^\circ - \tau_0 - \gamma = 112.5^\circ - 15^\circ\ u$ ($u =$ the elapsed time of flight in hours) and $\tau = 4.5 + u + \lambda$ (all computed in hours) in order that the resultant values of a and h may be taken from the azimuth and altitude tables.

After the first hour, during which the plane is flying on the unchanged east course, or with a course error assumed of $f = 0$, the longitude difference at 0530 equals :

$$\lambda = 6\ 2/3\ \text{min.}\ sec\ 50^\circ = 10.37\ \text{min.}$$

$$\text{therefore } \tau = 4.5\ \text{h.} + 1\ \text{h.} + 10.37\ \text{min.} = 5\ \text{h.}\ 40.4\ \text{min.}$$

With this value of τ and $\delta = 20^\circ$ N. and $\varphi = 50^\circ$ N., we find from EBSEN'S azimuth tables the azimuth $a = 73^\circ.27$ and from BALL'S Altitude Tables $h = 12^\circ5'.6$. Then ε is determined from the triangle ZSP_1 in which the arc $ZP_1 = 40^\circ$ remains constant, while h retains its previous value and $x = 112^\circ5 - 15^\circ u$ and therefore the value of $x = 97^\circ5$.

If we introduce the auxiliary angle μ, then we obtain the formula :-

$$\tan \mu = \cot x \sec. 40^\circ \quad \text{and} \quad \cos (\mu - \varepsilon) = \sin 40^\circ \tan h \tan x \sin \mu$$

We find $\mu = -3^\circ 45.1'$ and $\varepsilon = 70^\circ.05$, so that the course error becomes $f = a - \varepsilon = 3^\circ.22$. For the next hour therefore we hold the course $90^\circ + 3^\circ.22$ and the change in position is given by $\Delta \varphi = -100' \sin 3^\circ.22 = 5'.6$, and $\Delta \lambda = 6\ 2/3$ min. $sec.\ 50^\circ \cos 3^\circ.22 = 10.35$ min. At the position of the airplane at 0630 therefore $\varphi = 49^\circ 54'.4$ N. and the hour angle of the sun $\tau = 5\text{ h } 40.4^m + 1\text{ h} + 10.35^m = 6\text{ h } 50.75^m$.

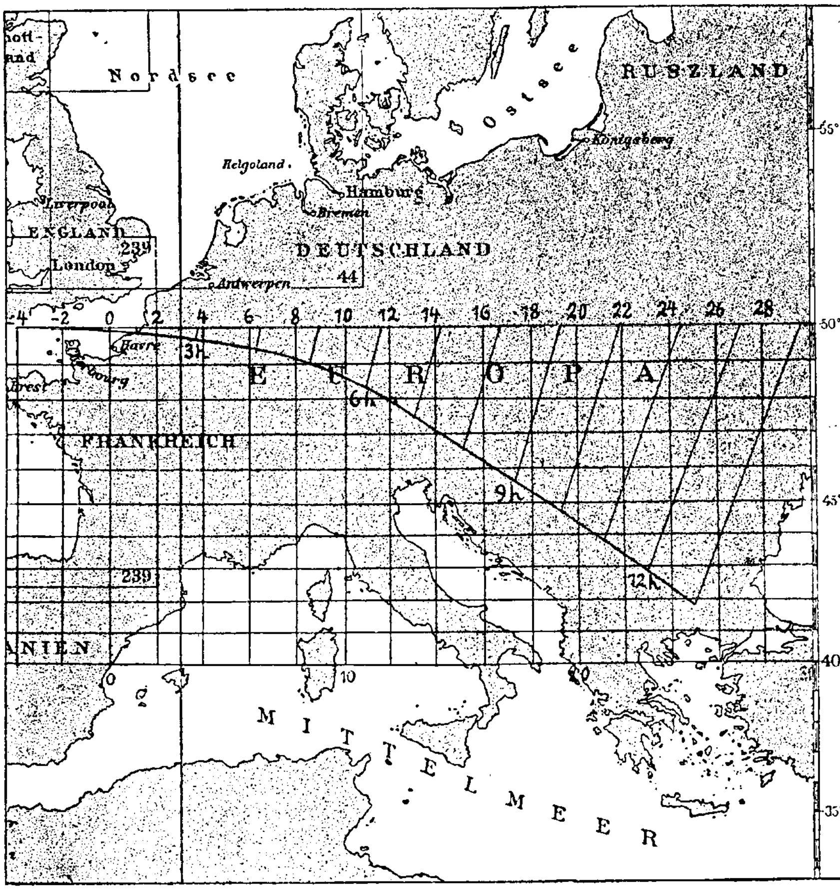
The results of the further step by step calculations are shown in the following Table :-

Course followed and positions reached by solar compass instead of a direct flight on the 50° latitude North at the speed of 100 nautical-miles per hour towards the East.

Hour of meridian at take off position τ	Latitude reached φ	Corrected Longitude λ	Hour angle of sun at position of airplane τ	Azimuth of sun at position of airplane a	Height of sun at position of airplane h	Apparent solar azimuth ε	Course by solar compass $90+a-\varepsilon$	Variation of latitude $\Delta\varphi$	Variation of longitude $\Delta\lambda$	LONGITUDE: Desired	Reached
0430	50° 0'	0 h 0 m	04 h 30,0m	60,30°	1° 46,2'	60,30	90,0°	0,0'	10,4 min.	- 4° 0'	- 4° 0'
0530	50 0	0 10,4	05 40,4	73,27	12 5,6	70,05	93,22	5,6	10,35	1 24,4	1 24,4
0630	49 54,4	0 20,75	06 50,75	86,14	23 15,2	78,94	97,20	12,5	10,20	1 11,1	1 11,
0730	49 41,9	0 30,95	08 01,1	100,01	34 29,5	87,93	102,08	20,9	10,05	3 46,7	3 44,
0830	49 29,8	0 41,0	09 11,0	116,37	45 9,7	98,43	107,94	30,8	9,75	6 22,3	6 15
0930	48 59,0	0 50,75	10 20,75	137,68	54 7,2	113,55	114,15	40,9	9,23	8 57,8	8 41
1030	48 18,1	0 59,98	11 29,98	166,00	59 24,7	137,47	118,53	47,8	8,82	11 33,4	10 59,8
1130	47 30,3	1 8,80	12 38,80	197,98	59 1,3	166,18	121,78	52,7	8,40	14 9,9	13 12,0
1230	46 37,6	1 17,20	13 47,20	225,12	53 14,3	192,23	122,89	54,3	8,14	16 44,6	15 18,0
1330	45 43,3	1 25,34	14 55,34	245,27	44 14,6	211,91	123,36	55,0	7,96	19 20,1	17 20,
1430	44 48,3	1 33,30	16 03,30	261,60	33 48,7	226 60	125,00	57,4	7,66	21 55,7	19 19,5
1530	43 50,9	1 40,96	17 10,96	274,19	22 57,8	239,50	124,69	56,9	7,57	24 31,3	21 14,4
1630	42 54,0	1 48,53	18 18,53	286,53	12 19,0	250,86	125,67	58,3	7,40	27 6,8	23 8,0
1730	41 55,7	1 53,93	19 25,93	298,93	2 20,3	262,17	126,76	—	—	29 42,4	24 59

It is evident from this how appreciably the course flown by the sun compass begins to deviate from 90° after a few hours; after three hours the error in the course exceeds 12°; after 5 hours, 24°; after eight hours 33° and in the last three hours before sunset it remains about 35° (Column 8). Consequently the positions reached by the airplane will differ greatly from those towards which the course was laid. In latitude (Column 2) the error amounts to 1° after 5 hours; 2.5° after 7 hours, and then increases rapidly to 8° after thirteen hours of sustained flight. In longitude (last column) the error amounts to 1° after about 7 hours' flight, 2° after 9 hours and 4° after 12 hours. Since, in these calculations, we have assumed that the initial course was held constant for one hour at a time, the actual errors in practice will be greater than those shown by these approximate calculations. The course plotted on the small chart (Fig. 2) would be followed in a calm according to the sun compass, assuming the plane to take off at 0430, at about sunrise, from a point 4° West and 50° N., at 100 nautical miles per hour, and assuming that the pilot attempts to hold to the exact loxodromic east course with the aid of the instrument. The points which it is assumed would be reached on the course east on the 50th parallel are connected by a straight line on the chart. In flight, however, one would arrive approximately :-

instead of	Mainz	at	Weissenburg,
—	Prag	at	Ischl,
—	Lemberg	at	Kragujevac,
—	Kiew	at	Philippopel.



This therefore constitutes the source of a very serious error, which must unquestionably be taken into account in the instructions for the use of the sun compass in addition to the corrections to the course ordinarily required due to the effect of the wind.

The progress of these errors during a long flight depends on the latitude of the point of departure, the course flown and the speed of the airplane over the ground ; it is therefore very different in the individual cases. Therefore, on the assumption that the course is to be followed exactly in flight, the course errors which result as a function of the flying time must be taken into consideration and calculated in advance. If the compensation for the wind is taken into consideration, such calculations may become quite complicated since, aside from the latitude and the longitude of the place reached, the duration of the flight must also be taken into account.

It would be more advisable to correct the error by instrumental means. If the inclination of the compass were adjusted each time to correspond to the latitude reached, this would not suffice except in the case of a flight along a meridian, since the changes in longitude also give rise to appreciable errors. In the example used in our illustration the deviation from the true course would be almost as great as we have shown. Therefore, in addition to the above, the speed of rotation of the sun compass should be made constantly variable in a uniform manner, within reasonable limits — a condition easily attained — and this speed might then be properly adjusted for the particular flight in question at departure and changed subsequently if necessary for variations in the wind.

If, for example, we wish to fly on a course NE at 100 nautical miles per hour with an East wind of 20 miles per hour, we have first to count on an off set of 8.1° in the course to allow for the wind, and consequently the vertical plane of the sun compass must be adjusted at (315° − 8.1°) instead of 315°. Under these conditions and provided the course NE is made good, the speed over the ground in this direction will be 84.86 miles/hr, that is, we shall have to readjust the instrument every 60 nautical miles to the North or every 60 *sec* φ minutes of arc to the East, corresponding to 4 *sec* φ minutes of time. If the flight is to be from 44° to 52° N. latitude, the mean value is $1/2\ (sec\ 44° + sec\ 52°) = 1.5$ and therefore $4\ sec\ \varphi = 6$ minutes of time. The rate of rotation about the axis of the solar compass must be changed to the ratio of $(60 + 6) : 60$, or must be 10 % greater than with the airplane stationary. Also, the axis of the compass which was set at 44° inclination at the start, must be set 1° higher above the horizontal every hour.

As long as these structural improvements are lacking in the instrument, one should not conclude, since it is suitable for use in the vicinity of the poles, where differences in direction on the earth are identical with differences in meridians, that this device is also capable of employment as a convenient directive device for long flights in the lower latitudes, where celestial bearings and hour angles are very different things.