DIAGRAM FOR POSITION FINDING BY LONG DISTANCE BEARINGS AND FOR FINDING THE BEARING OF ONE POINT FROM ANOTHER

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1. The diagram described in this communication was constructed in 1921, more especially with the view to providing a simple and accurate method of plotting the position of a point by means of its true bearings from two or more fixed wireless direction finding stations. Incidentally the system of curves which forms the basis of the diagram furnishes also a rather interesting method of finding the true bearing of one point from another. Circumstances prevented the further development of the diagram; and as regards the problem of finding true bearings there were so many methods already, both diagrammatic and tabular, that some diffidence was felt in adding to their number, although the method of the diagram appeared to have considerable advantages.

In connection with the problem of finding positions by means of long distance bearings, that is bearings from stations too distant to permit of the bearing lines drawn as straight lines on a chart, much has been written during the last ten years, but the direct and accurate method described in the following paragraphs is to be preferred to any hitherto proposed as far as the writer is aware. In this method no calculation is involved and the time required to fix a position is probably less than in the case of any of the methods requiring calculation or reference to tables, in most of which it is necessary to obtain a second approximation if great accuracy is desired.

2. Let A and B in Fig. 1 be two points on the sphere; PQ, PR their meridians and C the point in which the great circle AB meets the equator. Now the true bearing of B from A, that is the angle QAB, is the same as that of C from A; also the true bearing of A from B, that is the angle PBA, is equal in magnitude, but in the opposite quadrant, to that of C from B, since $PBA = RBC$.

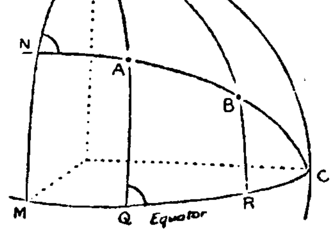

FIG. 1.

Let the latitude and longitude of A be φ and ω, the latter being measured from the meridian of C; also let α be the latitude of N, the point in which the great circle ABC meets a meridian at right angles, that is the vertex of the great circle. This point N is 90° of longitude away from C; therefore $MQ = 90^\circ - \omega$ if PM is the meridian of N.

From the triangle APN, right angled at N, we have:

$$\tan PN = \tan PA \cos NPA$$

that is: $$\cot \alpha = \cot \varphi \sin \omega$$

or: $$\tan \varphi = \tan \alpha \sin \omega \ldots\ldots \qquad \text{(I)}$$

Also from the triangle AQC, right angled at Q, we have:

$$\tan QC = \tan QAC \sin AQ$$

or if QAC, the true bearing of C or B from A, be denoted by β:

$$\tan \omega = \tan \beta \sin \varphi \ldots\ldots\ldots\ldots\ldots \qquad \text{(II)}$$

which equation is of the same form as (I), φ and ω being interchanged and β replacing α.

3. If now equation (I) be plotted in plane rectangular coordinates with φ and ω as variables and a given value of α a curve of the form $C\alpha$ in Fig. 2*a* will be obtained, the axes of φ and ω being as shown in the figure and the scale of φ being equal to that of ω.

If also equation (II) be plotted with these axes, the same value being taken for β as for α in the case of equation (I), a curve $C\beta$ of exactly the same form as $C\alpha$, will be obtained as shown in Fig. 2*b*. The curve $C\beta$ differs from $C\alpha$ only in position with respect to the axes of φ and ω.

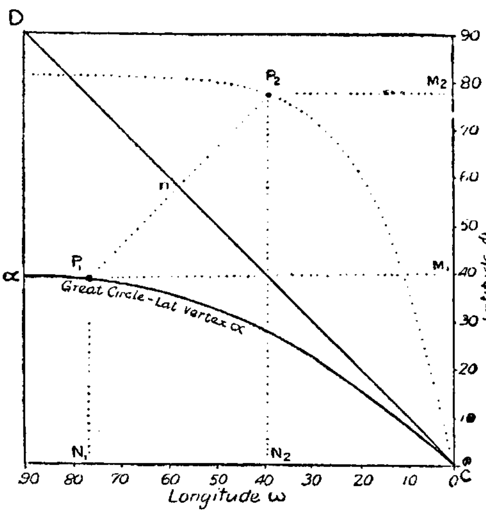

FIG. 2*a*.

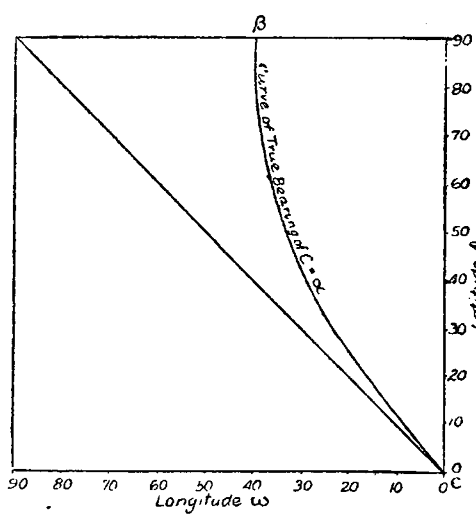

FIG. 2*b*.

Thus the representation of a great circle ABC on the above system is also that of the curve of constant true bearing of a point C on the equator, that is the locus of points from which this point has a given true bearing, provided that in the case of the latter the axes of φ and ω are interchanged and the value of the true bearing β being the value of α for the $C\alpha$ or great circle curve.

This identity of form of the representation of the great circle whose vertex is in latitude α and that of the curve of constant true bearing β of a point on the equator on the system of Fig. *2a* and *2b* is the basis of the proposed diagram.

From the above considerations it is seen that the true bearing of C from any point on the sphere is to be found as the value of α for the $C\alpha$ curves at the point whose coordinates are those of the point in question interchanged. Thus the true bearing of C from P_1 in Fig. *2a* is read from the $C\alpha$ curves at the point P_2 such that $P_1N_1 = P_2M_2$ and $P_1M_1 = P_2N_2$ or, what is the same thing, such that $P_1n = P_2n$, $P_1n\ P_2$ being the perpendicular from P_1 to CD, the diagonal of the quadrant of the diagram. In other words the true bearing of C from P_1 is read from the $C\alpha$ curves at the image of P_1 in this diagonal. Similarly the true bearing of C from another point Q_1 on the same great circle $C\alpha$ as P_1 is read at Q_2 the image of Q_1, in the diagonal CD. The true bearing of C from P_1, as noted in §. 2, is also that of Q_1 from P_1 and the true bearing of C from Q_1 is the same numerically as that of P_1 from Q_1.

4. Any two points on the sphere lie on some definite great circle. This great circle having been identified the true bearing of either point from the other is found as that of the point in which the great circle meets the equator as described in §. 3. The method of procedure in finding true bearings will be understood from Fig. 3 which shows the arrangement of the diagram for this purpose. The diagram consists of three quadrants of $C\alpha$ curves drawn on board, and, a transparency squared in latitude and longitude on the same scale as the diagram, so that the two points in question can be plotted on it. The transparency slides over the diagram between guides as shown in the figure so that its upper and lower lines are kept in coincidence with those of the diagram. This enables the great circle upon which the two points lie to be found, The method of finding a true bearing will be clear from the consideration of an example.

EXAMPLE. — P_1 is in latitude 60° N; Q_1 is in latitude 45° N. and 75° of longitude East of P_1. Find the true bearing of Q_1 from P_1.

This example is worked in Fig. 3. Referring to the figure the steps are as follow:

I) Plot P_1 and Q_1 on the transparency, the former on the 0° meridian in latitude 60° and the latter 75° of longitude away in latitude 45°.

II) Slide the transparency over the diagram until P_1 and Q_1 lie on the same curve of the diagram or similarly placed between two consecutive curves. This identifies the great circle upon which P_1 and Q_1 lie.

III) With the transparency in this position take the distance P_1n of P_1 from the diagonal of the quadrant of the diagram in which P_1 lies with dividers and set it off on the other side of the

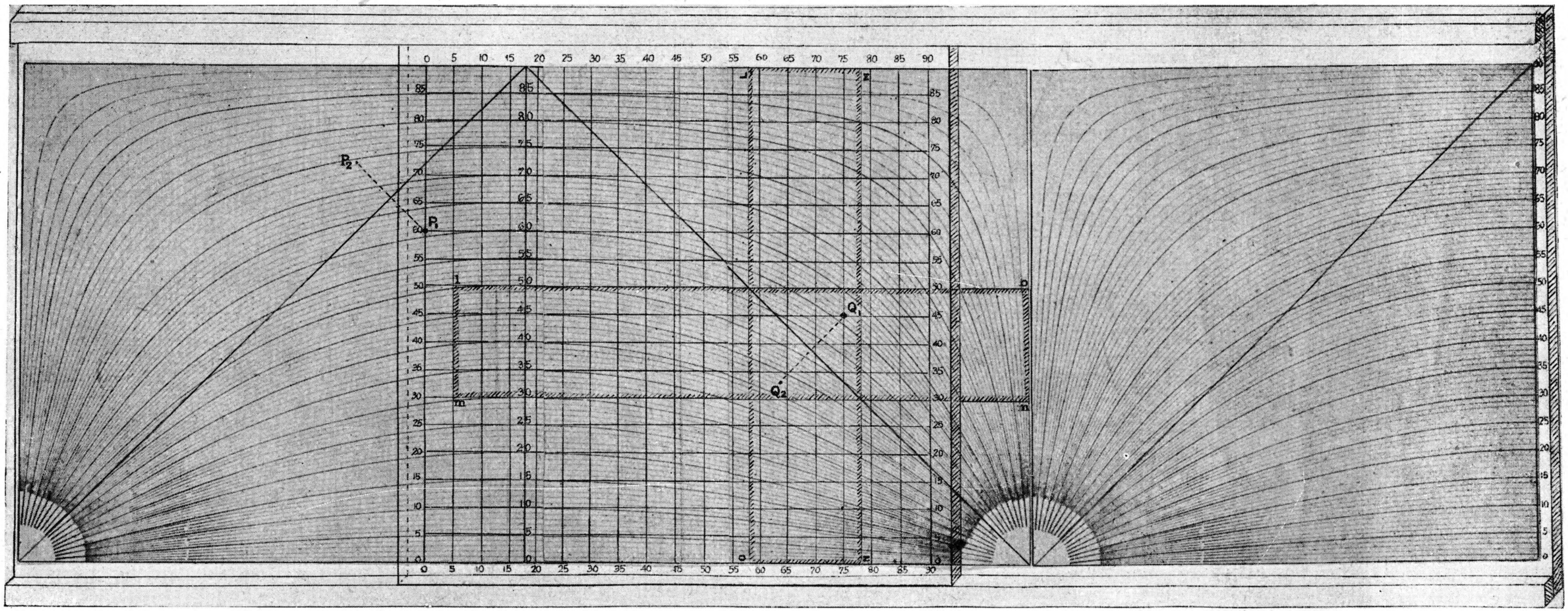

FIG. 3.

Diagram with example of finding the true bearing of one point P_1 from another point Q_1. The transparency is placed so that P_1 and Q_1 lie on the same curve or similarly placed between two curves.

Diagramme avec exemple montrant comment l'on trouve le relèvement vrai d'un point P_1 pris d'un autre point Q_1. On place le transparent de manière à ce que P_1 et Q_1 se trouvent situés sur la même courbe ou qu'ils soient placés homothétiquement entre deux courbes.

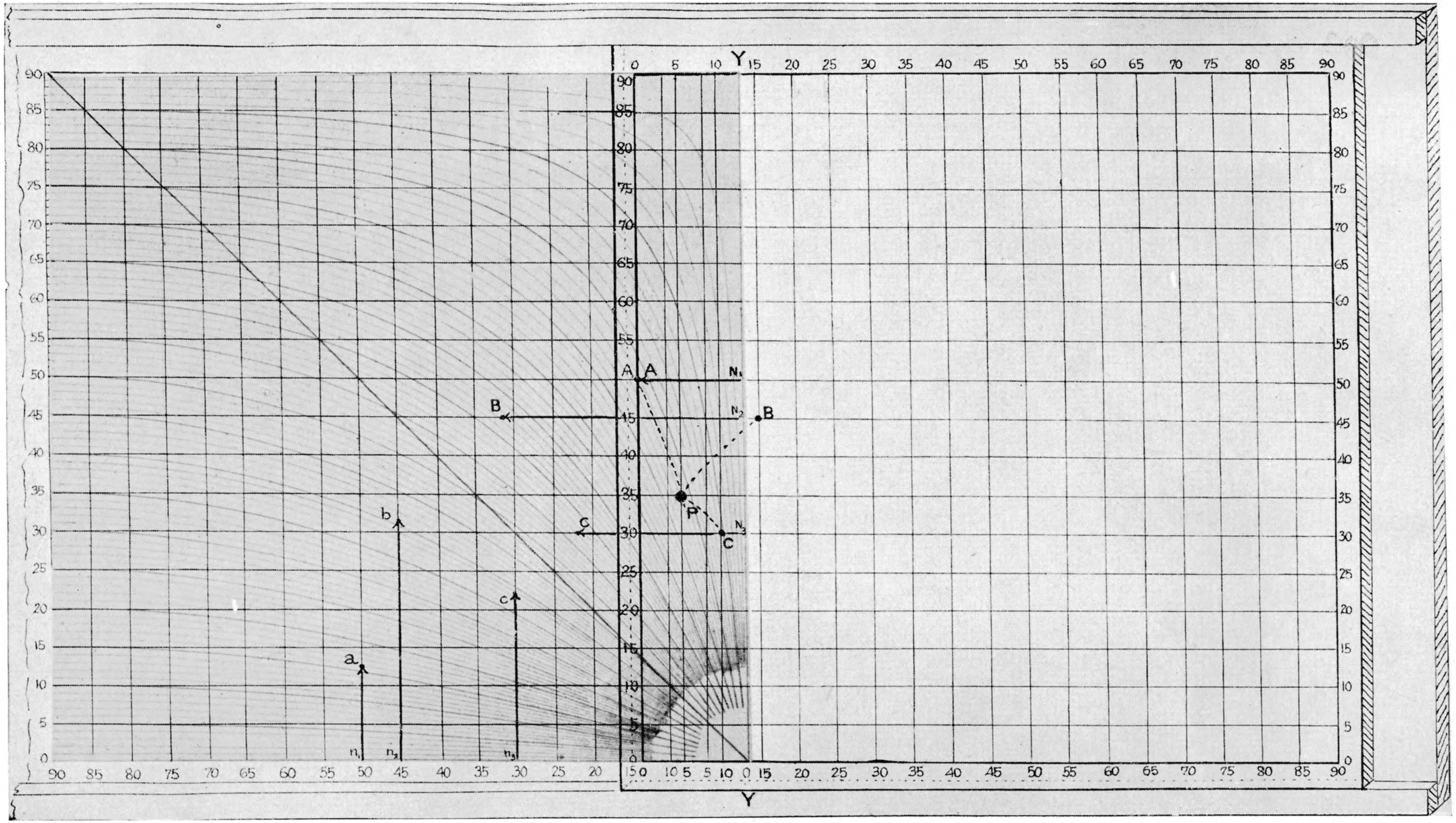

FIG. 4

Portion of diagram with example of plotting the position of a point P *by its true bearings from two or more stations. The points* a, b *and* c *are plotted on the diagram (see §. 5) and transferred to* A, B *and* C *similarly placed on the opposite side of the diagonal.* A, B *and* C *on the transparency are then in turn placed over* A, B *and* C *on the diagram and the necessary arcs traced or interpolated.*

Portion de diagramme avec exemple montrant comment on déduit la position d'un point P *de ses relèvements vrais pris à deux ou plusieurs stations. On porte sur le diagramme les points* a, b *et* c *(voir paragraphe 5), puis on les transporte aux points* A, B *et* C *symétriquement placés de l'autre côté de la diagonale. On place alors les points* A, B *et* C *du transparent au-dessus des points* A, B *et* C *du diagramme, et l'on trace ou l'on interpole les arcs nécessaires.*

diagonal. The point of the dividers is now at P_2; the true bearing of Q_1 from P_1 is read from the curves of the diagram at P_2.

Similarly the true bearing of P_1 from Q_1 is read at Q_2, the image of Q_1 in the diagonal of the quadrant of the diagram in which it lies.

The readings of the true bearings from Fig. 3 are:

$$Q_1 \text{ from } P_1 = 74^\circ \text{ NE};$$
$$P_1 \text{ from } Q_1 = 42^\circ\, 3/4 \text{ NW};$$

the quadrant in which the true bearings lie being obvious from the relative positions of the points; in this example P_1 and Q_1 are on opposite sides of the vertex of the great circle upon which they lie, and the true bearings are both northerly. The calculated values of these true bearings are 74°04' and 42°50'.

For points on opposite sides of the equator the middle and right hand quadrants of the diagram are used, the points being plotted on the transparency without regard to the sign of their latitudes. On the original form of the diagram curves of equal distance from the point on the equator were drawn. These enabled the distance P_1Q_1 to be read, either as a sum or a difference, at the same time as bearings; these curves are not shown in Fig. 3.

5. A point P whose true bearing from a given point A is β lies on the great circle inclined to the meridian of A at the angle β. As stated in §. 2 the true bearing of the point in which this great circle meets the equator is also β. To draw a true bearing from a given point it is therefore sufficient to consider the true bearing of a point on the equator, and to identify the great circle which passes through A and the point on the equator whose true bearing from A is β. The diagram described in the preceding paragraphs enables this great circle to be easily drawn on the transparency. We have thus one line upon which the point P lies, and the same procedure with the true bearing of P from a second given point B will fix the position of P which is then read from the graduation of the transparency. The method will be clear from an example.

EXAMPLE. — Suppose the positions of three stations A, B and C and the true bearings of P from them are as given in the following table. Find the position of P.

STATION.	*Latitude of Station.*	*Longitude of Station.*	*True Bearing of* P *from Station.*
A	50° N.	70° E.	15°35' SE.
B	45° N.	85° E.	40°45' SW.
C	30° N.	80° E.	38°50' NW.

The steps are as follow:

I) Plot the stations A, B and C on the transparency: A on the 0° meridian in latitude 50°; B and C in their latitudes 45° and 30°, and 15° and 10° of longitude respectively away from A.

II) Mark the point in which the 50° meridian of the diagram meets the $15^\circ 35'$ curve; also that in which the 45° meridian of the diagram meets the $40^\circ 45'$ curve and that in which the 30° meridian of the diagram meets the $38^\circ 50'$ curve; as at a, b and c in Fig. 4.

III) Transfer the points a, b and c to equal distances on the other side of the diagonal of the diagram; this is most easily effected by setting off the lengths an_1, bn_2 and cn_3 from the edge YY along the 50°, 45° and 30° parallels of latitude respectively. This gives the points A, B and C on the diagram as in Fig. 4.

IV) Slide the transparency along the diagram until A as plotted in (I) falls on A as marked on the diagram in (III) and trace the necessary portion of the curve upon which A lies.

V) Having turned the transparency over from left to right, slide it along until B falls on B on the diagram and trace a tick where the curve upon which B lies crosses the curve already traced for A. This gives the position of P, which is then read from the transparency: the latitude directly and the longitude by difference from A.

The true bearing from C would be treated similarly, the arc CP being drawn with the transparency right way up, as for A. Two true bearings of course suffice to fix P, but a third is desirable as a check. In the case of B of the example the transparency was turned over before drawing the arc corresponding to the true bearing from this point; this is in order to have the arc in the right direction for a SW bearing; whether this turning over is necessary in any particular case will easily be seen. In the case of the example the true bearings, being calculated for the point 35° N, 75° E, give an accurate fix for P as shown in Fig. 4. In general, of course, on account of the errors affecting true bearings from D. F. stations, three true bearings would give a "cocked hat", which would be dealt with in the usual way.

In finding a position the points A, B and C as plotted on the diagram will usually not fall exactly on a curve of the diagram but between two curves; the arc required is however easily interpolated on the transparency by eye.

6. The scale of the diagram as originally drawn was 1 inch = 10° of latitude or longitude, and showed the curves at 1° intervals as in Fig. 3 and 4. This is sufficient to enable true bearings to be found to the accuracy necessary for the purposes of navigation. For position finding larger scale drawings of zones of 10° or 20° of latitude were contemplated: for instance a large scale drawing of the portions marked $L\,M\,N\,O$ and $l\,m\,n\,o$ in Fig. 3

for the zone 30° - 50° was made. On a scale which would allow a position to be found to within a minute of latitude, for bearings from stations up to 300 miles distant, the size of the sheet, if folded in the middle, would be about 30 inches square. The portion *l m n o* would extend from *n o* to 95° of longitude, *i. e.* about 300 miles beyond the vertices of the great circles included in the zone. The arrangement of the two parts of the diagram is indicated in the reduced sketch of the zone 30° - 50° in Fig. 5, the portion *L M N O* being reversed so that the true bearing curves slope down from left to right.

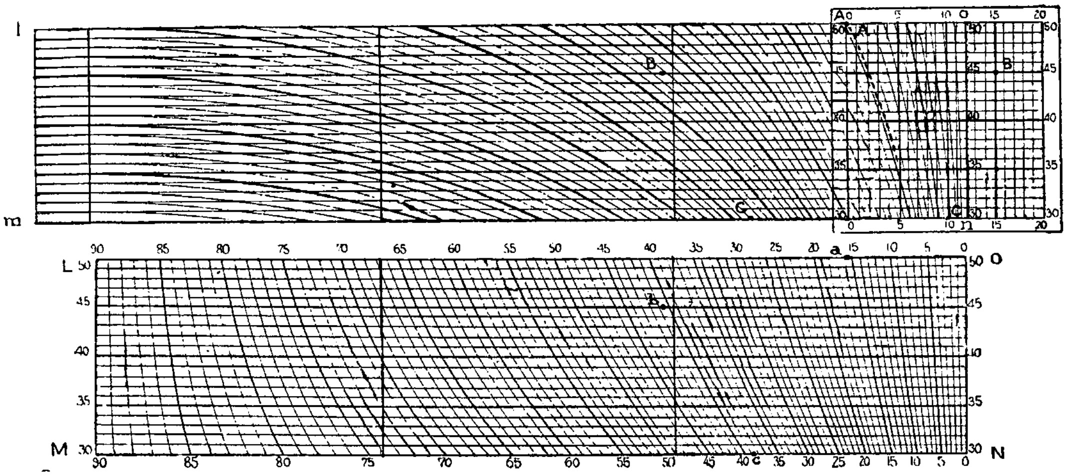

FIG. 5.

Arrangement of large scale position finding chart: zone of latitude 30° – 50°.

The method of using this position finding chart is as described in §. 4; thus, referring again to the example:

I) Mark *a*, *b* and *c* on *L M N O* where the true bearing curves 15°35', 40°45' and 38°50' cross the parallels of latitude 50°, 45° and 30°.

II) Transfer *a*, *b* and *c* to the same distances from *n o* on *l m n o* as from *N O* on *L M N O* on the same parallels of latitude; this gives *A*, *B* and *C* on the diagram.

The position of *P* is then plotted on the transparency by placing *A*, *B* and *C* on the transparency over *A*, *B* and *C* on *l m n o* in turn and tracing the necessary arcs, noting that for the point *B* the transparency is to be turned over before tracing the arc. In order to facilitate the plotting of *A*, *B* and *C* on *l m n o*, lines are drawn parallel to *N O* and *n o*, so that the position of *a*, *b* and *c* and *A*, *B* and *C* may be referred to the nearest corresponding lines on *L M N O* and *l m n o*, as shown in Fig. 5.

In conclusion it may be noted that the curves of the diagram have been used before for the solution of a spherical triangle for navigation purposes. The dual property of the curves and its use in the construction of the diagram however, as far as the writer is aware, have not been noticed hitherto.