ON THE ALTITUDE OBSERVATIONS OF A HEAVENLY BODY NEAR HORIZON.

by

S. OGURA, Hydrographic Engineer, Imperial Japanese Navy,

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SUMMARY:

The most serious errors in the observed altitude of a heavenly body measured from the sea horizon at sea arise from the uncertainty of the values of the dip of sea horizon and atmospheric refraction. The amount of the atmospheric refraction varies with the conditions of the atmosphere, the variation becoming smaller as the altitude of the heavenly body increases. Consequently, for the determination of a ship's position at sea, navigators as a general rule make no observation in low altitude, say lower than 6°. Recently it has been noted by navigators that the error due to uncertainty of the amount of atmospheric refraction can not be so great as was formerly supposed, provided that suitable corrections for the temperature and pressure of the atmosphere are applied. In this paper the observations by Rear Admiral S. Yonemura, I. J. N., Lieutenant A. Sone, I. J. N., and Mr. Y. Kasiwano are discussed. The limited number of these observations, 240 in total, does not permit definite conclusions to be deduced therefrom, but the author has shown that the observations at very low altitude can be utilized for the determination of a ship's position at sea, and that the amount of refraction may differ from place to place at the same temperature and pressure.

1. *INTRODUCTION.*

The most serious errors in the observed altitude of a heavenly body measured from the sea horizon at sea, arise from the uncertainty of the values of the dip of sea horizon and of atmospheric refraction. The error of dip is great, not only when the observation is made at low height of eye above the surface of the sea in great tossing of the ship, arising from the uncertainty of the height of eye, but when the conditions of the atmosphere are abnormal, especially when the difference of temperature between sea water and air at the height of the eye is great. The error of dip due to abnormal conditions of atmosphere is common to all observed altitudes of heavenly bodies, and it is treated by the present writer in the "*Suiro Yôhô*" Vol. 5, pp. 147-154, 1926. The atmospheric refraction for a given altitude of a heavenly body varies principally with the variations of temperature and pressure of the air. If the altitude is higher than a certain value, say 6°, the variation of refraction with the conditions of atmosphere is very small, but if the altitude is very low the variation of refraction is great and very uncertain. Accordingly, for the determination of a ship's position at sea, it is desirable to avoid the observations of a heavenly body in very low altitude.

For instance, the "*Admiralty Manual of Navigation*" advises observations to be made as low down in the ship as possible, say, at a height of eye of 10 feet or so, the bodies observed having altitude of at least 20°; MARGUET'S "*Cours de Navigation*" for the French Naval School and BOWDITCH'S "*Practical Navigator*", published by the U. S. Hydrographic Office, teach us to avoid altitudes less than 8° and 10° respectively. The nautical tables now widely used by navigators give tables for the correction of observed altitude higher than 5° to 7° of a heavenly body for mean state of atmospheric conditions. Some of the nautical tables, however, give the corrections of atmospheric refraction for the variations of temperature and pressure of the atmosphere. For instance, the "*Manual of Navigation*" (in Japanese) by Rear Admiral YONEMURA, and the Nautical Tables published by the Japanese Naval College (both of them for altitudes higher than 0°), the IMMAN'S Nautical Tables (for altitudes higher than 4°), the NORIE'S Nautical Tables (higher than 2°), the "*Practical Navigator*" by BOWDITCH (higher than 5°), the Nautical Tables of the Italian Navy (higher than 6°), and the Nautical Tables of the German Navy, give such corrections. The second and the English Editions of the "*New Altitude and Azimuth Tables*" published by the Japanese Hydrographic Department give the tables (Table 2 A) for corrections of the observed altitude of the Sun's lower limb for the altitudes 0° - 6°, which are composed of three parts, *i. e.* mean correction, correction for temperature and correction for atmospheric pressure. It is true that the observations of a heavenly body in low altitude should be avoided if possible, but recently certain Japanese navigators noticed that if we apply suitable corrections to the observed altitude, the observation of a heavenly body in low altitude gives a better result than was supposed hitherto, and it is evident now that the observation in low altitude is by no means worthless. The data of observations in low altitude are very scarce, and the present writer was not able to get such data, except from those observed by Japanese navigators. The following paragraphs give some results obtained by Japanese navigators.

2. *OBSERVATIONS MADE BY REAR ADMIRAL YONEMURA.*

Rear Admiral YONEMURA (later Vice Admiral), the present Director of the Japanese Hydrographic Department, had great interest in the observation of a heavenly body in low altitude and he made a great number of observations, especially in 1923 when commanding the Training Squadron, he observed the Sun in a low altitude in the South China Sea, the Australian Seas and the Sea South of Japan, and arrived at the conclusion that observations in low altitude give better results than formerly supposed. It is because of his good results that a new table, Table 2 A is added to the second and the English editions of the "*New Altitude and Azimuth Tables*". In his journey round the world in the following years he observed the Sun's low altitude in various seas and he ascertained that the observations of the Sun's low altitude can be utilized for the determination of a ship's position at sea. The following table gives the results of his observations.

OBSERVATIONS OF THE SUN IN LOW ALTITUDE BY REAR ADMIRAL YONEMURA (1923-1925).

LOCALITY.	*Mean Altitude*	*Mean of Errors*	*Maximum Altitude*	*Minimum Altitude*	*Number of Observ.*
IN A TRAINING SHIP (1)	1°45'	1'33	4°00'	0°10'	49
INDIAN OCEAN	1 38	1.73	4 50	0 50	7
RED SEA	1 35	1.10	3 00	0 13.5	6
MEDITERRANEAN SEA	1 04	1.10	2 45	0 24	8
BAY OF BISCAY TO ENGLISH CHANNEL	1 56	0.76	4 22	0 44	5
ATLANTIC OCEAN	1 25	1.02	1 25	1 09	4
PACIFIC OCEAN	0 58	1.19	2 08	0 43	10

The observed altitude of the Sun was corrected by the Table 2 A of the "*New Altitude and Azimuth Tables*".

When Rear Admiral YONEMURA visited the Hydrographic Office of the U. S. Navy in Washington, D. C., he showed the results of observations he had made during his voyage from Singapore to London to Mr. G. LITTLEHALES. The latter quoted YONEMURA's observations in the "*Hydrographic Bulletin*" of the 4th. February, 1925, and called the attention of mariners to the fact that observations of the Sun's low altitude give fairly good results.

3. *OBSERVATIONS MADE BY LIEUTENANT SONE.*

The results of observations of the Sun's altitude made by Lieutenant A. SONE, I. J. N., for the purpose of studying the error of observations in the area of "Kurosio" in winter, are given in the "*Suiro Yôhô*" (*Hydrographic Bulletin*), Vol. 5, pp. 155-158. Seven out of these observations are made in low altitudes, excluding one that made use of a ship's water line instead of the horizon. All of these observations being made in the sea where the difference in temperature of air and sea water is considerable, the present writer, after correcting the dip for temperature difference by the KOHLSCHÜTTER-BREHMER's table, obtained the following results as to the error of altitude observations ("*Suiro Yôhô*", Vol. 5, p. 153):

Maximum altitude observed	5°45'
Minimum » »	0°48'
Mean » »	3°21'
N° of observations	7
Average of errors of altitude observations	1'.4
Mean of errors (taking the sign into account)	0.0

The observations having been made in the sea where sea water temperature is considerably higher than air temperature (7°.3 to 13°.1 C), the error of

(1) Japan - South China Sea - W. Australia - Tasmania - New Zealand - New-Caledonia - Caroline Is. - Japan.

the corrected altitude of the Sun must have been far greater than in the ordinary case, owing to the uncertainty of the value of dip of sea horizon and of atmospheric refraction.

4. *OBSERVATIONS MADE BY MR. Y. KASIWANO.*

As an officer of a ship of the Kawasaki Kisen Kaisha, Mr. KASIWANO observed the Sun's low altitudes in the seas south of Japan from December 1925 to May 1926, and studied the errors of observations referring to the ship's most probable position as derived from other methods of astronomical observations or from cross bearings of terrestrial objects. The number of such observations was 13 while the ship was at anchor, 131 while steaming, being a total of 144. His results are published in the "*Bulletin of the Higher Mercantile Marine School*", Tokyo (in Japanese), N° 321 (1926).

He used Table 2 A of the "*New Altitude and Azimuth Tables*" to correct the observed altitude. With the corrected altitude thus obtained, he calculated the longitude and then compared this longitude with that of the most probable position of the ship, giving the difference between the two sign + when the longitude derived from the Sun's low altitude is towards the Sun in comparison with the other, and sign – in the contrary case. Assuming the position taken as the most probable to be quite accurate, the difference of longitudes thus obtained is equal to the error of observation of the Sun's low altitude if the Sun is in the E. or W., and it becomes greater as the azimuth from E. or W. increases. In the area in which the observations were made, the maximum rising and setting amplitudes of the Sun's azimuth being equal to 28°, the maximum value of the ratio of error of longitude to the error of altitude observations is 1.13. Accordingly, the maximum difference of the two is about 1', corresponding to the longitude error of 10', the difference being less than 0.'3 in most cases. Mr. KASIWANO gives in Table 1 his results of observations classifying them into every 1° of altitude, in Tables 2 and 3 the statistic results of his observations, and finally he describes his opinion on his results.

The difference of temperature between air and sea water as shown in KASIWANO's Table 1, being frequently conspicuous (10 times above 6° F, the maximum 11° F), the present writer applied the correction for difference of temperature to the error of longitude. In this case it is assumed that the temperature of air is measured at the height of eye (the height of temperature measurement is not given in KASIWANO's paper), and that the correction to longitude error is supposed to be equal to $+ 0'.18\ (t - t_w)$ where t and t_w are the temperatures of air and of sea water respectively in FAHRENHEIT. This corresponds to $+ 0'.33\ (t - t_w)$ if t and t_w are expressed in Centigrade, and is equal to 1' when the difference of temperature is 5°5 FAHRENHEIT. The error of longitude thus corrected is classified in the same way as in the KASIWANO's Tables 2 and 3, with the results as shown in the following tables. The error is taken as + when the longitude determined from the Sun's low altitude is towards the Sun in comparison with the other, and vice versa.

TABLE 2. ERRORS IN LONGITUDE.

Obs.	*Whole Obs.*			$\underline{\odot}$				$\overline{\odot}$			
Alt.	*N°*	*A*	*B*	*N°*	*A*	*B*	*Range of errors*	*N°*	*A*	*B*	*Range of errors*
0°	20	-3.'0	3.'1	9	-3.'6	3.'6	-10.'4 to +0.'1	11	-2.'5	2.'7	-7.'5 to +0.'8
0°-1	35	-2.1	2.2	17	-2.8	2.9	- 6.8 +0.6	18	-1.4	1.5	-4.2 +0.4
1 -2	30	-1.0	1.2	20	-0.7	1.0	- 3.6 +0.6	10	-1.5	1.7	-3.2 +0.8
2 -3	30	-0.8	1.2	14	-0.8	0.9	- 4.9 +0.7	16	-0.7	1.5	-4.1 +2.8
3 -4	18	-0.9	1.0	11	-0.6	0.9	- 2.8 +1.3	7	-1.2	1.3	-2.4 +0.3
4 -5	12	-0.2	1.1	7	-0.1	1.1	- 1.5 +1.9	5	-0.5	1.2	-2.9 +0.9

In the above table *A* denotes the mean of errors taking account of their signs, and *B* the average error not taking account of signs.

TABLE 3. ERRORS IN LONGITUDE LOCALLY CLASSIFIED.

Observ.	*Whole observations.*			$\underline{\odot}$			$\overline{\odot}$		
Altit.	*N°*	*A*	*B*	*N°*	*A*	*B*	*N°*	*A*	*B*
	South of Ogasawara Guntô (Lat. 28° N.).								
0°	16	—1.'6	1.'7	7	—1.'7	1.'7	9	—1.'5	1.'7
0°- 1	23	—1.1	1.2	10	—1.5	1.6	13	—0.8	0.9
1 - 2	25	—0.7	1.0	19	—0.6	0.9	6	—0.9	1.3
2 - 3	27	—0.6	0.9	13	—0.9	0.6	14	—0.3	1.2
3 - 4	12	—0.5	0.8	6	+0.1	0.4	6	—1.0	1.1
4 - 5	9	+0.2	1.0	6	+0.4	1.1	3	—0.2	0.7
	In the vicinity of Japan (North of 27° N.).								
0°	4	—8.'7	8.'6	2	—10.'1	10.'1	2	—7.'2	7.'2
0°- 1	12	—3.9	3.9	7	— 4.7	4.7	5	—2.9	2.9
1 - 2	5	—2.4	2.4	1	— 2.6	2.6	4	—2.3	2.3
2 - 3	3	—3.9	3.9	1	— 4.9	4.9	2	—3.4	3.3
3 - 4	6	—1.6	1.6	5	— 1.3	1.5	1	—2.4	2.4
4 - 5	3	—0.3	1.6	1	+ 1.1	1.1	2	—1.0	1.9

Mr. Kasiwano describes his experiments in the observations of the Sun's low altitude and their results as follows :

1. The error of observation of the Sun's low altitude is smaller for the upper limb than for the lower.

2. The time of contact of the Sun's limb with the horizon can be observed by the aid of a telescope and shade glass of a sextant, and the upper limb gives better results than the lower.

3. The error is greater in the seas near Japan than in the South of Ogasawara Guntô, and becomes smaller with increasing altitude, being fairly small in altitude 4°. If we assume the error permissible by a navigator to be about 1', the observations of the Sun's altitude down to 1° can be utilized for practical purposes.

4. The error is negative in most cases and its value decreases with increasing altitude.

5. The observations of the Sun in low altitudes have been considered to be dangerous by navigators. But the writer's experiments show that the observations of altitude down to 4° give good results.

6. The error in longitude for altitudes lower than 4° being negative in most cases, it is to be doubted that the first or mean correction to the observed altitude of the Sun in the Table 2 A of the "*New Altitude and Azimuth Tables*" is too small (in other words, too great in negative value). Moreover, judging from the fact that the longitude error is greater when negative in the vicinity of Japan than in the South of Ogasawara Guntô, it is to be suspected that the second correction or the correction for temperature in Table 2 A is not suitable.

7. The study on the observations of the Sun's low altitude in different seas and different seasons is required.

As is pointed out by Mr. KASIWANO, it is worth remarking that the error in longitude has negative sign in most cases, decreasing with the increase of the Sun's altitude and that negative error is remarkably greater in the vicinity of Japan than in the sea South of Ogasawara Guntô. If we pick up KASIWANO's observations in which the ship's position is determined by cross bearings of terrestrial objects, we have the following results as to the error in longitude :

Obs.	*Whole Obs.*			$\underline{\odot}$				$\overline{\odot}$			
Alt.	*N°*	*A*	*B*	*N*	*A*	*B*	*Range of errors*	*N°*	*A*	*B*	*Range of errors*
0°	5	−3'9	3.'9	3	−5'0	5.'0	−10.'4 to −0.'9	2	−2'2	2.'2	−3.'7 to −0.'6
* 0	4	−2.2	2.2	2	−2.3	2.3	− 3.7 −0.9	,,	,,	,,	,, ,,
0°−1	7	−0.3	0.7	2	−0.1	0.7	− 0.7 +0.6	5	−0.5	0.7	−2.4 +0.4
1 −2	5	0.0	0.5	4	−0.1	0.5	− 1.1 +0.5	1	+0.5	0.5	
2 −3	6	+0.2	0.4	3	+0.1	0.4	− 0.5 +0.7	3	+0.3	0.4	−0.1 +1.0
3 −4	5	−1.0	1.5	3	−0.7	1.5	− 2.8 +1.3	2	−1.6	1.6	−2.4 −0.7
4 −5	2	−0.2	1.3	2	−0.2	1.3	− 1.5 +1.1	0			

In the above table *A* denotes the mean of longitude errors taking account of their signs and *B* the mean of their absolute values. * denotes that one observation with longitude error −10'.4 is omitted.

We can see from the above table that the error in longitude is smaller and less systematic compared with Table 2. In these observations the ship's position being determined referring to terrestrial objects, the error in ship's position must be very small and the error in longitude may principally be attributed to the error of observation of the Sun's low altitude. On the other hand, in the observations in the open sea the longitude error is the sum of errors arising from the error of altitude observation and of the ship's most probable position derived from astronomical observations. As to the causes of the result that the error in longitude is smaller when the ship's position is determined by terrestrial objects than when derived from astronomical observations, we may consider the existence of systematic errors in ship's position determined astronomically and of the inconsistence in the amount of atmospheric refraction in the vicinity of land and in open sea. We need much more data of observations.

5. *ON THE TABLE FOR THE CORRECTION OF OBSERVED LOW ALTITUDE.*

Since the amount of atmospheric refraction varies irregularly with the conditions of the air, especially with those of temperature and pressure, the correction to the observed low altitude of a heavenly body is very uncertain. If we want to know an accurate value of refraction we have to know the conditions of air at different layers. In the actual case, however, at least for navigators at sea, being able to observe the conditions of the air only at the height of observer, we have to assume the conditions in the upper layers of atmosphere to vary according to some regular law. Consequently, it is inevitable that the amount of refraction calculated basing upon the conditions of the air, principally temperature and pressure, at the height of an observer does not give accurate value. The difference between calculated and actual values of refraction might sometimes reach a few minutes of arc in low altitude, decreasing rapidly with increase of altitude. The amount of atmospheric refraction has been studied by many investigators and different values are obtained for low altitudes. The Table 2 A, correction of the observed altitude of the Sun for the altitude $0^{\circ} - 6^{\circ}$, in the "*New Altitude and Azimuth Tables*", is an abridgement of the RADAU's refraction tables publishes in 1899. Most of refraction tables compiled by various authorities being principally based upon observed data on land, the values of refraction given in these tables may have systematic differences with those in the sea at different localities and in different seasons. If we can get data from a great number of observations, it will be possible to compile a better correction table for the observed altitude of a heavenly body at sea. At any rate, it is extremely desirable to get data of altitude observations of the Sun very near to the horizon made at different seas and in different seasons. Navigators are earnestly requested to supply such data of observations.