RANGE OF VISIBILITY BY DAY AND OPTICAL RANGE BY NIGHT

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I. *INTRODUCTION OF THE COEFFICIENT OF ABSORPTION IN THE THEORY OF VISIBILITY.*

So far in communications made on the theory of the range of visibility (1) [1], (2) and their verification by experiment (3), (4), (5), the question of absorption of light, *i. e.* the transformation of light into a sort of energy [(1), page 54], has merely been touched upon, for the reason that, as concerns range of visibility in daylight, it is of but little importance. In the question of ascertaining what losses of light occur as the light spreads out from a luminous source at night, it would seem that absorption should play a greater part; as far as diffusion is concerned, we will at least abstain here from considering it as the *only* effective process and from *totally* ignoring absorption. We shall therefore proceed to set forth the influence of absorption on the formula of the theory on visibility of light.

Let a be the coefficient of diffusion, b the coefficient of absorption, r the distance of an element (of volume $d\tau$) of a pyramid of visual rays from the eye, df the spacial angle within which the object appears from the eye. The increase of light $d\Phi$ ensuing from $d\tau$ will then be given by:

$$d\Phi = a.\ c.\ df.\ r^2.\ dr \frac{e^{-(a+b)\,r}}{r^2}, \tag{1}$$

where c is constant, the relationship of which to the brightness of the sky i_g and the function of diffusion Z is given by:

$$c = \int_0^{2\pi} d\alpha \int_0^{\pi/2} i_g\ (\alpha,\ \zeta)\ Z\ (\alpha,\ \zeta)\ sin\ \zeta\ d\ \zeta, \tag{2}$$

where α is the azimuth and ζ the zenith distance.

If the pyramid of visual rays be integrated with respect to the length l, we obtain as the apparent surface brightness h_s of a black surface of albedo o day with sky cloudless, lower layer regular and atmosphere horizontally homogenous:

$$h_s = \int_0^l d\Phi = -\bar{c}. \frac{a}{a+b} \Big[e^{-(a+b)\,r} \Big]_0^l = -\bar{\bar{c}} \left(e^{-(a+b)\,r} - 1\right).$$

If we pass on to $l \rightarrow \infty$, h_s must become h_h, the horizontal brightness; then, in accordance with the latter equation, $\bar{\bar{c}} = h_h$ and the formula for atmospheric light is obtained in the form

$$h_s = h_h \left(1 - e^{-(a+b)\,l}\right). \tag{3}$$

In this formula, l becomes the range of visibility s_s of the black object, when the difference of brightness between the object and the horizon is equal to the threshold of excitation of the eye ε, which, for average brightnesses to which the Weber-Fechner law is applicable, is 0.02 in round numbers. Consequently, according to equation (3), the range of visibility of the black surface is

$$s_s = \frac{1}{a+b}\ ln\ \frac{1}{\varepsilon} \tag{4}$$

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[1] *Bibliographical information will be found at the end of the article.*

From this, it is evident that the brightness of the surface of a black body in free atmosphere depends solely on the sum of the coefficients of absorption and diffusion, which means that, conversely, *from the range of visibility of defined objects, the sum of the coefficients of absorption and diffusion will be obtained from equation* (4). This *sum* is the expression which determines the attenuation of light from a luminous source by night. We are therefore able, depending on whether diffusion or absorption, or both, play a prominent part, to find, by observing the visibility of defined objects, the determining value for the attenuation of light from a luminous source by night; this value will be called the *coefficient of attenuation*

$$\sigma = a + b.$$

II. *OBTAINING THE COEFFICIENT OF ATTENUATION FROM DAILY DETERMINATION OF RANGES OF VISIBILITY.*

The photometric methods thus far employed for its determination cannot be considered at the majority of stations because of the high price of the necessary apparatus and the multifarious precautions which have to be taken in using it. Thus the problem with which we are confronted is to determine, superficially at least, the coefficient of attenuation from visual observations.

These visual determinations of the range of visibility also are subject to a set of precautions, already indicated in (3), (5), (6), (7), and which it will suffice therefore to recall briefly. It is particularly necessary that: 1. The visual angle of the sighted object utilised be not much less than 1° [(1) p. 54]. 2. Along the whole of the pyramid of visual rays employed, an upper and lower lighting of fairly marked constancy should obtain [(3) p. 11]. Hence, it should not occur, for instance, that the pyramid of visual rays be situated in the shadow of clouds, whilst other parts, on the contrary, are illuminated by the sun. In like manner, in snowy weather, the layer of snow must fill the entire lower side of the pyramid of visual rays. 3. The pyramid of visual rays are horizontal, because atmospheric disturbance varies, as a rule, very greatly with altitude. For example, it is particularly inadmissible to choose a mountain peak as the sighted object for determining the range of visibility ; the parts of the mountain situated at about the same level as the sighting point should be utilised [(1) p. 43].

It should be noted furthermore that it is advisable to choose the sighting point as high as possible above the ground, for, undoubtedly, the greatest lack of homogeneity in the air prevails near the ground, where forest, meadow, swamp or water surface each engender their special atmospheres with the corresponding atmospheric disturbance. The higher we go, the more the rather considerable differences which exist at ground level disappear, the more also it may be expected that the atmospheric layer utilised for the determination of the range of visibility will be sufficiently homogenous in the horizontal direction. The height above the surface of the earth to which it is necessary to go in order to get a satisfactory homogeneity depends on the accuracy aimed at and on the local structure of the earth's surface. At any rate, it is recommended to avoid the lower 3 metres (10 feet).

Without any further explanation it is obvious that the sighted object and its surroundings must be sufficiently well defined in their structure. Since artificial marks could not be constructed for visual observations dealt with in this article, natural marks had to be resorted to. We are indebted to F. Löhle (5), (7), for having first directed attention to the possibility of using them and for having done so.

As natural landmarks, there are available :

	Albedos according to			*Mean*
	Dorno (11)	*Stuchtey* (9)	*Lukiesh* (10)	
Dark coniferous forest	—	0.043 }	0.03 — 0.05	{ 0.04
Light coloured forest	—	0.076 }		{ —
Snow field	0.637 — 0.893	—	—	0.75
Field	0.066 — 0.077 }	0.132 — 0.156	—	0.1
Meadow	0.058 — 0.064 }		—	

a) Mark in front of the horizon.

From the remoteness of an object of albedo o situated in front of the horizon and located just within the range of visibility, the coefficient of attenuation σ may be obtained direct from the equation (4), or, more conveniently, from its graphic representation on a double logarithmic scale (Fig. 1, curve 1).

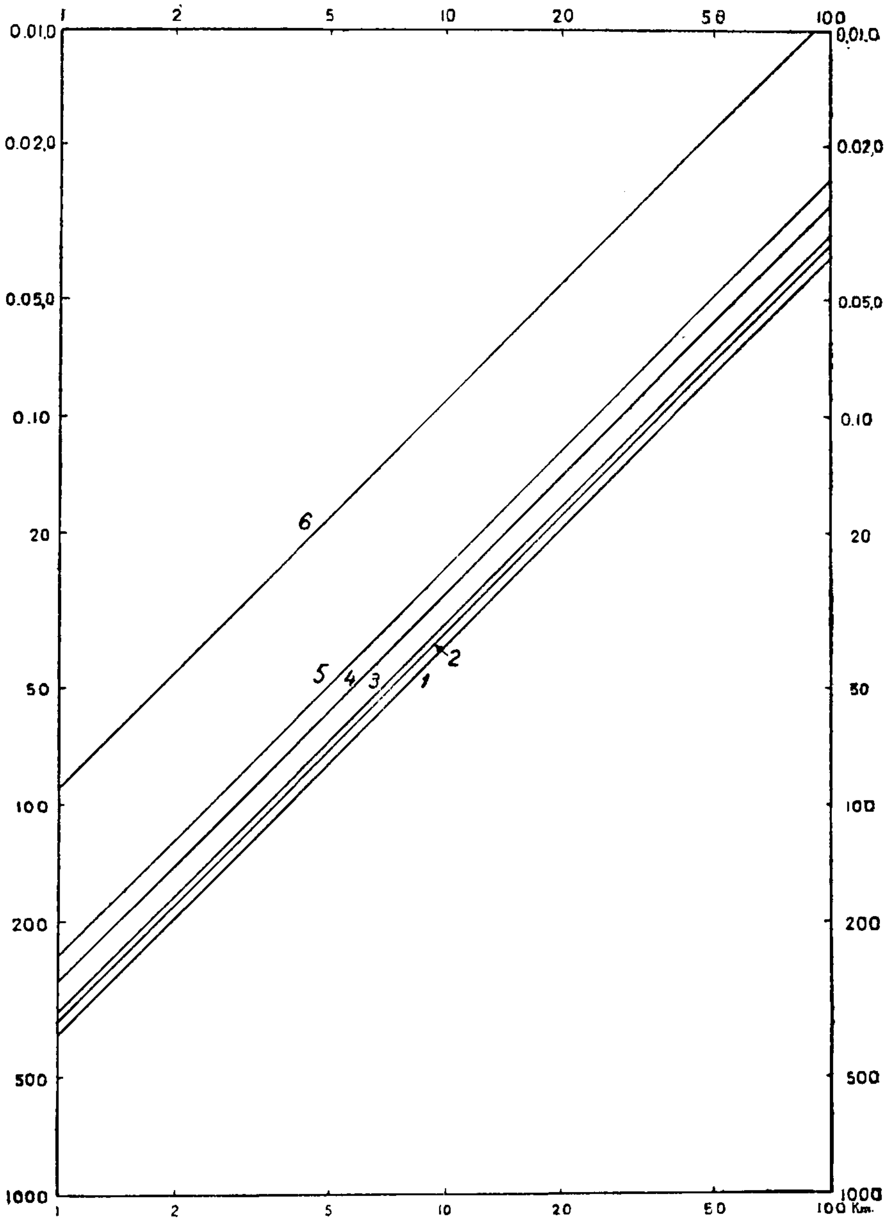

Fig. 1

Coefficient of attenuation σ as a function of the range of visibility s *of various objects (for* 1 *to* 3, *towards the sky from the horizon)* : 1. Dark coniferous forest (albedo o) ; 2. White house ($A = 0.5$) ; 3. Snow field ($A = 0.75$) ; 4. Forest ($A = 0.04$) in front of a snow field. 2 to 6 is valid for a uniformly overcast sky only, 1 is valid for a cloudless sky also.

In actual fact, it is immaterial at what azimuthal distance the mark is situated, and whether the sky be clear [1] or uniformly overcast; it is necessary only that clouds disappear in mist on the horizon.

If the albedo of a sighted object differs greatly from 0, the reflected light of the object becomes apparent and equation (3) receives an additional term which allows for the surface brightness h^o of the object. The apparent brightness h_1 of a grey object will therefore be

$$h_1 = \overset{o}{h}_1\, e^{-\sigma l} + h_h \,(1 - e^{-\sigma l}) \tag{5}$$

$\overset{o}{h}_1$ depends on the albedo A, the mark and the brightness of the sky. A simple relation between the surface brightness and that of the horizon exists only when the sky is uniformly overcast. In this case, in effect

$$\overset{o}{h}_1 = \frac{A_1}{2}\, h_h.$$

By introducing this value into equation (5), we find that, for $\frac{h_h - h_1}{h_h} = \varepsilon$, for a known range of visibility of a grey object [2], the coefficient is

$$\sigma = \frac{1}{s}\, ln \left[\frac{1}{\varepsilon}\left(1 - \frac{A_1}{2}\right)\right] \tag{6}$$

The curves 2 and 3 of Fig. 1 are graphic representations of these equations for different marks, and permit the coefficient of attenuation of marks of known albedo to be read directly in the measurement of visibility with an overcast sky and when the marks stand out on the horizon.

b) Mark in front of a forest, fields or a snowy surface.

If a mark be used whose back-ground is not formed by the celestial horizon but by a forest, fields or a snowy surface, the range of visibility of the mark is given by the apparent difference of brightness between the mark and the back-ground. For the apparent brightness h_2 of the back-ground there is the following equation which is similar to equation (5):

$$h_2 = \overset{o}{h}_2\, e^{-\sigma l} + h_h \,(1 - e^{-\sigma l}), \tag{7}$$

and when $\overset{o}{h}_2$ represents the surface brightness of the back-ground of albedo A_2 and the sky is uniformly overcast, it is given by

$$\overset{o}{h}_2 = \frac{A_2}{2}\, h_h.$$

If the mark and the back-ground are situated at about the same distance, in actual fact at the full range of visibility of the mark, by combining equations (5) and (7), we find for the coefficient of attenuation

$$\sigma = \frac{1}{s}\, ln\, \frac{1}{2\varepsilon}\Big((2 - A_2)\,(1 + \varepsilon) - (2 - A_1)\Big).$$

ε is small as compared with unity and may therefore generally [3] be neglected. The above equation can thus be simplified into

$$\sigma = \frac{1}{s}\, ln\, \frac{A_1 - A_2}{2\,\varepsilon}. \tag{8}$$

In Fig. 1, groupings are given for various cases which frequently occur in practice, and which enable the results to be treated conveniently. The details of the elements are

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(1) *When making visibility observations towards the sun, the eye should be carefully protected from direct sun-light.*

(2) *For coloured landmarks, the threshold of sensation ε is lower than for colourless (grey) landmarks, but the range of visibility is greater; in reality ε depends on the tint and depth of the colour in this case. As the exact value of ε is generally not known for coloured marks, it is advisable, as far as practicable, to choose colourless marks for observations of visibility. A dark forest, snow fields and white or grey houses may be considered as such with a sufficient degree of approximation.*

(3) *When $A_1 \backsim A_2$, ε must not be neglected.*

brought out by the figure itself. However, it should be noted that the values given for the albedo are merely approximate and, from the above table, it will be realized that often the departures from the average are fairly considerable. As the results of the measurements contain the difference of the albedos of the mark and the back-ground, equation (8), the errors due to the uncertainty of the albedos will be the more considerable the less the albedos (mean values) differ from each other, *i. e.* as the true difference of brightness between the mark and the back-ground is less; consequently it is advisable to select marks whose true brightness contrasts very greatly with the back-ground, for example, a dark forest in front of a snow field, or a white house in front of a meadow or a dark coniferous forest.

III. *COEFFICIENT OF ATTENUATION AND OPTICAL RANGE OF LIGHTS.*

We can now turn to the problem of determining the optical ranges of lights by night in relation to the coefficient of attenuation determined by day. As a preliminary hypothesis for obtaining reliable results, it is naturally necessary, on such occasion, that the state of the atmosphere be not sensibly modified in passing from day to night.

If I represents the candle-power of a light, l its distance, Φ_0 the luminous pencil emitted by the light within the angle of the aperture of the eye, Φ the pencil of light which reaches the eye, then

$$\Phi = \Phi_0 \, e^{-\sigma l},$$

and the lighting B of the pupil of the eye becomes

$$B = \frac{I}{l^2} \cdot e^{-\sigma l} \text{ (}^1\text{)} \qquad (9)$$

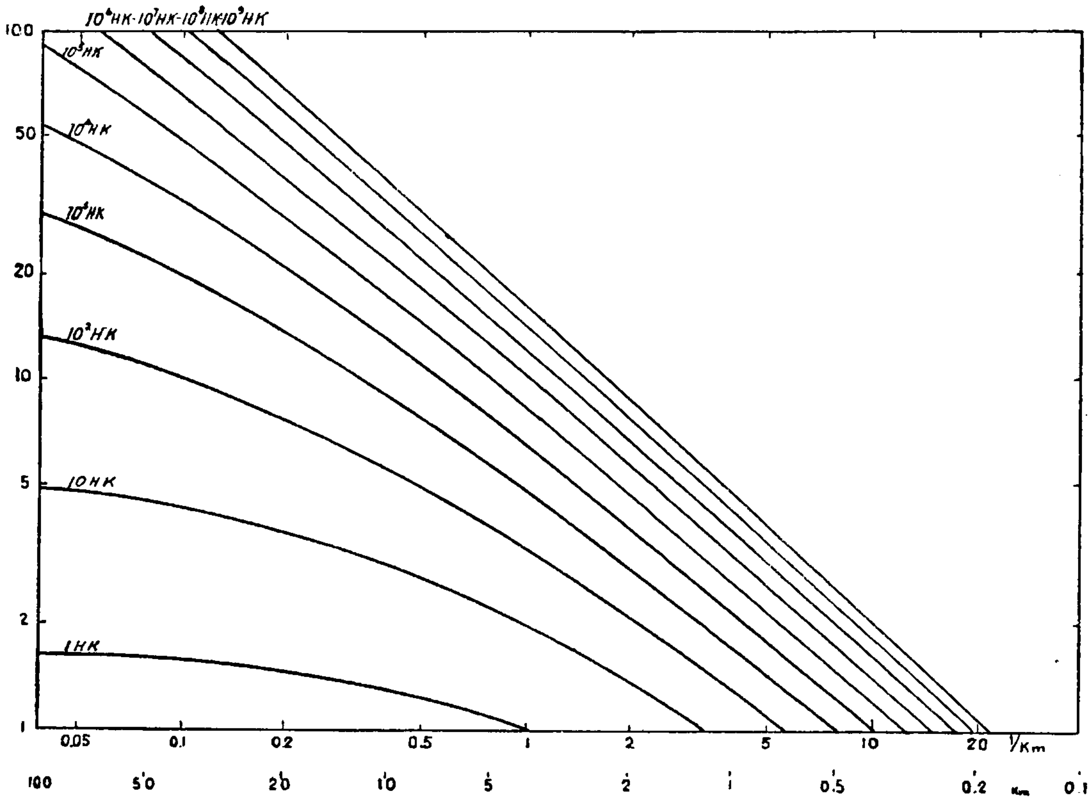

FIG. 2

Optical ranges of lights at night in relation to the coefficients of attenuation σ for different luminous values.

(1) *Equation* (10) *is identical to the* ALLARD *Formula,* $\frac{p^t}{t^2} = \frac{\lambda}{L}$, *employed in navigation, if* $e^{-\sigma}$ *be replaced by the coefficient of transmission* p.

In this relation, when the horizon is totally dark, l coincides with the optical range t of the light, when B reaches the absolute threshold value of the eye $\lambda = 3.5 \times 10^{-7}$ lux. For flashing lights, I should be replaced by the luminous value L (12), introduced by VAN BRAAM VAN VLOTEN, which contains the duration of the flashes and, in accordance with experiment, gives a smaller optical range than the corresponding maximum luminous values of the flashing light. Equation (9) thus becomes

$$\frac{e^{-\sigma t}}{t^2} = \frac{\lambda}{L} \qquad (10)$$

This equation determines for each light, *i. e.* with L constant, an unambiguous relation between the optical range and the coefficient of attenuation and may be graphically represented, as is done in Fig. 2, for lights of different luminous values.

Fig. 2 shows that, both with increasing distance and increasing coefficient of attenuation, the necessary luminous value, and thus also the candle-power, increases very rapidly, and in both cases gives a limit defined by the technical and economical possibilities of attaining the maximum candle-power.

BIBLIOGRAPHY.

(1) H. KOSCHMIEDER, Beitr.z.Phys.d.fr.Atm. 12, 33, 1925.
(2) H. KOSCHMIEDER, Beitr.z.Phys.d.fr.Atm. 12, 171, 1925.
(3) H. KOSCHMIEDER, Danziger Sichtmessungen I, Forschungsarbeiten des Danziger Observatoriums, Heft 2, 1930.
(4) H. RÜHLE, Danziger Sichtmessungen II, Forschungsarbeiten d.D.O., Heft 3, 1930.
(5) F. LÖHLE, Meteorol.Zeitschr. 1929, p. 49.
(6) T. BERGERON, Geofysiske Publikasjoner, Vol. V, N° 6, Oslo 1928.
(7) F. LÖHLE, Ann.d.Hydr. 1929, p. 327.
(8) I. RUNGE, Phys.Zeitschr. 30, 76, 1929.
(9) K. STUCHTEY u. A. WEGNER, Nacht.d.Klg.Gesell.d.Wiss. zu Göttingen 1911, p. 209.
(10) M. LUKIESH, Astrophys. Journ. 29, 108, 1919.
(11) C. DORNO, Veröff.d.Preuss.Meteorol.Inst., Abh. VI, 1919, p. 124.
(12) VAN BRAAM VAN VLOTEN, *La valeur lumineuse des Feux à éclats*, Scheveningen, 1926 (reprint).