JOHANN HEINRICH LAMBERT

BIOGRAPHY BY H. MAURER.

Genius is not confined to any particular social class, and it makes its own way in the world even under the most unfavourable circumstances.

A marvellous example is given by the life of the great mathematician, astronomer and philosopher, Johann Heinrich LAMBERT. Descendant of a Palatine family, he was born on 26th August, 1728, the eldest of the children of a poor tailor of Mulhausen in Alsace. At that time this town belonged to Switzerland. The information given below is taken from the book (*) by Professor D. HUBER of Basle, which was published on the occasion of the centenary of the birth of LAMBERT. This centenary was celebrated in 1828 by the formal unveiling of a monument at Mulhausen commemorating the life of the scientist. The book contains a biography written by the Protestant Pastor GRAF of Mulhausen, a tribute to LAMBERT as a philosopher by Professor S. ERHARDT of Heidelberg, and a tribute to his memory as a mathematician and physicist by HUBER himself.

LAMBERT'S scholastic days ceased at his twelfth year and he was then compelled to work in his father's tailor-shop, in order to help in supporting his younger brothers and sisters. But the child, eager to learn and whose parents even lacked oil for the evening lamp, used to read at night by the light of the moon or of candles which he obtained by selling drawings, while the others slept. At school he had already learned calligraphy, latin and a little geometry; his most prized possession was a book on the science of calculation which he had received from one of his father's customers. From this he learned the calculation of the ecclesiastic calendar and was even able to discover mistakes in the calculations. On the basis of the knowledge acquired from this book he propounded such intelligent questions to the workmen engaged in the wretched shop of his father, that one of them gave the young boy a second volume on arithmetic and geometry. Gradually the attention of other persons was drawn to this infant prodigy. A certain Professor ZÜRCHER gave him free instruction in French and the ancient languages and the mayor's clerck REBER procured for him a position as recorder in the mayor's office, later as accountant in a metal factory in Sept, and finally with the Professor of Law ISELIN, at Basle. The latter permitted LAMBERT to attend his own lectures on law, and also gave him the time to study for himself. LAMBERT never attended other lectures but, on the other hand, he read all kinds of books: on philosophy by WOLF, MALLEBRANCHE, LOCKE, and on mathematics including algebra and mechanics.

Being later recommended by ISELIN, LAMBERT became in 1748 tutor to the nephew of the Count DE SALIS and two young men who were related to him. He

(*) Johann Heinrich LAMBERT, nach seinem Leben und Wirken aus Anlass der zu seinem Andenken begangenen Secularfeier in drei Abhandlungen dargestellt, published by D. HUBER, Prof. of Mathematics, Basle, 1829.

J.-H. LAMBERT
1728 - 1779

instructed them in the catechism, languages, geometry, military architecture, geography and history, and meanwhile he himself studied physics, meteorology, mathematics, astronomy, mechanics, metaphysics and rhetoric from the books in the library of the house. He also perfected himself in the German, French, Italian, Latin and Greek languages. As early as 1749 he outlined the plan of his *Lettres cosmologiques*. In 1750 he commenced his meteorological observations. In 1753 he surveyed the region of Coire and then became a member of the Society for Mathematics and Physics at Basle. His first printed work appeared in the Proceedings of this Society in 1755.

Beginning in 1756, LAMBERT undertook several scientific voyages with his pupils. These took him to Göttingen, where he became a corresponding member of the Scientific Society, to Utrecht and the Hague where he published his book on light track in 1758, and to Paris, where he made the acquaintance of D'ALEMBERT, to Marseilles, Nice, Turin and Milan. In the years which followed we find him in Zurich, Augsburg, Munich, Erlangen and Leipzig, entering everywhere into close relations with the scientific bodies in those places. In 1764 he arrived in Berlin where since 1761 he had been a corresponding member of the Academy, and where, eventually, he was to settle down. His first audience with FREDERICK THE GREAT took the following curious turn :-

THE KING : "Good evening, sir ! Will you kindly inform me which of the sciences you have studied in particular ?"

LAMBERT : "All."

THE KING : "You are also a very able mathematician ?"

LAMBERT : "Yes."

THE KING : "Which professor instructed you in mathematics ?"

LAMBERT : "Myself."

THE KING : "You are then a second Pascal ?"

LAMBERT : "Yes, Your Majesty."

Upon which the King returned to his chamber laughing and later, at table, he made the remark that they had proposed for his Academy one of the greatest fools he had ever seen. But about three quarters of a year later, the King expressed his opinion as follows :- "In judging this man one should consider only the breadth of his vision and not the minor details." He made him titulary member of the Academy with honorary grants, placing him with EULER and three other scientists on the Directing Committee of the Academy, and nominated him Chief Counsellor for Construction, for the General Inspection of the Public Works of the realm. With regard to the latter nomination, LAMBERT made the following remark to the royal Ministers :- "Your Excellencies need not think that I shall revise and correct the ordinary memoranda for the construction works. You may employ clerks for this work if you do not wish to attend to it yourselves. I shall not meddle with questions which concern everybody, as that would be a waste of time for me. If, however, you find difficulties which you cannot solve, then you may call upon me."

We here see LAMBERT at the apogee of his activity, in active intellectual relations with EULER, LAGRANGE, NIKOLAI, ERMAN, KANT, MENDELSSOHN and BODE. LAMBERT brought about the nomination of BODE as an astronomer at the Berlin Observatory and instigated him to calculate, for many years, the

Berlin Ephemerides. If, nowadays, one is inclined to regard LAMBERT above all as a representative scientist in mathematics and physics, in his day he was also considered a great philosopher whose imposing work, the *Novum Organon* was greatly admired. The high regard in which he was held by KANT is shown by the latter's remark that he would not pass on any proposition which did not conform absolutely to the judgment of LAMBERT. "If it is not possible to win his assent it is impossible to base this science on incontestable premises."

One can do no better than adhere to the order of precedence which LAMBERT himself assigned to the mathematicians of his generation :- "In the first rank stand EULER and D'ALEMBERT ; in the second, LAGRANGE, who will soon equal the first two ; I myself am in the third rank."

The mind of LAMBERT was constantly engaged in scientific matters without regard for external conditions. In order to study the laws of the reflection of light he simply walked into the finest café of Berlin, drew his sword and commenced his studies by making all kinds of movements in front of a large mirror, without taking any notice of the public, who certainly must have considered him a lunatic.

The engraving by ENGELMANN, reproduced here, after a drawing by VIGNERON, gives a good idea of his physiognomy. The face is intellectual and benevolent, and this incited LAVATER to write his work on physiognomics. LAMBERT was a devout Christian and attended church assiduously ; his standpoint is contained in his confession of faith :- "If Christianity had no mysteries I should doubt it. It would be a poor principle to refuse to believe in anything we cannot conceive ; a thing which we are compelled to do daily in so many other matters."

It is characteristic that this same man should think so highly of logical-mathematical reasoning, to which he sought to subordinate everything. If he sought to make general reasoning into a system of mathematics, in which ideas should appear as magnitudes, he demanded that, in morals and in art, the magnitude of every good thing should be comparable with others ; or in other words made measurable. He advocated the establishment of a scientific language comprising 106080 syllables, such that the sense of each word might be logically recognised according to its syllabic construction. At the age of 49 the vital forces of this great man had been exhausted as a result of his incessant labour and following a disease of the respiratory organs an attack of apoplexy put an end to his days on 18th September, 1779.

The prodigious diversity of LAMBERT'S labours in the mathematical and physical sciences is revealed in the amplitude of the bibliographical index given at the conclusion of this article. Those works which are of particular interest in the field of activity of the International Hydrographic Bureau are indicated by the bold characters of the numerals giving the order of the works. The following remarks might be of interest.

Among the *astronomical* works we find alongside of the famous work *Lettres cosmologiques*, in which he describes the system of the Milky Way of the universe in a manner generally accepted even to-day, numerous memoranda on comets (at the age of 16, LAMBERT sought to calculate the path of the comet of 1744), and on the planets ; in particular, the presumed satellite of Venus and the reciprocal perturbations of Jupiter and Saturn captivated his imagination. The publication of the Berlin Astronomical Ephemerides was due to his initiative; he contributed to the collection of astronomical tables which the Berlin Academy published in 1779 in three volumes. Science in recognition of its gratitude gave the name of LAMBERT to one of the craters of the Moon.

Pure Mathematics is indebted to the methods of LAMBERT which increased the convergence of series, for his arrangement of tables of divisors of numbers, for his method of interpolation as well as his work on the fundamental principles of the calculation of probabilities, and for the *Lambert series*, which stimulated LAGRANGE and LAPLACE in the elaboration of their theories on the development of functions in series. His works on geometry were numerous, both on the laws of perspective and on the quadrature and rectification of curves; they also comprise works which were epoch-making for cartography. He also worked on slide-rules and compiled logarithmic and trigonometrical tables.

Physics owes to him the fundamental principles of photometry, the LAMBERT law of cosines, the conception of the *Albedo*, and his research into the loss of light by reflection. Very justly it has given the name "Lambert" to an absolute unit, that of surface illumination. In the theory of heat, the "Pyrometry" of LAMBERT already shows the beginnings of the notion of specific heat and his pyramid of colours is the prototype of the "colouring body" of to-day.

From the point of view of *Geophysics*, we should note the importance of his efforts to serve *Meteorology*: his experiments on the graphical representation of the meteorological elements over a period of time, the variation of the barometric height with the seasons and the movements of the moon, the calculation of the average wind over an interval of time and attempts to develop hygrometry. From the standpoint of *terrestrial magnetism*, it is interesting to note a memorandum by LAMBERT on the point of intersection of the isogon of 15° in Africa on the isogonic chart for the year 1770. This point probably corresponds to the one which is now in the Atlantic, from which point the magnetic variation increases towards the North and South and decreases towards the West and East.

The astounding mass of work accomplished by this mathematical genius which applies directly to the affairs of everyday life, may be readily seen from the list of his works. Among them we find articles on lighting apparatus, ink and paper, wind-mills and water-mills, four-wheeled vehicles, beds for invalids and bellows.

For *geographers* and *seamen*, LAMBERT's cartographic work was extremely fruitful. We find these principally in the memoirs entitled: *Anmerkungen und Zusätze zur Entwerfung der Land und Himmelskarten* (Notes and addenda on the projection of terrestrial and celestial charts). In this memorandum LAMBERT compares the different projections with each other and adds other important ones of his own. These may be briefly classified as follows:-

For the *equidistant azimuthal projection*, which is more often attributed to POSTEL, but which had been employed even before by Gerhard MERCATOR, LAMBERT calculated the tables giving the azimuth and the distance from the centre of the chart for every five degrees of latitude and longitude.

The *equivalent azimuthal projection* originated with LAMBERT himself. The law of the radius in this is $\rho = 2r \sin \frac{\delta}{2}$, in which r is the radius of the sphere and δ the angular distance from the centre of the chart.

Among the cylindrical projections the *equivalent projection* was invented by LAMBERT. For a position of the axis coinciding with the axis of the earth, the law for the scale at each meridian, for latitude φ, is $y = r \sin \varphi$.

For a transverse position of the axis (axis of the cylinder perpendicular to the axis of the earth) it is called the *Lambert transverse isocylindric projection.* LAMBERT calculated a table for the rectangular coordinates x and y, from the formulae :-

$$\cot \frac{x}{r} = \cot \varphi \cos \lambda \quad ; \quad y = r \cos \varphi \sin \lambda \ .$$

The *conformal cylindrical projection* was also described by LAMBERT, under the name *Lambert conformal cylindrical projection*, for a transverse position of the axis. Whereas, where the axis of the cylinder lies in the axis of the earth, it represents the MERCATOR projection, the equations for the coordinates for the transverse position (assuming $r = 1$) become :-

$$\cos \varphi \sin \lambda = \frac{e^y - e^{-y}}{e^y + e^{-y}} \quad ; \quad \cot \varphi \cos \lambda = \cot x \ .$$

The loxodromes (rhumbs), which on MERCATOR'S chart are straight lines, are not so on this projection.

In the ordinary *conical projection*, in which the axis is oriented in line with the earth's axis, the meridians become straight lines at angles $u = m \lambda$, where λ is the difference in longitude and m is a constant, while the parallels of latitude and usually the pole also, are the arcs of concentric circles. LAMBERT has also proposed an *equivalent projection* of this type (also called the isospheric stenoteric conical projection) in which the pole is represented by a point.

Besides the equation $u = m \lambda$ for the lie of the meridians, the law of the radius may be applied to each meridian :

$$\rho = \frac{2r}{\sqrt{m}} \sin \left(45 - \frac{\varphi}{2}\right);$$

r is the radius of the sphere and φ = geographic latitude.

If the angles are to be conformal throughout the length of the parallel of latitude φ_o, we should take :

$$m = \cos^2\left(45 - \frac{\varphi_o}{2}\right)$$

LAMBERT also developed the formulae for the ellipsoid :- for a semi-major axis A and the eccentricity ε we have :-

$$\rho = 2A \sqrt{\frac{1-\varepsilon^2}{m}} \; sin\left(45 - \frac{\varphi}{2}\right)\left[1 + \frac{\varepsilon^2}{3}(1 + sin\,\varphi + sin^2\varphi)\right]$$

For the *conformal conical projection*, also discovered by LAMBERT, the formulae to be applied are :-

$$u = m\lambda \,; \quad \rho = a\left[tang\,\frac{p}{2}\;cot\,\frac{p_o}{2}\right]^m ; \quad k = \frac{m\rho}{r\,sin\,p}\,.$$

Here ρ is the distance of the apex of the cone for the polar distance p; p_o a constant, the original polar distance, r the radius of the sphere, m and a are constants and k is the scale ratio between the chart and the sphere for the polar distance p.

If the cone is to touch the sphere at the polar distance p_o we must take:

$$m = cos\;p_o\,; \qquad a = r\;tan\;p_o\,.$$

We can also determine m and a in such a manner that $k = 1$ on two parallels of polar distances p_1 and p_2. We should then have :-

$$m = \frac{log\;sin\;p_1 - log\;sin\;p_2}{log\;tg\;\frac{p_1}{2} - log\;tg\;\frac{p_2}{2}}\,.$$

The chart may then be considered as having been constructed for a cone which cuts the sphere at the parallels at polar distances p_1 and p_2, but also for a cone tangent to the sphere at the circle having a polar distance of p_o = arc $cos\;m$. The scale constant $\frac{a}{r}$ only is differently conceived in the two cases.

For $p_o = 0$ the cone becomes reduced to the plane tangent to the pole and the projection becomes stereographic in accordance with the laws :-

$$u = \lambda\,; \quad \rho = 2\,r\;tg\,\frac{p}{2}.$$

For $p_o = 90°$, it becomes the MERCATOR projection. The complicated equations for the conformal conical projection for the ellipsoid will not be cited here.

The *Lambert conformal conical projection* represents the general case of a conformal projection with the meridians as straight lines; LAMBERT pointed out also the still more general case of the conformal projection in which the meridians and the circles of latitude are orthogonal circles (including the limiting case, in which they become straight lines). Ordinarily this projection is called the projection of LAGRANGE who published a description of it in the

Memoirs of the Academy of Berlin in 1781; but it had already been reported by LAMBERT in 1772.

In this projection the meridians form a group of circles passing through the representations of the two poles and intersecting each other at angles $m\lambda$, where λ is the difference of longitude. The representation of the circles of latitude is provided by the corresponding group of orthogonal circles. In a system of rectangular coordinates (x, y) of which the origin is one of the poles and the axis of the ordinates is the central meridian, the following formulae apply :-

$$x = \frac{-A \sin m\lambda}{N}; \quad y = \frac{A \cos m\lambda + B \cot^m\left(45 + \frac{\varphi}{2}\right)}{N};$$

$$N = A^2 tg^2\left(45 + \frac{\varphi}{2}\right) + 2AB \cos m\lambda + B^2 \cot^m\left(45 + \frac{\varphi}{2}\right)$$

in which A, B and m are constants; $1/B = 2b$ is the distance of the representations of the poles on the rectilinear meridian $\lambda = 0$. If we take $A/B = \alpha$, the coordinates of the centre of the circle representing the meridian λ are $y = b$; $x = b \cot m\lambda$ and its radius $\rho = b \operatorname{cosec} m\lambda$. The coordinates of the centre of the circle representing the parallel of latitude φ are

$$x = 0, \quad y = \frac{2b}{\alpha^2 tg^{2m}\left(45 + \frac{\varphi}{2}\right) - 1},$$

$$\text{and its radius } \rho = \frac{2\alpha b\, tg^m\left(45 + \frac{\varphi}{2}\right)}{\alpha^2 tg^{2m}\left(45 + \frac{\varphi}{2}\right) - 1}.$$

If the parallel φ_0 is to be a straight line, then we must have

$$\alpha = \cot^m\left(45 + \frac{\varphi_0}{2}\right).$$

b, φ_0 and m are therefore still available.

$\varphi_0 = 0$ gives the charts with a rectilinear equator. If $m = 1$ the result is a transverse stereographic projection; while with $m = 1/2$ a chart is produced which gives the whole world conformal within a circle.

If we assume that one of the pole is moved to infinity; *i. e.*, if we assume $B = 0$ and $A = 1$, we obtain the conical conformal projection of LAMBERT with rectilinear meridians and, in its limiting cases, the MERCATOR projection in one case and, in the other, the stereographic polar projection. We see therefore in the *Lambert projection with groups of orthogonal circles* a form which is the general origin of conformal projections. The LAMBERT groups of orthogonal circles are introduced into other conformal charts of a quite different nature also. In the LITTROW (1833) conformal chart, where the meridians and the parallels of latitude form a system of hyperbolae and homofocal ellipses, in which all the straight lines are lines of equal azimuth, and of which the identity with the curves of the WEIR azimuth diagram was proved by the author of this article in 1905, the system of arcs of great circles passing from the centre of the chart, through points on the equator 90° apart, as well as that of the horizontal circles described about them, is a system of groups of LAMBERT orthogonal circles, as shown by A. WEDEMEYER in 1918. Consequently the LITTROW chart is also a chart with the LAMBERT system of circles but conceived from another point of view.

More than 200 years have passed since this great genius appeared on earth. Even to-day we are filled with admiration for what this absolutely self-taught man was able of himself to produce during his short eixstence. Endowed with the faculty of submitting practically all questions in which he was interested to mathematical treatment and to plumb them by means of systematically ordered thought, we find him in the most varied fields, and always as a pioneer in new scientific conceptions.

We do not wish to close this short sketch of the life of this great man without pausing an instant to consider his magnanimity. Always taking for granted the benevolence of the Creator, he advanced the audacious hypothesis, in his *Lettres cosmologiques*, that the comets of our system must be inhabited by happy creatures. In accordance with the motto of creation recognized on earth "As much life as possible", the existence in the solar system of as many comets as there are stable trajectories, the number of which LAMBERT calculated as approximately 12,000, must be assumed. This conception corresponds to the maxim which he wrote in his own hand which we find below the appended portrait. The translation thereof is as follows :-

"On our earth, organic bodies are, among all others, those which are created most abundantly and most easily...... Everything in this world for which the means are most abundantly provided, must be considered as being part of the scheme of creation..."

The inscription on the monument of LAMBERT at Mulhausen is :-

JOHANNES HENRICUS LAMBERT,
natus Mulhusii 26 Aug. 1728, denatus Berolini 25 Sept. 1777.

Dem durch Selbsttätigkeit entwickelten grossen Geiste. (*)
Ingenio et studio
Sa cendre repose à Berlin, son nom est écrit dans les Fastes d'Uranie (**)

(*) To the great spirit developed by his own efforts.
(**) His ashes repose at Berlin, his name is written in the immortal records of Urania.