NOTES ON SUBMARINE PHONOTELEMETRY

(*SOUND RANGING*)

by

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(See *Hydrographic Review*, Vol. IV, N° 2, November 1927, page 139 ; Vol. V, N° 1, May 1928, page 91 ; Vol. VI, N° 1, May 1929, page 111).

1. Let (Fig. 1) A, B, C, be the known positions of three hydrophones receiving a signal emitted from a source of sound at P. Let t_a, t_b and t_c be the instants of arrival of the signal at A, B, and C. Let $\tau_{ab} = t_a - t_b$ and $\tau_{ac} = t_a - t_c$, *i. e.* the differences between the times of arrival of the signal at A and B, and at A and C respectively.

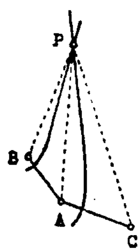

Fig. 1.

Assuming that the sound is propagated with the same velocity in all horizontal directions about the point P, the intervals τ_{ab} and τ_{ac} are proportional to the differences in the distances $PA - PB = \delta_{ab}$ and $PA - PC = \delta_{ac}$. Let v be the actual velocity of sound, then :-

$$\delta_{ab} = v.\ \tau_{ab} \quad , \quad \delta_{ac} = v.\ \tau_{ac}.$$

Each of the *parameters* δ_{ab} and δ_{ac} determines one branch of the hyperbola (of which the foci are located respectively at A,B and A,C), the geometric locus of the source of sound P. In the preceding Note (1) it was decided to call this locus the *phono-hyperbola*. The intersection of the two phono-hyperbolas determines the point P. In reality, and strictly speaking, the actual value of the velocity of the sound is not known ; an approximate value only can be assumed and thus the field of possible variations is restricted. This being granted the velocities may be given the various values v', v'', v''',... comprised between the above-mentioned limits of variation, the pair of phono-hyperbolas suited to each of these may be determined and the

(1) See *Hydrographic Review*, Vol. V, N° 1, May 1928 : *Submarine Phonotelemetry*, para. 27.

points P', P'', P''',... may be found pertaining to the curve defined by the general formula :-

$$1) \quad \frac{\delta_{ab}}{\delta_{ac}} = \frac{\tau_{ab}}{\tau_{ac}} = \text{constant.}$$

In other words, this curve is *the geometrical locus of the points on the plane for which the relations between the distances to three fixed points in this plane, taken in pairs in a given order, has a given constant value.*

Consequently, the measure of the intervals τ_{ab} and τ_{ac}, on which the problem of phonotelemetry is based, exactly determines not the position of the source of sound, but only the *locus of position* of this point. It may be added that the determination of the position from an approximate value of the velocity of sound is generally sufficient for all practical purposes; but it is also of interest to ascertain the line along which the point would be displaced as a result of a small variation in the velocity of sound. To obtain this valuable information it is not necessary to construct the curve by points as indicated above, this solution being neither rapid nor elegant.

It is but necessary to construct the tangent to the curve at the point P which is determined from an estimated value of the velocity of sound and to assume, as appears permissible, that in the vicinity of P the tangent to the curve may be taken as practically coinciding with the curve itself. The tangent may be constructed, without recourse to differential calculus, by using an elementary geometrical method. The solution obtained in this manner is certainly more rapid and simple than that which analysis gives. I would not venture to deny that the study of the curve defined by the relation 1, of its equation and of its properties may not be interesting (and this not only for the solution of problems of phonotelemetry), but I must confess that I refrained from the attempt in the hope that some mathematician might undertake this.

Remarks. — There are two special cases which should be treated separately and for which, however, the solution is immediate :-

a) When the instants of arrival of the sound signal at two of the three hydrophones coincide, that is, when the source of the sound is located at equal distances from the two hydrophones. In this case, the locus of the point P corresponding to various values of the velocity of sound, *is the bisector of the angle formed by the two equal distances*, or, if preferred, *the perpendicular erected at the centre of the straight line which joins the two equidistant hydrophones.*

b) When the signal arrives simultaneously at the three hydrophones, or, in other words, when the source of sound is equidistant from the three points. In this case the position of the point P is determined independently of the velocity of the sound, which means that, by definition, it coincides with the centre of the circle on the circumference of which the three hydrophones are located.

2. Let one of the three distances which separate the point P from the given hydrophonic stations be chosen *arbitrarily*, for example the distance AP (Fig. 2), and let this distance be d_1, let d_2 and d_3 be the other two distances.

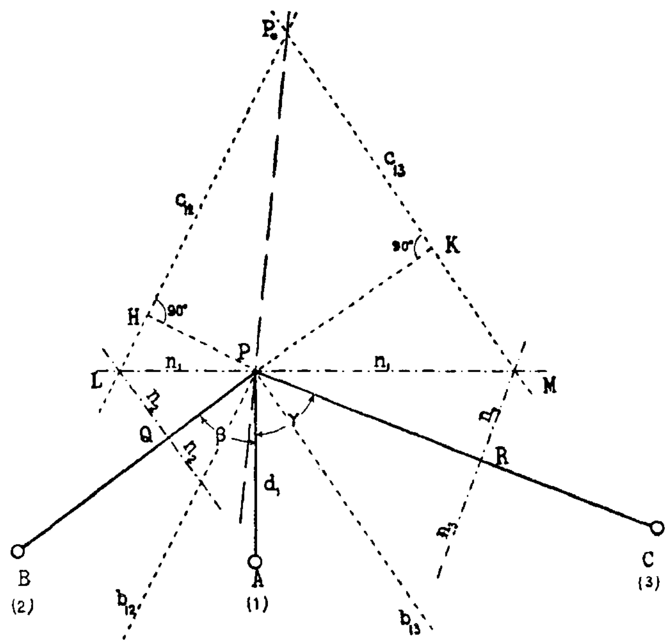

Fig. 2.

Let the distance d_1 on d_2 and d_3 be laid off, from the points B and C respectively, in the direction of the point P.

Let b_{12} be the bisector of the angle contained between d_1 and d_2 ; b_{13} the bisector of the angle between d_1 and d_3.

From the point P draw the straight line n_1 perpendicular to d_1 ; from Q the straight line n_2 perpendicular to d_2 and from R the straight line n_3 perpendicular to d_3. Through the point L, the intersection of n_1 and of n_2, draw the straight line c_{12} parallel to b_{12}, and through the point M, the intersection of n_1 and n_3, draw the straight line c_{13} parallel to b_{13}. Let P_0 be the point of intersection of c_{12} and c_{13}. The straight line $P\,P_0$ then coincides with the tangent sought.

3. Proof : That which was demonstrated in the preceding Note (1) must be recalled here :

1) The phono-hyperbolas which correspond to the values $\delta_{12} = v.\ \tau_{12}$, $\delta_{13} = v.\ \tau_{13}$ of the parameters and which define the point P by their intersection, coincide in the vicinity of the point P with the bisectors b_{12} and b_{13}.

2) The phono-hyperbolas corresponding to the values :-

$$\delta'_{12} = (v+dv)\ \tau_{12} \quad , \quad \delta'_{13} = (v+dv)\ \tau_{13}$$

of the parameters, and which are obtained by assuming the value of het velocity of sound to be $v + dv$, differing extremely little from v, are straitgh

(1) See *Hydrographic Review*, Vol.V, N° 1, May, 1928, loc. cit. para. 27 set eq.

lines parallel to b_{12} and b_{13}, differing respectively therefrom by the quantities

$$2)\quad \begin{cases} e_{12} = \dfrac{dv.\ \tau_{12}}{2\sin\dfrac{\beta}{2}}\ , \left(\beta = \widehat{APB}\right) \\[2ex] e_{13} = \dfrac{dv.\ \tau_{13}}{2\sin\dfrac{\gamma}{2}}\ , \left(\gamma = \widehat{APC}\right) \end{cases}$$

In fact the quantities $dv.\ \tau_{12}$ and $dv.\ \tau_{13}$ are the infinitesimal increments of the parameters due to the infinitesimal increment dv which was assigned to the value of v.

The direction of the lateral displacements e_{12} and e_{13} is obtainable from very simple considerations: a *positive* increase of dv produces an *increase in the absolute value of the parameter* (difference in distance) *and consequently a displacement, or a parallel sliding, of the hyperbola* (*bisector*) *towards the nearest focus* (*hydrophone*). The contrary occurs when the increment dv is *negative.*

This being granted, it should be noted that the construction of the tangent at P is equivalent to the construction of the point P' which is infinitely close to P, or, in other words, obtained by assuming an increment dv to the velocity and, consequently, giving the parameters of the two hyperbolas the infinitesimal increases $dv.\tau_{12}$ and $dv.\tau_{13}$ respectively. The straight line PP' is the tangent required.

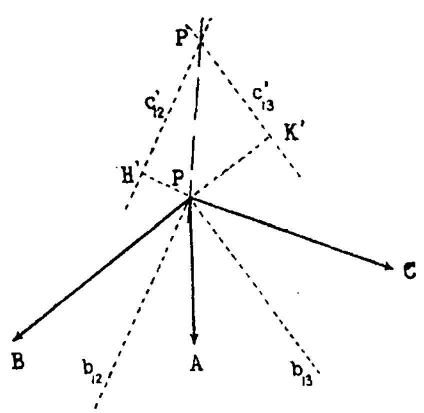

Fig. 3.

In the case under consideration, if v be given an infinitesimal negative increment dv, the phonohyperbolas corresponding to the new values of the parameters are (Fig. 3) the straight lines c'_{12} and c'_{13} respectively parallel to b_{12} and b_{13}, of which the distances PH' and PK' to these bisectors are given by formula 2. The intersection of c'_{12} and c'_{13} determines the point P'.

Let the infinitely small quadrilateral $PH'\,P'\,K'$ thus formed (Fig. 3) be compared with the quadrilateral of finite dimensions $PHP_{a}K$ in Fig. 2, *i. e.*

that formed by the straight lines c_{12}, c_{13} and by the perpendiculars PH and PK dropped from P onto these straight lines. The two figures are homothetic, that is, similar and similarly placed.

It is a fact that the angles at P, H', P' and K' of the infinitely small quadrilateral are equal to the angles at P, H, P_0 and K in the quadrilateral of finite dimensions. Besides, it is easy to establish the following equations, see figure 2.

$$PH = \frac{PQ}{2 \sin \frac{\beta}{2}}, \qquad PK = \frac{PR}{2 \sin \frac{\gamma}{2}},$$

but $= PQ = v.\ \tau_{12}$, $PR = v.\ \tau_{13}$; and consequently,

$$3) \qquad PH = \frac{v.\ \tau_{12}}{2 \sin \frac{\beta}{2}}, \qquad PK = \frac{v.\ \tau_{13}}{2 \sin \frac{\gamma}{2}}.$$

Also (Fig. 3) :

$$4) \qquad PH' = \frac{dv.\ \tau_{12}}{2 \sin \frac{\beta}{2}}, \qquad PK' = \frac{dv.\ \tau_{13}}{2 \sin \frac{\gamma}{2}}.$$

The equations 3 and 4 show that the sides PH and PK of the finite quadrilateral are proportional to the sides PH' and PK' respectively of the infinitely small quadrilateral. Finally, it should be noted that the sides PH and PK coincide respectively with PH' and PK'. Thus the homotheticity of the infinitely small figure and the finite figure is evident. P is the centre of homotheticity. This demonstrates the coincidence of the straight line PP_0 with the straight line PP' or, in other words, with the tangent to the curve at point P.

4. Influence of errors in the positions of hydrophones on phonotelemetric determinations. — This question was not dealt with in the preceding Note.

Let A,B, represent the exact positions of the foci of the hyperbola ; let P be the source of sound. Let us consider (Fig. 4) a *small* displacement AA' at

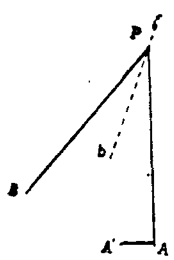

Fig. 4.

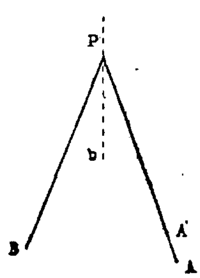

Fig. 5.

Fig. 6.

right angle to the straight line AP, due to imperfect knowledge of the position of the hydrophone A. This displacement will not introduce any variation in the parameter (difference in distance) but the phonohyperbola, which coincides with the bisector of the angle APB, now coincides with the bisector of $A'PB$. As a result a rotation of the geometric locus of position is produced about point P; when the error AA' is very small relative to the distance AP, this rotation is extremely slight, and consequently *negligible.*

Secondly let a displacement AA' along the straight line AP be considered (Fig. 5). Such displacement causes a variation of equal amount in the value of the parameter. In other words, the phonohyperbola constructed on the foci A and B coincides with the hyperbola constructed on the foci A' and B, on condition that $v.\,\tau_{ab}$ is taken as the parameter of the first and $v.\,(\tau_{ab} + \theta)$ as the parameter of the second; in this θ is the time required for the sound to cover the distance $A'A$. The error in position of the hydrophone A may therefore be corrected by applying a correction to the instant t_a at which the sound is perceived at point A, this correction evidently being equal to the time required for the sound to cover the distance $A'A$. Without this correction the parameter will be in error by the quantity $A'A$ and in consequence will produce a *parallel* displacement of the phonohyperbola (bisector of the angle APB) equal to

$$\frac{\overline{A'A}}{2 \sin \frac{\beta}{2}} \quad , \qquad (\beta = \widehat{APB})$$

An error of position of the hydrophone in any direction other than AP or its normal, may be resolved into its orthogonal components (Fig. 6), Aa and $A'a$ along these two directions. The reasoning given above applies to each of these components. Hence it is concluded *that a small error in the position of the hydrophones does not influence the geometrical locus of position except when it affects the distance of the hydrophone from the source of sound.* Taking the figure of 1,500 metres per second as the mean value for the velocity of sound in sea-water, it will be found that an error of one mètre and a half in the position of the hydrophóne, along the line joining the hydrophone with the source of sound, has the same influence (not taking the sign into consideration) as an error of one thousandth of a second in the determination of the instant of arrival of the signal at the hydrophone under consideration; *an error of 15 metres* will produce the same effect as an error of one hundredth of a second, etc. This line of reasoning applied in the case of hydrophone A applies equally to each of the other hydrophones.

These considerations allow the measurement of the time intervals to be adapted to the accuracy with which the positions of the hydrophones are known. Tı it is considered possible to measure the time interval to within one thousandth of a second, the order of magnitude of the permissible error in the position of the hydrophones should not exceed 1.5 metres. An error in position of, or greater than, 15 metres is compatible with an accuracy in the time interval of one hundredth of a second, etc...

Finally, the following important point must be noted :- *The conclusions just enunciated are applicable not only to the determination of position by phono-hyperbolas, but also to all other phonotelemetric determinations (radio acoustic measurements of distances or of relative distances).* The reader will readily understand this.

5. All the geometrical problems in connection with phonotelemetry are solved *on a plane,* that is, on a flat representation of the earth's surface. It should be noted, however, that the substitution of the plane for the earth's surface is permissible only if, within the limits of the area under consideration, the geodetic lines are represented by straight lines (or very nearly straight lines) and if the distances and the angles are unchanged (or at most with negligible differences only). These conditions are not all satisfied by MERCATOR'S cylindrical projection *except in one special case,* that is, on the equator, away from which the only condition which is fulfilled is that of conformity, which is insufficient in itself. This projection should therefore not be used in other latitudes. All of the conditions are fulfilled by LAMBERT'S conformal conical projection (1) based on the middle latitude of the area under consideration (or at least on a parallel close to this latitude). Other projections might also be used ; however, preference is given to LAMBERT'S because, amongst numerous other advantages, it possesses that of being very easy to construct.

6. Before concluding this brief note on phonotelemetry we should like to take this opportunity to justify our insistance in treating such a question. This insistance is, in fact, in contrast (in spite of several brilliant exceptions) to the limited favour accorded to, and rare use made of, these methods, up to the present, in hydrographic surveys. The most prominent of these exceptions is well known. The whole world knows of the very extensive use of radio-acoustic methods made by the Coast and Geodetic Survey of the United States of America. In the report of the Director of this survey we read, in connection with the recent survey of Georges Bank, which covers an area of about 15,000 square miles, about 200 miles off the coast of New England, most explicit and significant statements with regards to the utility of phonotelemetry.

> "The employment of *echo sounding* and *radio acoustic range finding* methods making practical use of the accurate timing of sound in water, coupled with the recent acquisition of the ship *Oceanographer,* will make it possible, despite handicaps and hazards, to accomplish this survey with an accuracy and dispatch that was impossible five years ago."
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> (*Annual Report of the Director, United States Coast and Geodetic Survey, for the Fiscal Year ended June 1930, page 4*).

Further, it is probable that the phonotelemetric methods are employed in practice in Great Britain. In fact, certain paragraphs of the *Admiralty Manual of Navigation 1928,* (Vol. 1, Chapter VIII, p. 82, *et seq.*) are devoted

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(1) See *Hydrographic Review,* Vol. VII, N° 1, May 1930 : *Notes on Practical Hydrography.*

to a description of the methods of sound ranging (*Submarine sound ranging ; radio acoustic sound ranging*) (1). The fact that this subject has been dealt with in an essentially practical manual, published nine years after the report made on the same subject by the British Delegation to the International Hydrographic Conference of London (2) shows, if I am not mistaken, that phonotelemetric methods have not by any means fallen into disuse.

These important exceptions having been noted (without taking into account several other insignificant trials), it may be said that phonotelemetric methods have met with no favour in Hydrographic Services. This may possibly be explained on the grounds that, up to now, these methods have required very costly apparatus, very delicate instruments and a staff of specialists. It is realized that Hydrographic Services have not always got abundant staff and funds !! (3). It is considered, however, that, in the present state of technique (and in particular of that of radio-electricity), methods and instruments might be considerably simplified. We are convinced that, by working in this direction, results of great practical importance could be attained. It is with the aid of submarine phonotelemetry only (and, if I may be permitted to say so, certainly not with the *present-day* methods of radiogoniometry) that a good survey of a continental shelf may be made, because it is only in this manner that the desired accuracy in the positions of the soundings can be obtained in places far from shore where the ordinary visual methods of geodetic surveying cannot be employed. It is only by means of phonotelemetry that the positions of certain important dangers can be determined ; these, at present, are poorly located on charts because the positions are imperfectly determined by inadequate methods, the extreme elasticity of which all we hydrographic surveyors fully realise... That is the reason for our special insistance : *Gutta cavat lapidem.*

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(1) At the time when the preceding Note was published (*Hydrographic Review*, May 1928), this new edition of the *Admiralty Manual of Navigation* was completely unknown to us. However, we have found nothing new therein with respect to the subject of the communication made to the London Conference.

(2) See : *Proceedings of the International Hydrographic Conference*, London 1919 : *Sound ranging. - Report of the British Delegation*, page 168 et seq.

(3) I should like to suggest to our hydrographic colleagues a ruse which seems to me certain to succeed :- that they attempt to arouse the interest of the maritime defence services (the Submarine Service, Communications Service and the Artillery), by reminding them that phonotelemetry underwent its greatest development and found its most interesting applications in the course of the World War. These services are not often subjected to difficulties in balancing their budgets and generally have abundant personnel and means for special purposes at their disposal.