MEASURES FOR THE SETTLEMENT OF REPORTS OF DOUBTFUL DANGERS IN THE OCEAN

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This discussion bears a relationship to Special Publication N° 20 of the International Hydrographic Bureau, entitled "General List, arranged by Oceans of Shoals of Doubtful Existence and of Shoals the Positions of which are Doubtful or Approximate", especially with regard to those instances in which the shoal is located in the open ocean and must, therefore, be the culmination near the surface of the sea of a submerged mountainous formation. The International Hydrographic Bureau has been diligent in setting forth the present state of information respecting these dangers to navigation, often including an estimate of the degree of doubt or approximation to be attributed to the assigned geographical position and the nature of the evidence upon which the validity of the report is to be received, and, furthermore, stating the surveys and searches which have occasionally cleared away the doubt of existence by establishing definite geographical location of the danger and thus narrowing, from the preceding extended state, the tract where the exercise of caution is demanded in navigation. Other instances there are in which searching examination has disproved the existence of the reported danger, and yet others which remain in a state of inconclusiveness because of the evidence bearing upon the discovery and the nature of the operations which have been conducted in search of them.

There is little chance of finding a reported shoal in the open ocean by directing the course of the searching vessel to the geographical position assigned in the report and endeavouring to rediscover the shoal by passing in its vicinity with lookouts posted aloft or with a sounding-line suspended in the depths. Unsuccessful proceedings of this kind have sometimes been cited as evidence in disproof, and it is, therefore, of interest to arrive at an estimate of the expectation that would thus be justified.

Supposing that the assigned latitude and longitude are subject to extreme errors of a miles in longitude and b miles in latitude and that the extent of the shoal has a radius of r miles. What probability of finding the shoal, by attempting to proceed to the geographical position defined by the assigned latitude and longitude has a navigator who can determine his geographical position at sea within the same limits of extreme error as the original determination is subject to ?

If the discoverer should revisit the shoal a great number of times and should deduce the latitude and longitude under like circumstances at each visit, the latitudes would all differ from the true latitude, and, likewise, the

longitude from the true longitude. If we call the differences between the true latitude and each deduced latitude errors of latitude, and lay them off, according to their algebraical signs, to the right and left of an assumed origin, and then, corresponding to each of these abscissae, erect an ordinate of a length proportional to the probability of that error, these ordinates and abscissae will be coordinates of the probability curve. And, in like manner, if the errors in longitude were found and plotted in conjunction with their probabilities, a similar curve would be developed.

In this investigation, the probability curve, ordinarily represented by LAPLACE's formula, $y = ce^{-a^2x^2}$, will be replaced by two equally inclined straight lines AB and AB' as shown in figure I.

This substitution introduces a law of errors constituting a departure from the law of errors upon which the method of least squares is founded and causes the results to be reached in this solution to differ by an appreciable but extremely small amount from those which would be reached by the employment of LAPLACE's formula. This, however, will have no practical significance when it is considered that, from the nature of the calculations about to be made, absolute precision is not to be sought.

The probability of having an error between $OC = x$ and $x + \Delta x$, to the right of the axis OS, is equal to $s.dx$. As, in this case, OB and OB' measure the extreme errors, all possible errors are comprised between zero and OB, and zero and OB'; and the sum of all the elements which are singly represented by $s.dx$, or the area of the triangle ABB', should be equal to unity, which is the measure of certainty. The equation to the straight line AB will be, calling a the extreme error OB and p the intercept on the axis of s,

$$\frac{s}{p} + \frac{x}{a} = 1 \qquad \text{or} \qquad s = \frac{a - x}{a^2}.$$

And, since x can only vary between zero and a, the probability of having an error between x and $x + \Delta x$ is

$$p_1 = \frac{a - x}{a^2} dx \qquad\qquad (1)$$

Fig. I

Fig. 2
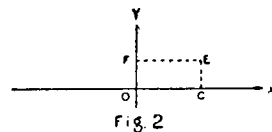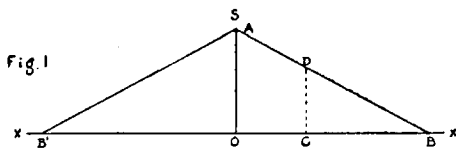
The causes which produce the grouping of a number of deduced geographical positions around the true one are of two kinds: one tending to place the deduced latitude to the north or south of the true latitude, and the other tending to place the deduced longitude to the east or west of the true longitude. So that a particular deduced geographical position E will be the result of having an error OF in latitude and OC in longitude.

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The probability that the geographical position deduced by the discoverer occupies a certain position with reference to the true geographical position of the sounding is, therefore, easily stated. Through the true geographical position of the sea-mark conceive two rectangular axes, OX and OY, to be passed as shown in figure 2. Upon the former let errors in longitude be measured, and, upon the latter, errors in latitude. The position E, of which the coordinates are x and y, results from the concurrence of two conditions, the error of x miles in longitude and the error of y miles in latitude. The probability p_1 of the error x is, as shown in equation (1),

$$p_1 = \frac{a - x}{a^2}\, dx$$

and in like manner, the probability p_2 of the error y will be

$$p_2 = \frac{b - y}{b^2}\, dy \qquad\qquad (2)$$

In these formulas, a and b represent respectively the extreme errors in longitude and latitude in miles.

The probability p of having, at the same time, the error x and the error y, or of deducing the geographical position E as the position of the sounding, will be the product of p_1 and p_2, or

$$p = \frac{(a - x)\ (b - y)}{a^2\, b^2}\, dx.\, dy \qquad\qquad (3)$$

an equation in which x can vary from zero to a, and y from zero to b. It is, therefore, applicable to the first right angle of the axes OX and OY, but, in order to make it applicable to the other quadrants, it is only necessary appropriately to change the signs of x and y.

Therefore, p is the expression of the probability that the geographical position reported for the sounding by the discoverer will be in error x miles in longitude and y miles in latitude, and, if we take one mile as the unit of measuring in denoting the coordinates of geographical position, the integration of (3) between the limits of x and $x + 1$ and of y and $y + 1$ will give the probability of having an error between these limits. The expression for this is :

$$p = \frac{(x - a + \frac{1}{2})\ (y - b + \frac{1}{2})}{a^2\, b^2} \qquad\qquad (4)$$

And the probability that a searcher will find the shoal is the probability that the happenings will so concur that his calculations will bring him within a circle whose radius is r miles and whose center is removed x miles in longitude and y miles in latitude in directions reversed to the directions of the errors of the geographical position to which the searcher is directed by the assigned coordinates. To find the probability P of coming within any portion of the rectangle of extreme errors inclosed by the circle whose radius is r and whose center is the true position of the shoal, it is sufficient to integrate the expression (3) between limits depending only upon the equation to the circle $x^2 + y^2 = r^2$.

For the first quadrant, the integral is

$$P = \frac{1}{a^2 b^2} \int_0^r (a - x) \, dx \int_0^{\sqrt{r^2 - x^2}} (b - y) \, dy$$

and for the whole circle

$$P = \frac{4}{a^2 b^2} \int_0^r (a - x) \, dx \int_0^{\sqrt{r^2 - x^2}} (b - y) \, dy \tag{5}$$

and since y must satisfy all points of the circle, the first integration will be between the limits of zero and $\sqrt{r^2 - x^2}$; that with reference to x will be between the limits of zero and r. The integration is effected by ordinary methods, and is found to give:

$$P = \frac{2r^2}{ab} \left(\frac{\pi}{2} - \frac{2r}{3a} - \frac{2r}{3b} + \frac{r^2}{4ab} \right) \tag{6}$$

And the probability of having the concurrence of the reverse errors of the latitude and longitude assigned by the discoverer in order to come within this circle would be the product of P and p, whose expressions are given in (4) and (6).

This result relates merely to the sole undertaking of attempting to proceed directly to the assigned position, and does not take account of any extension of measures of search beyond that alone.

When evaluated with numerical quantities within the range which would be appropriate to a, b, x, y, and r, it appears that there would not be one chance in hundreds of finding the shoal by thus performing the search for it.

The uncertainties that may surround reports of such discoveries, the conflict of testimony in regard to them, the costliness of search for them, and the verification or disproof of their existence are reflected in the history of the so-called Reed or Redfield Rocks.

For fifty years mariners continued to believe, or to be unable to disbelieve, in the existence of dangerous rocks in the Pacific Ocean, in a position about seven hundred miles westward of San Francisco. In 1850 Captain REED, of the brig *Emma*, reported that he had seen two rocks, measuring from six hundred to nine hundred feet long, and from two hundred and fifty to four hundred feet broad, and having a depth of about three fathoms of water over them, in latitude 37°24' North and longitude 137°22' West Greenwich; and in 1851 a report came from the U. S. sloop-of-war *Falmouth* confirming the existence of rocks in this geographical position. Captain REDFIELD, of the whaler *Susan Abigail*, also reported in 1856 that he had discovered some rocks, with about ten feet of water over them, in a geographical position eleven miles to the northward of the geographical position stated by Captain REED; and in 1866 rocks were again said to have been seen by Captain CAVE, whose report assigned to them a position one mile to the northward and five miles to the eastward of the geographical position agreed upon by the original authorities. Commander FRANKLIN, of the U. S. S. *Mohican*, reporte d in 1870 that the office of the harbour-master at Honolulu contained records showing

that the ships *Kuttosoff* and *Eliza Kimball* had found these rocks, which by that time had come to be known as Reed or Redfield Rocks, and that the latter ship had anchored in five fathoms of water there, and described three of the rocks as extending considerably above the surface of the water.

In the meantime the bark *What Cheer* had, in 1858, passed over the locality in which the rocks were reported to exist, and her master stated that he could discover no evidence of their existence, and in 1865 the master of the bark *Live Yankee* had made a similar report; and, although it was well understood, on account of the uncertainty in the reckoning of these vessels and the margin of error inherent in the position assigned to the rocks, how slight a foundation these reports afforded in disproof of the existence of what had been seen by six apparently reliable and careful shipmasters, and by five of them examined and described, yet they served to stimulate investigation. So that in the years immediately afterwards we find that searches were made by the U. S. S. sloop-of-war *Kearsarge* in 1870, the U. S. Coast Survey schooner *Marcy* in 1871, the U. S. Coast Survey steamer *Hassler* in 1873, and the U. S. S. *Narragansett* in 1875. All of them, however, employed the inconclusive method of proceeding to the reported locality of the rocks and crusing back and forth over the region, with lookouts posted aloft and a certain amount of deep-sea sounding-line hanging over the side of the ship. The four vessels just named must have spent altogether 31 days in this way, and the total distance which they cruised back and forth over the locality is about 2,800 miles.

The reconnaissance made by the *Kearsarge* took place on a beautifully clear day in September. The ship had proceeded from Honolulu, where the chronometers had been verified, and had every facility for determining the accuracy of her position in the field of search; but, although the range of view extended to a radius of ten miles around the ship as she steamed back and forth, visiting in turn each of the positions in which the rocks had been reported, the observed conditions did not reveal the slightest indication of the proximity of shoals or rocks, either by the breaking of the waves or by the presence of birds or fishes.

The examination made by the *Narragansett* took place on a day in June which was most favourable for the work, as the atmosphere was remarkably clear, and there was a heavy swell that would have caused the sea to break in a depth of five or six fathoms; but the lookouts that were posted aloft and on the deck of the vessel during the day's search could discover no indications of the reported danger.

In the search by the *Marcy* the surface conditions were examined in great detail from July 23 to August 5, under favourable circumstances of wind and weather, and with a swell large enough at all times to cause the sea to break in a depth of three fathoms. During this search the distance run by the vessel was one thousand and fifty miles, of which seven hundred and seventy miles were made in daylight, when breakers or any discoloured water should have been visible within at least one mile. A constant lookout was maintained from the masthead, but no breakers were seen, nor any indications of shoal water, such as the presence of gulls, seals, fish or kelp. The water was uni-

formly of a deep blue, and at various places the deep-sea sounding-line was lowered to depths varying from three hundred to eighteen hundred fathoms without touching bottom.

The site of the reported dangers was again and still further examined between the 24th of May and the middle of June, 1873, by the U. S. Coast Survey steamer *Hassler*, but without finding any feature corresponding to Reed or Redfield Rocks. The steamer ran fourteen hundred miles in traversing the region in the vicinity of the reported geographical position. Lookouts were constantly aloft, but no indications of shoal water could be discovered.

It is plain that the force of evidence in regard to the existence of the Rocks, proceeding from these four searches, depended almost altogether upon the event that the observers did not see them. A doubt accordingly remained which was strengthened from time to time by the accounts of incoming mariners, who asserted that they had seen these dangers. In 1888, the master of the ship *British Yeoman* reported that he had seen these rocks in passing them; and in 1889 the master of the schooner *Una* reported that he had observed a peaked rock standing four feet above the surface of the sea in latitude 37°24' North and longitude 137°26' West, saying: "There is not any room for doubt as to this being a rock, but the position may vary a little on account of the sights not being extra good."

The whole question was finally set at rest by sounding down to the bottom with plummet and piano-forte wire throughout the rectangle of extreme errors reasonably assignable in latitude and longitude to the average of the reported geographical positions. Notwithstanding the curious accordance among them, seeing that their range was only seven minutes of arc in longitude and seventeen minutes in latitude, the area searched by deep-sea soundings was marked out by assigning extreme errors of $\pm$ 60' in longitude and $\pm$ 40' in latitude.

Theoretically, the shape of an isolated submarine peak would be that of a solid of revolution in which the resistance to crushing of any horizontal section is equal to the combined weight of the portion of the formation above that section and of the superincumbent body of water.

Let y denote the radius of any horizontal section and z its distance from the top of the formation. Let K denote the coefficient of resistance to crushing of the material composing the formation; w, the weight of a unit of its volume; and w', the weight of a unit volume of sea water.

Accordingly, $\pi w \int y^2 dz =$ the weight of the formation above any section whose distance from the top is z, $2 \pi w' \int y.z.dy - \pi w' \int y^2 dz =$ the weight of the water upon the formation above any section whose distance from the top is z, assuming the top of the formation to reach to the surface, and $\pi K y^2 =$ the strength of any section to resist crushing. Then

$$\pi w \int y^2 \, dz + 2 \pi w' \int y.z.dy - \pi w' \int y^2 dz = \pi K y^2 \qquad (\text{I})$$

By differentiation, equation (1) becomes

$$\pi\, wy^2 dz + 2\,\pi\, w'y.z.dy - \pi\, w'.y^2.dz = 2\,\pi\, K.y.dy \qquad (2)$$

which expresses the condition that the increase of strength of any section in excess of that of the section next above is equal to the sum of the increases of the weight of the formation and the weight of the water upon any section in excess of their combined weight imposed upon the section next above.

Letting S denote the area of any horizontal section whose radius is y, and dS, the differential of S, equation (2) may be written in the following forms :

$$w.S.dz + w'.z.dS - w'S.dz = K.dS$$

$$(w - w')\; S.dz = (K - w'z)\; dS$$

$$\frac{dS}{S} = (w - w')\,\frac{dz}{K - w'z} = \frac{w - w'}{w'}\,\frac{dz}{\dfrac{K}{w'} - z} \qquad (3)$$

By integration, equation (3) becomes

$$Log\; S = -\,\frac{w - w'}{w'}\; Log\left(\frac{K}{w'} - z\right) + C$$

in which C is the constant of integration.

$$\text{Hence} \qquad Log\left(\frac{K}{w'} - z\right) = \frac{w'}{w - w'}\,C - \frac{w'}{w - w'}\; Log\, S$$

$$\text{or} \qquad \frac{K}{w'} - z = \frac{e^{\frac{w'}{w-w'}\cdot C}}{e^{\frac{w'}{w-w'}\cdot Log\, S}}$$

In the absence of knowledge of the value that should be assigned to K, the coefficient of resistance to crushing, this equation has been used in the generalized form:

$$A - z = \frac{B}{e^{\frac{w'}{w-w'}\cdot Log\, S}} = \frac{B}{e^{\frac{1.03}{2.67-1.03}\, Log\, S}} = \frac{B}{2.71828^{\,1.446\, log\, S}}$$

to find the equation of their average form from the observed bathymetrical data on Seine Bank in latitude 33°50' N. and longitude 14°20' W., Cocos or Keeling Island in latitude 12°06' S. and longitude 96°53' E., Enderbury Island in latitude 3°10' S. and longitude 171°10' W., Funafuti Atoll in latitude 8°25' S. and longitude 179°07' E., Taviumi Bank in latitude 12°05' S. and longitude 174°35' W., and the shoal near Midway Island in the North Pacific Ocean in latitude 28°00' N. and longitude 177°40' W.

For this purpose the values of z and y, expressed in nautical miles, were inserted in the above equation, and a conditional equation was formed for each pair of coordinates relating to each of the submarine formations. From these conditional equations normal equations were found by the method of least squares, which gave the values of the constants A and B. The resulting equation is

$$1.87 - z = \frac{1.87}{2.71828^{1.446 \, log \, \pi y^2}}$$

and the corresponding curve, which by revolution around the vertical axis would generate the average form, is shown in figure 3, together with others which have been plotted for purposes of comparison from measured data.
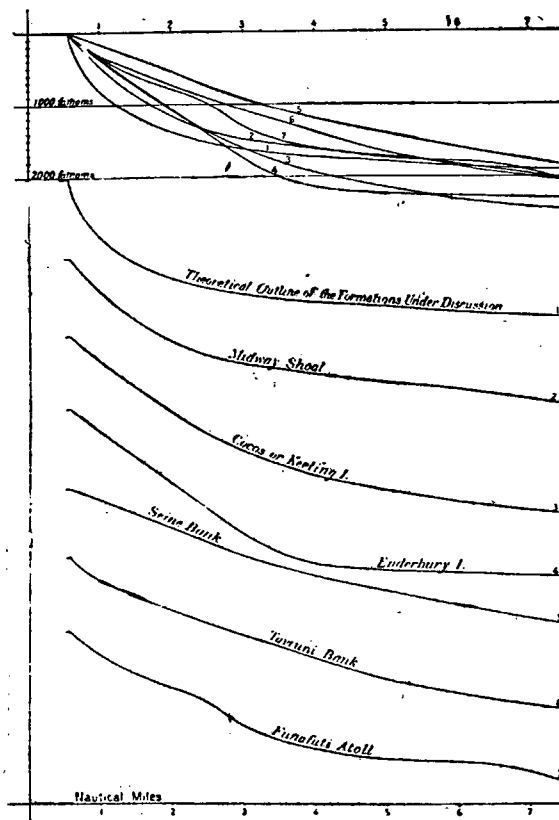

Fig. 3

It is of interest to extend the comparisons given in fig. 3 by those that were made by the late Captain AULT, notwithstanding their different appearance on account of the exaggeration of the vertical scale of his diagram, figure 4. They were made during the last cruise of the ill-fated non-magnetic ship *Carnegie*, for the purpose of comparing the outlines of the structure of

Midway Island and of Wake Island with the theoretical outline of submarine peaks. (*)
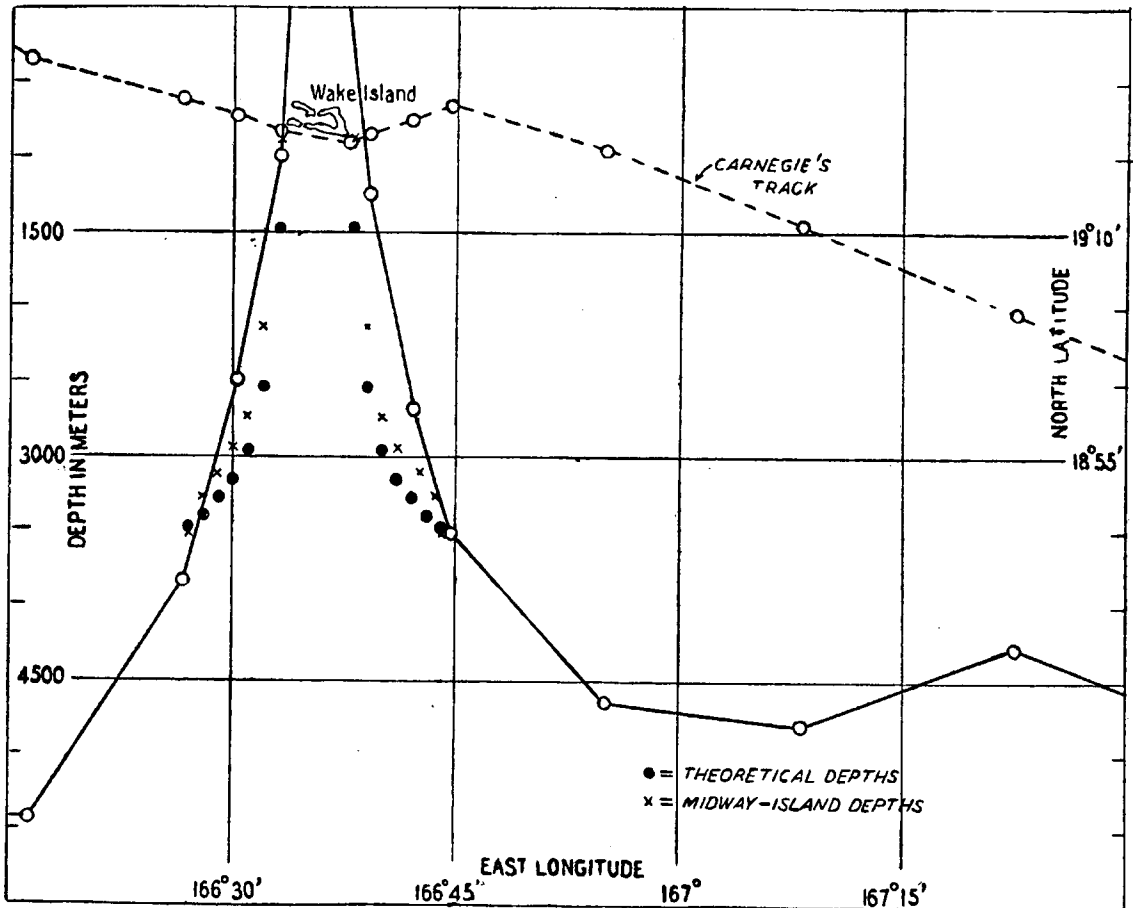

Figure 4

(*) *FORM OF THE SLOPE OF WAKE ISLAND.* — This island is one of the most isolated peaks in the North Pacific Ocean. It is in latitude 19°19' North and longitude 165°36' East of Greenwich, and is four miles long by two miles wide. Since it is only 21 feet high and is located along the route for vessels going between Honolulu and Guan, it constitutes a very great danger to navigation. While the *Carnegie* sailed past it on May 11, soundings were taken with the sonic depth-finder, both in approaching and leaving the island, at intervals of about every three miles near the island.

The profile as developed from these soundings is very similar to that of Midway Island (1), both for the eastern slope and for the western. It is not so steep as Midway for the first 2,500 metres, but is steeper from 2,500 metres to 3,500 metres. The following are the radii of the cone for various depths: at surface, 2 nautical miles; at 1,000 metres, 2.7 miles; at 2,000 metres, 3.5 miles; at 3,000 metres, 6.7 miles; at 4,000 metres, 11.1 miles.

<div style="text-align:right">

Signed : J. P. AULT,

at sea, on board the *Carnegie* (May 14, 1929).

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(1)   The theoretical form shown is from LITTLEHALES.

The concluding search for Reed or Redfield Rocks, which was made by the U. S. S. *Wheeling* and the U. S. S. *Ranger*, with intervals between the soundings gauged by the necessary minimum of the dimensions of the orographical feature whose presence or absence it was intended to disclose, demonstrated that there is no submarine mountain culminating near the surface of the ocean in this region, and, with equal force, that there never were any rocks here nor any shoal upon which a vessel could have anchored.

Within the rectangle of extreme errors attributed to the latitude and longitude of the reported locality, 88 deep-sea soundings were measured — 59 by the U. S. S. *Wheeling* ranging in depth from 2,850 to 3,352 fathoms, and 29 by the U. S. S. *Ranger* ranging in depth from 2,776 to 3,097 fathoms. The distribution of these measurements, at alternate intervals of 8 and 2 nautical miles, was such that, if there existed any isolated submarine peak composed of materials constituting the earth's crust and shaped with sufficient strength to resist crushing under the stresses of its own weight and the weight of the superincumbent ocean, its presence would have been disclosed.