NOMOGRAPHIC DETERMINATION OF THE INCLINATION CORRECTION OF SOUNDINGS TAKEN BY MEANS OF A FISH LEAD (PLOMB POISSON) [1]

by

M. A. GOUGENHEIM, Ingénieur Hydrographe Principal, French Navy.

---

1) In an article entitled "On the Curvature of the Lead Line & the Correction for its Inclination" (*Hydrographic Review*, Vol. IV, N° 2) Captain Tonta compared the corrections for inclination obtained theoretically by him, by M. Courtier, Ingénieur Hydrographe en Chef, and by Professor de Marchi and showed that, in the three hypotheses considered, the correction for inclination k may be reduced to a single formula:

$$(1) \qquad k = l\,[1 - \psi(\alpha)] + C_0 B(\omega)\,[\psi(\omega) - \psi(\alpha)]$$

in which l represents the length of the submerged line,

α its angle from the vertical at the point at which it enters the water,

ω its angle from the vertical at the point at which the lead is attached,

and C_0 a constant of the lead and wire system, approximately measuring the length of the arc of the curve of the sounding line produced from the lead to the apex of this curve (with a horizontal axis).

Besides the angle ω is related to the measurable quantities α and l by the following formula:

$$(2) \qquad f(\omega) = A\,f(\alpha) \qquad \text{by taking} \quad A = \frac{C_0}{C_0 + l}.$$

The constant C_0 is determined by measuring, at the same speed of the vessel, the inclinations of the line ω, α_1 and α_2 corresponding to the lengths of line submerged 0 (lead awash), l_1 and l_2.

$$\text{Then} \quad C_0 = \frac{l_2 - l_1}{f(\alpha_2) - f(\alpha_1)}\,\frac{f(\omega)}{B(\omega)}.$$

---

(1) This note may be considered as a generalisation of the results of an investigation of the problem which we have made, in the particular case where the lead line is assumed to take the form of an arc of a circle. This first examination, made with a purely practical object, will appear in the *Annales Hydrographiques* issued by the Hydrographic Service of the French Navy.

2) By formula (1), the correction is not determinable nomographically, but as the second term of k is small relative to the first, and as ω is smaller than α, which itself very rarely exceeds 30 degrees, and if the functions of ψ, f and B be taken as a basis, formula (1) may be simplified and can be expressed at first thus:

$$(1\ bis) \qquad k = [1 - \psi(\alpha)]\left[l + C_0 B(\omega)\left(1 - \frac{1-\psi(\omega)}{1-\psi(\alpha)}\right)\right].$$

By developing the functions of ψ, f and B relative to α and ω, in series, and limiting ourselves to the α^4 terms for $\psi(\alpha)$, the α^3 terms for $f(\alpha)$ and the ω^2 terms for $B(\omega)$:

$\psi(\alpha) = 1 - a\alpha^2(1 + b\alpha^2)$, in which a is always equal to $\frac{1}{6}$;

$f(\alpha) = \alpha(1 + c\alpha^2)$;

$B(\omega) = 1 + m\,\omega^2$.

Eliminate ω between (2) and (1 *bis*) by means of these developments.
Then:

$$(3) \qquad k = [1 - \psi(\alpha)]\left[l + C_0(1 - A^2)\,[1 + (b + m - 2c)A^2\alpha^2]\right]$$

Find the order of importance of the term

$$(4) \qquad [1 - \psi(\alpha)]\,C_0(1 - A^2)A^2(b + m - 2c)\alpha^2.$$

$A = \frac{C_0}{C_0 + l}$ being between 0 and 1, the maximum of $A^2(1 - A^2)$ is $\frac{1}{4}$.

On the other hand, $1 - \psi(\alpha)$ is of the order of $\frac{\alpha^2}{6}$.

Consequently, an upper limit of $\frac{C_0(b + m - 2c)\alpha^4}{24}$ must be found.

If α does not exceed 30 degrees, α^4, expressed in radians, will attain $\frac{1}{16}$ at most.

C_0 theoretically may be infinite, with an infinitely heavy lead or an infinitely thin sounding line; but in this case, α is always very small and $C_0\,\alpha^4$ is of the order of one metre.

In practice C_0 is less than 20 metres, which is the maximum which we shall consider.

The quantity $b + m - 2c$ is, at most, equal to $\frac{1}{2}$ in absolute value (1).

If all these maxima be included in the term (4), it will be found that it is still less than 3 centimetres. It is, however, considerably smaller in the hypotheses of TONTA and COURTIER, even when α exceeds 30 degrees.

---

(1) The values of $b + m - 2c$ in various cases are:

(a) *Tonta Hypothesis:*

$$\psi(\alpha) = \frac{Log\ tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}{tan\ \alpha} = 1 - \frac{\alpha^2}{6} - \frac{13\ \alpha^4}{360} \qquad b = \frac{13}{60}$$

$$B\ (\omega) = sec\ \ \omega = 1 + \frac{\omega^2}{2} \qquad m = \frac{1}{2}$$

$$f\ (\alpha) = tan\ \ \alpha = \alpha\left(1 + \frac{\alpha^2}{3}\right) \qquad c = \frac{1}{3}$$

whence $b + m - 2c = \frac{1}{20}$.

(b) *Courtier Hypothesis:*

$$\psi\ (\alpha) = \frac{\alpha}{Log\ tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)} = 1 - \frac{\alpha^2}{6} - \frac{\alpha^4}{72} \qquad b = \frac{1}{12}$$

$$B\ (\omega) = \frac{Log\ tan\left(\frac{\pi}{4} + \frac{\omega}{2}\right)}{sin\ \omega} = 1 + \frac{\omega^2}{3} \qquad m = \frac{1}{3}$$

$$f\ (\alpha) = Log\ tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) = \alpha\left(1 + \frac{\alpha^2}{6}\right) \qquad c = \frac{1}{6}$$

whence $b + m - 2c = \frac{1}{12}$.

(c) *de Marchi Hypothesis:*

Let $\gamma\ (\alpha) = tan\ \alpha\ sec\ \alpha + Log\ tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$

$$\psi\ (\alpha) = \frac{2\ tan\alpha}{\gamma\ (\alpha)} = 1 - \frac{\alpha^2}{6} - \frac{7\ \alpha^4}{120} \qquad b = \frac{7}{20}$$

$$B\ (\omega) = \frac{1}{\psi\ (\omega)} = 1 + \frac{\omega^2}{6}. \qquad m = \frac{1}{6}$$

$$f\ (\alpha) = \frac{1}{2}\gamma\ (\alpha) = \alpha\left(1 + \frac{\alpha^2}{2}\right) \qquad c = \frac{1}{2}$$

whence $b + m - 2c = -\frac{29}{60}$.

It can, then, be neglected (1) without introducing any appreciable error in k which thus becomes:

$$(5) \qquad k = [1 - \psi(\alpha)]\ [l + C_0 (1 - A^2)].$$

3) In this simple form, the product of a function of α by a function of l, the correction for inclination may be determined readily by means of a slide rule or of a transversal straight-line nomogram.

The calculation of the graduation $l + C_0 (1 - A^2) = C_0 \dfrac{1 - A^3}{A}$ of the scale (l) is fairly long. Further, if the sounding gear be modified, the constant C_0 will vary and this graduation will have to be recalculated.

This may be avoided by using a diagram constructed as follows :-

Take $l = C_0 u$, u being a variable auxiliary, which gives:

$$(6) \qquad k = l\,[1 - \psi(\alpha)]\ \frac{3 + 3u + u^2}{(1 + u)^2}.$$

On a fixed plane P, construct the curves

$$(\alpha) \quad \begin{cases} x = -\log 10\,[1 - \psi(\alpha)] \\ y = 0 \end{cases}$$ divided scale α,

and
$$(u) \quad \begin{cases} x = \log \dfrac{3 + 3u + u^2}{(1 + u)^2} \\ y = -\log u \end{cases}$$ auxiliary curve, not divided.

On a transparent movable sheet P', draw the straight line (D) $x' = y'$ the first bisector, and make the graduations

(l) $\quad y' = \log l \quad$ according to l along the y' axis,

and (k) $\quad x' = \log 10\, k \quad$ according to k on the ordinate straight line $y' = \log C_0$.

---

(1) The formula (2) $f(\omega) = A\, f(\alpha)$ is approximate only, the accurate relation between ω and α being

$$(2\ bis) \qquad f(\omega) = \frac{C_0 B(\omega)}{l + C_0 B(\omega)}\, f(\alpha).$$

The elimination of ω between (1 *bis*) and (2 *bis*) gives the following expression for the correction

$$(3\ bis) \qquad k = [1 - \psi(\alpha)] \left[l + C_0 (1 - A^2) \left[1 + \left(b + m - 2c - \frac{2\, m\, A^2}{1 + A} \right) A^2\, \alpha^2 \right] \right].$$

A lying between 0 and 1, $\dfrac{A^2}{1 + A}$ is between 0 and $\dfrac{1}{2}$.

To find the upper limit of the α^2 term, the greatest of the two numbers $b + m - 2c$ and $b - 2c$ at their absolute values must be considered.

In the three hypotheses under consideration, $b - 2c$ is equal to $-\dfrac{9}{20}$, $-\dfrac{3}{12}$, $-\dfrac{13}{20}$ respectively. Its absolute value exceeds $\dfrac{1}{2}$ once only and then but little. Thus our conclusion remains valid.

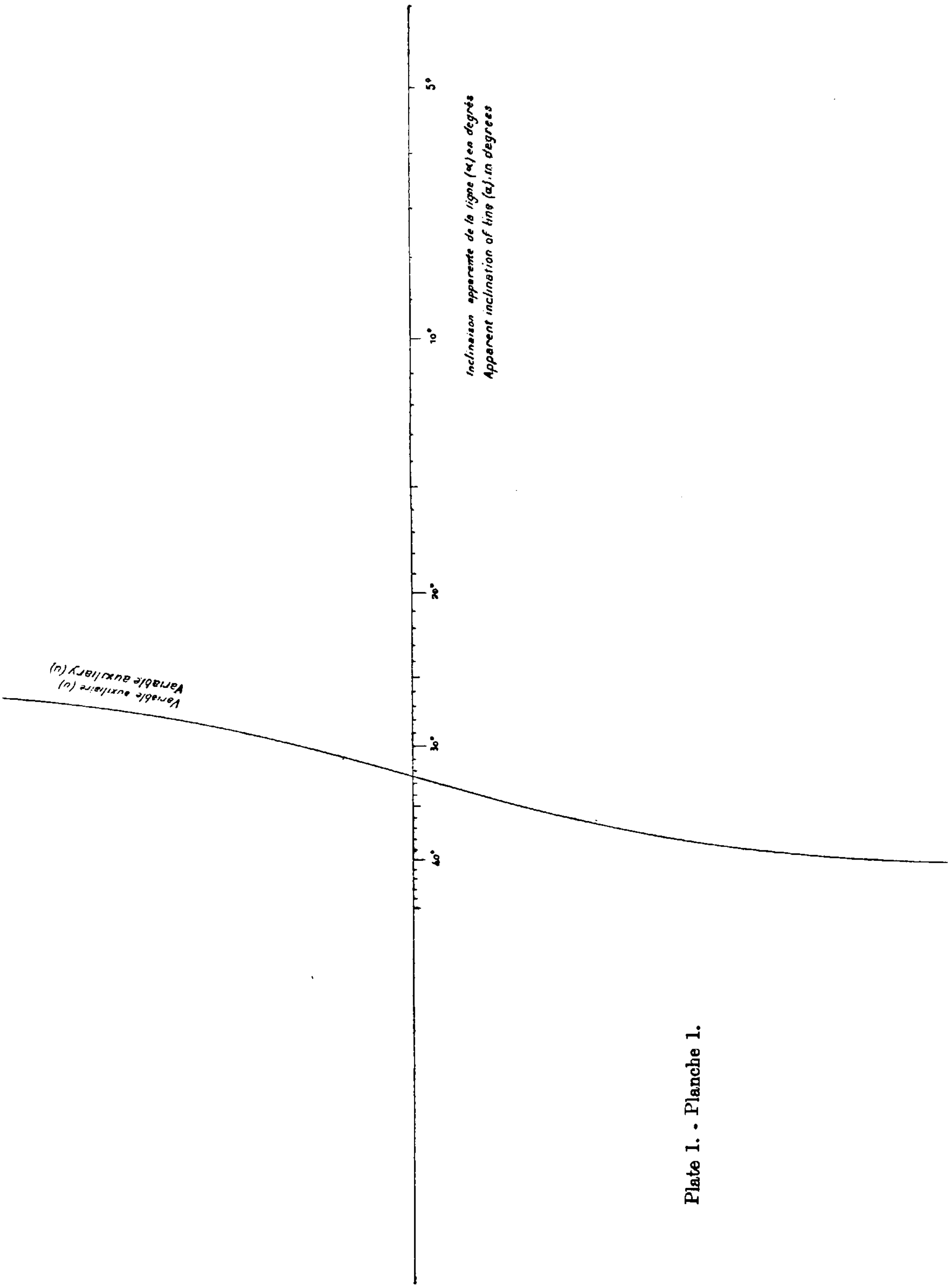

Plate 1. - Planche 1.

To obtain any correction for inclination, place the transparent sheet P' on the plane P keeping the axes of the coordinates parallel and making:

1) the scale (α) pass through the given value l_0 on the scale (l),

2) the straight line (D) pass through the given value α_0 on the scale (α).

Then k_0 is read off at the intersection of the auxiliary curve (u) and the graduation (k).

When thus set, the differences of the coordinates of the origins of the two systems are:

$$x - x' = -\log 10\,[1 - \psi(\alpha_0)] - \log l_0 = \log \frac{3 + 3u + u^2}{(1 + u)^2} - \log 10\ k_0,$$

$$y - y' = 0 - \log l_0 = -\log u - \log C_0,$$

from which we deduce

$$k_0 = l_0\,[1 - \psi(\alpha_0)]\,\frac{3 + 3u + u^2}{(1 + u)^2} \qquad \text{with } l_0 = C_0\,u.$$

Thus k_0 corresponds exactly to the correction given by formula (6).

Plates I and I *bis* represent this nomogram in Captain TONTA's hypothesis:

$$\psi(\alpha) = \frac{Log\,tan\left(\dfrac{\pi}{4} + \dfrac{\alpha}{2}\right)}{tan\,\alpha} \qquad \text{and in the case where } C_0 = 10 \text{ metres.}$$

The graduation (l) has been repeated on three parallels to $o'y'$* so that the parallelism of the axes of coordinates may be obtained by making the scale (α) pass through corresponding points of the markings of two of these graduated lines.

The only calculation necessary for marking this diagram is that for the division of the scale (α), the scales (l) and (k) being simply logarithmic. The (u) curve is then drawn by means of the table below which gives the coordinates x and y in millimetres, for a certain number of points on the curve, calculated with the modulus of 100 mm. adopted in constructing Plates I and I *bis*.**

u	x	y	u	x	y	u	x	y	u	x	y
0.10	+43.7	+100.0	0.60	+30.45	+22.2	1.60	+18.5	−20.4	6.00	+6.6	−77.8
0.15	41.9	82.4	0.70	28.65	15.5	1.80	17.2	25.5	7.00	5.7	84.5
0.20	40.3	69.9	0.80	27.05	9.7	2.00	16.0	30.1	8.00	5.1	90.3
0.25	38.7	60.2	0.90	25.6	+ 4.6	2.50	13.6	39.8	10.00	4.1	100.0
0.30	37.3	52.3	1.00	24.3	0.0	3.00	11.8	47.7	12.00	3.5	107.9
0.40	34.7	39.8	1.20	22.0	— 7.9	4.00	9.3	60.2	15.00	2.8	117.6
0.50	32.45	30.1	1.40	20.1	—14.6	5.00	7.7	69.9	20.00	2.1	130.1

* One of the three graduations has been suppressed on Plate 1 *bis* to suit the paging (NOTE OF THE EDITOR).

** The scale of Plates I and I *bis* has been arbitrarily modified to suit the paging (NOTE OF THE EDITOR).

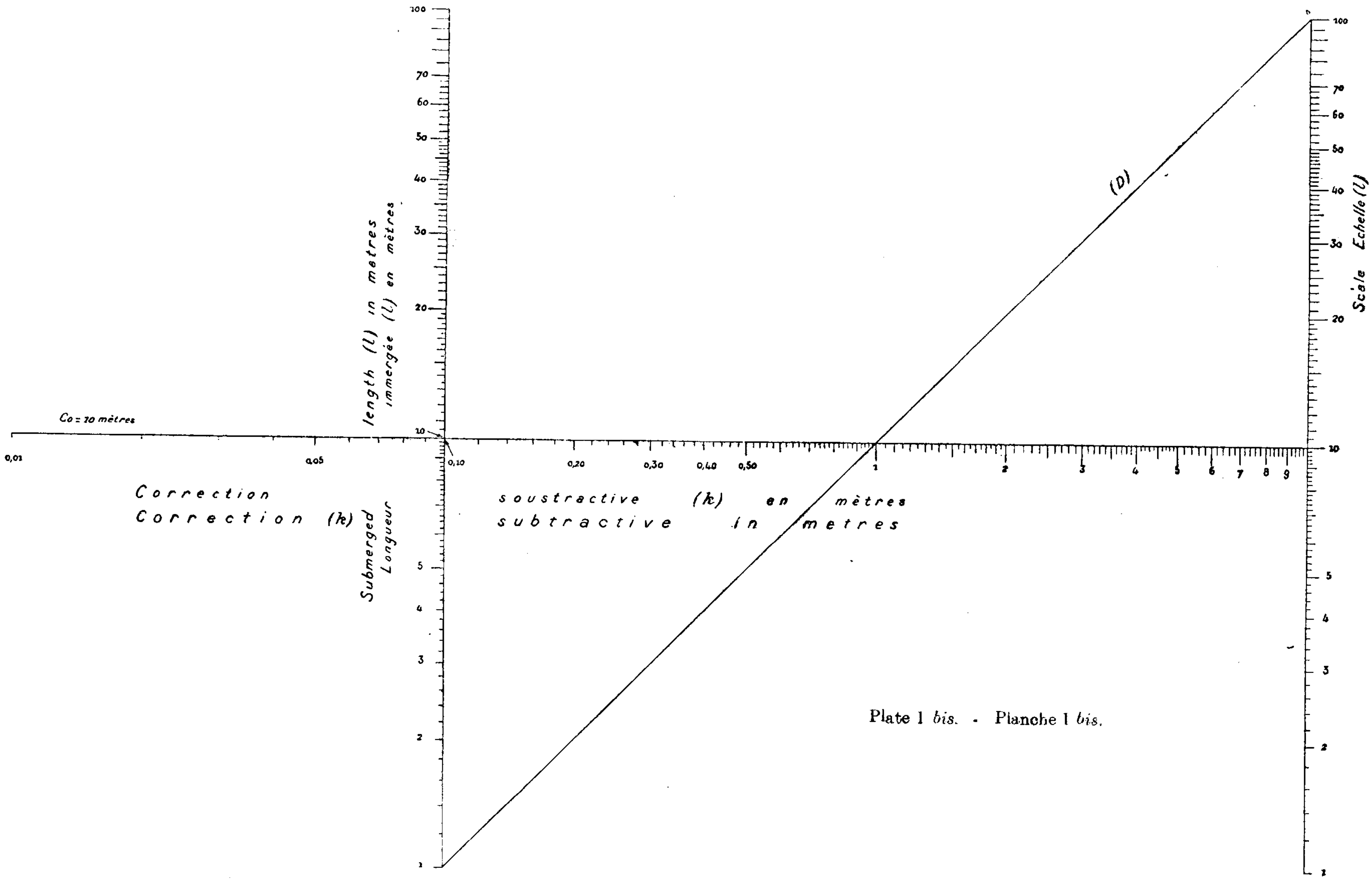

Plate 1 *bis*. - Planche 1 *bis*.

For a different value of C_0, it is sufficient to draw from the division $l = C_0$ on the scale (l), a parallel to $o'x'$ which is then graduated exactly as is the straight line C_0 itself, either by transferring the divisions by tracing, or by means of a table of logarithms.

If another hypothesis as to the form of the curve be adopted, the scale (α) only need be altered. The alteration is not appreciable except in the larger values of the inclination, the function $1 - \psi(\alpha)$ being of the form $\frac{\alpha^2}{6}(1 + b\,\alpha^2 + ...)$ in the case of a parabola, a catenary and an arc of a circle.

As an example, the following are the abscissae (modulus 100 mm.) of the points corresponding to 30° and 45° in the various hypotheses:

	$\alpha = 30°$	$\alpha = 45°$
TONTA hypothesis	+ 31.4 mm.	— 7.4 mm.
COURTIER hypothesis	+ 33.0	— 3.7
DE MARCHI hypothesis	+ 29.8	— 4.3
Arc of Circle $\psi(\alpha) = \frac{\sin\alpha}{\alpha}$	+ 34.6	+ 0.1

Thus a change in the law for the correction involves a relative displacement of the scale (α), which, in this part of the scale, corresponds to a change of 2 to 3 degrees in α, which is not very much in view of the relative inaccuracy with which the inclination is measured.

4) The correction for inclination as in form (3) can be solved nomographically by an analogous procedure, without neglecting term (4), as we have.

But this solution has the disadvantage, when the value of C_0 is altered, of necessitating the calculation of fairly complex functions for making the new divisions.

Therefore it will be but briefly referred to here.

Taking:

$$\begin{cases} g(\alpha) = 10\,[1 - \psi(\alpha)] \\ h(\alpha) = \alpha^2 \end{cases} \qquad \begin{cases} p(l) = l + C_0\,(1 - A^2) \\ q(l) = \dfrac{C_0\,(1 - A^2)\,A^2\,(b + m - 2c)}{l + C_0\,(1 - A^2)}, \end{cases}$$

the correction would be:

$$10\,k = g(\alpha)\,p(l)\,[1 + h(\alpha)\,q(l)] = v\,g(\alpha)\,p(l),$$

the auxiliary variable $v = 1 + h(\alpha)\,q(l)$ being introduced.

On a fixed plane P the curve:

$$(\alpha) \quad \begin{cases} x = \log g(\alpha) \\ y = -\log h(\alpha) \end{cases} \qquad \text{divided according to } \alpha,$$

and the scale:

$$(k) \quad \begin{cases} x = \log 10\,k \\ y = o \end{cases} \qquad \text{divided according to } k,$$

are drawn.

On a free transparent sheet P' the curves

$$(l) \quad \begin{cases} x' = -\log p\,(l) \\ y' = \log q\,(l) \end{cases}$$ divided according to l,

and $$(v) \quad \begin{cases} x' = \log v \\ y' = \log (v-1) \end{cases}$$ the auxiliary curve, undivided,

are drawn.

To obtain the inclination correction, whilst maintaining the axes of the two planes parallel, the given point on the curve (l) is made to coincide with the given point on the curve (α). The correction k is determined by the intersection of the scale (k) and the auxiliary curve (v).

No example of this nomogram will be given, as it is tedious to make and is of little interest in view of the smallness of the retained term (4).

5) Formula (5) giving the inclination correction may also be written:

$$(7) \qquad k = [1 - \psi(\alpha)]\ (C_0 + l)\ (1 - A^3).$$

If soundings are to be taken in depths at least equal to C_0, $A = \dfrac{C_0}{C_0 + l}$ is less than $\dfrac{1}{2}$ and A^3 than $\dfrac{1}{8}$, consequently A^3 may be neglected with reference to 1, a relative error not exceeding $\dfrac{1}{8}$ being made in the correction, which error decreases rapidly as l increases.

The absolute error remains small, for, in small depths, the inclination is never great; when $l = C_0$ this absolute error is about $\dfrac{C_0\,\alpha^2}{24}$.

If $C_0 = 20$ metres and $\alpha = 30$ degrees (which assumes a fairly considerable speed for the sounding vessel), this term scarcely exceeds 20 centimetres. In practice, the neglected term will always be less than this quantity.

Further, the fact of not taking this term in A^3 into account increases the correction for inclination and consequently diminishes the depth according to the lead. From an hydrographic point of view, this is a less serious disadvantage than an error in the opposite direction, for, when it is impossible to obtain accurate soundings, it is preferable to show the depth as being less than it really is.

It will be seen therefore that in most cases, in practice, the formula for the correction

$$(8) \qquad k = [1 - \psi(\alpha)]\ (C_0 + l)$$ is sufficient.

It is this formula that we have studied, more particularly in the case of the arc of circle, in an article which will appear in the *Annales Hydrographiques*.

The following is the nomographic solution which we have reached:

On a straight line uu' (Fig. 1) from a point C

a metric scale (l) is graduated in values of l towards u,

a metric scale (C_0) is graduated in values of C_0 towards u'.

On a parallel vv' at a distance D from uu' the metric scale (10 k) is graduated in values of k from a point B towards v'.

Then a system (α) of parallels to uu' is drawn at a distance x from this straight line, x being $= \dfrac{D}{10\,[1 - \psi(\alpha)] + 1}$.

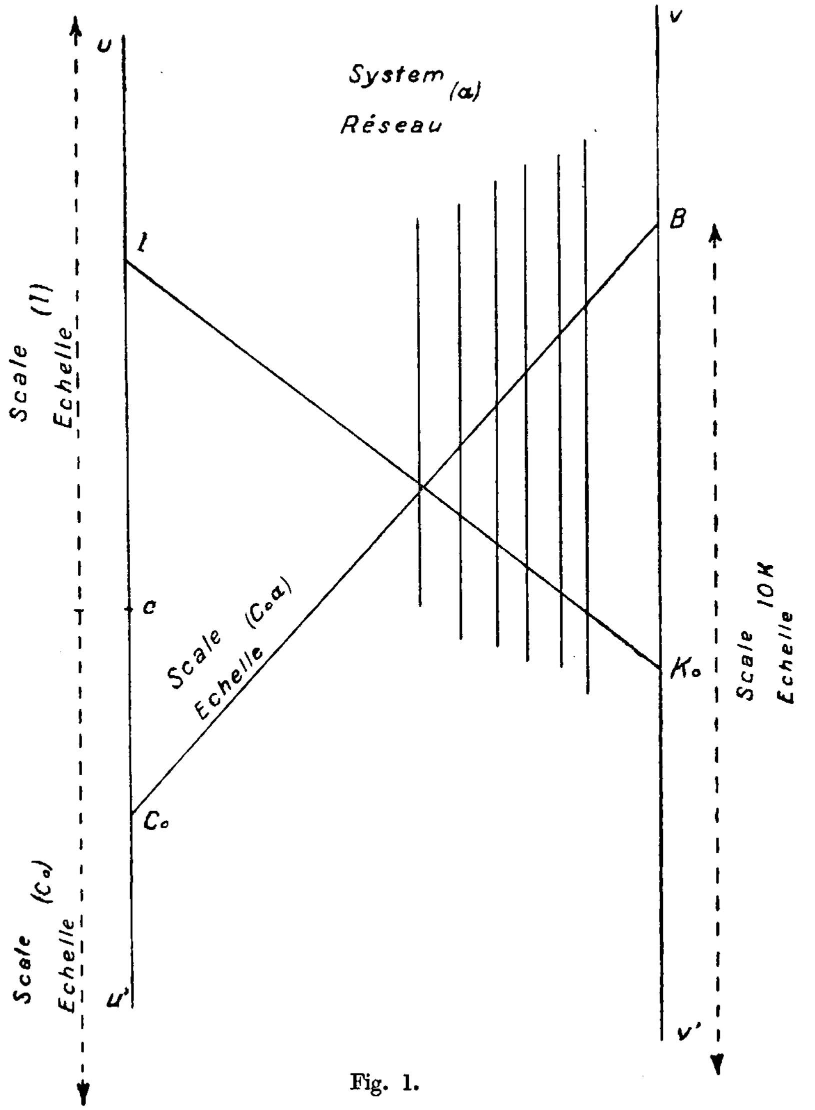

Fig. 1.

For a given value of C_0, B is joined to the graduation C_0 on the scale (C_0) and this straight line is divided into α by means of its intersections with the system (α). Thus a scale (C_0, α) graduated according to α is obtained.

To obtain a correction, a thread is stretched through the given graduation points l_0 and α_0 on the scales (l) and (C_0, α) and k_0 is read off at the intersection of the thread and the scale (10 k).

CORRECTION FOR INCLINATION
CORRECTION D'INCLINAISON

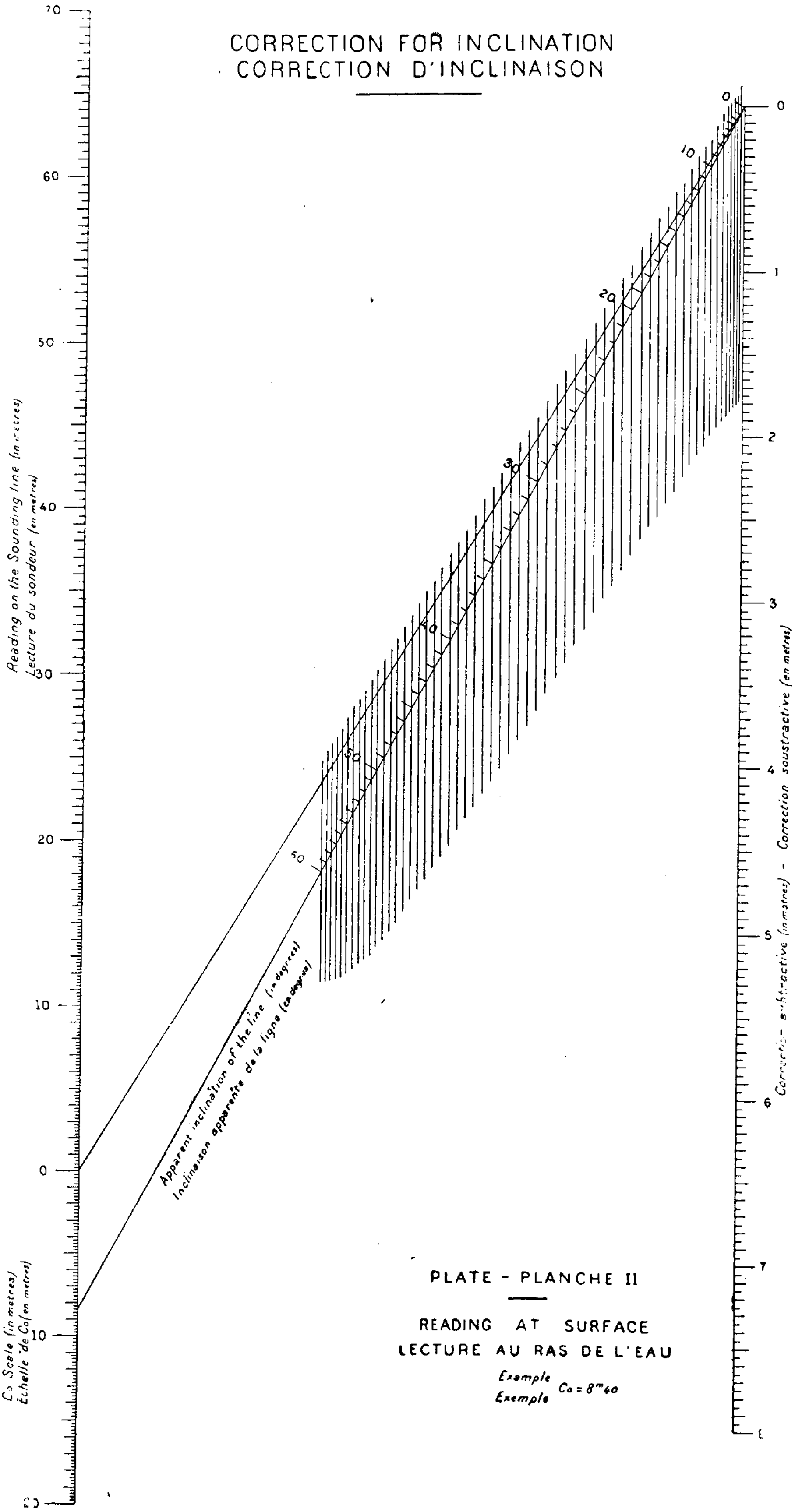

PLATE - PLANCHE II

READING AT SURFACE
LECTURE AU RAS DE L'EAU

Example
Exemple $Co = 8^{m}40$

This diagram, composed of transversal straight lines, is very easy to construct with metric scales, and is equally easy to use.

Plate II gives an example on the supposition that $\psi(\alpha) = \frac{\sin\alpha}{\alpha}$.

6) *Conclusion.*

As Captain TONTA has remarked, the various hypotheses put forward for determining the law of correction for soundings give results, in practice, which differ but little (as long as the inclination does not exceed 30 degrees).

It may be said even that they are equivalent, in view of the inaccuracy with which the inclination of the line is measured at present.

It has been seen that, in neglecting the term (4), as may legitimately be done, the functions $f(\alpha)$ and $B(\omega)$ do not enter except in the calculation of C_0. The results depend very little upon the method employed. As an example, the following results were obtained in the same experiment:

TONTA hypothesis	$C_0 = 11.03$ metres
COURTIER »	$C_0 = 11.28$ »
DE MARCHI »	$C_0 = 10.27$ »
Arc of circle	$C_0 = 11,84$ »

Once C_0 is determined, the correction for inclination depends on $\psi(\alpha)$ only; here also the hypothesis adopted has little influence.

Practically, when $\alpha \leqslant 30^\circ$, any of the above hypotheses may be employed.

The diagrams which we have constructed allow the correction for inclination to be obtained within a much greater approximation than that due to the uncertainty as to α.

As long as the determination of this angle is not improved, we would advise, if the nomographic method be used for obtaining the correction k, that the transversal straight line nomogram described in paragraph (5) be employed, on the sole condition that l is greater than C_0.