NEW INSTRUMENTS FOR ESTIMATING TOPOGRAPHICAL DISTANCES

from an article by Professor G. CASSINIS,
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Apart from rangefinders properly so-called, the optical processes used in topography for the evaluation of distances between 10 and 250 metres, approximately, may be taken to be two in number and be classified as follows:—

1. The MONTANARI-REICHENBACH-PORRO method, using a telescope of fixed parallactic angle. This method employs a goniometer or tacheometer with horizontal and vertical circles and an astronomical telescope the graticule of which consists of two or several horizontal wires or hair-lines which are used in conjunction with a graduated vertical pole. When the horizontal wires are three in number, taking L_1 and L_2 as the respective readings on the pole, α being the angle of altitude above the horizon of the sighting line of the central wire, the distance D (reduced to the horizontal) between the centre of the tacheometer and the position of the pole, is given by the formula:—

$$D = c \cos \alpha + k (L_2 - L_1) \cos^2 \alpha$$

where the coefficients c and k are constants depending on the telescope and the graticule used. c is an additive constant and k is a multiplying constant. The additive constant c is nil when the PORRO anallatic telescope is used; it is also practically nil when the WILD constant length telescope is used.

For a telescope the focal length of which is f, fitted with a RAMSDEN eye-piece, the multiplicative factor k is given by the formula:—

$$k = f/h$$

h being the distance apart of the outermost cross-wires of the graticule. In order to facilitate calculation, the coefficient k is generally chosen = 50, 100, etc.

By this means, using a telescope the magnifying power of which is between 15 and 30 and setting the stadia correctly with a plumb-line, a distance of 100 metres will be determined with a mean accuracy of from 20 to 10 %, provided that the angle α does not exceed 15° to 20°. It is not necessary to reach very great accuracy in measuring the angle α, it is sufficient to read it to within a minute of arc.

2. Method of variable parallactic angle. This method makes use of a goniometer with vertical and horizontal circles and an astronomical telescope fitted with a simple graticule. The same vertical sighting pole as above is used.

Let L_1 and L_2 be the readings on the pole corresponding to the angles of elevation α_1 and α_2 respectively; the distance reduced to the horizontal is given by the formula:—

$$D = \frac{L_2 - L_1}{\tan \alpha_2 - \tan \alpha_1}$$

This method is used either by obtaining beforehand the difference $L_2 - L_1$ or the difference $\tan \alpha_2 - \tan \alpha_1$. In the first case a pole with two marks, set at an invariable distance a from each other, is used. In the second an instrument is used which permits the adjustment of the suspension of the telescope so that

$$\tan \alpha_2 - \tan \alpha_1 = m$$

and consequently:—

$$D = \frac{L_2 - L_1}{m}$$

In order to obtain sufficiently accurate results, it is necessary to measure either the angles α_1 and α_2 or the difference of suspension m very accurately. With such instruments, and under the same conditions as above, a distance of 100 metres may be measured with an approximation of 12 to 8 %.

The above processes may be used also with a horizontal pole perpendicular to the vertical plane of sight. Until quite recently only the variable parallax system was used in this way, the angle 2φ formed by successive observations of horizontal poles, of length $2a$, being measured by azimuth circle or by some other instrument. The distance, reduced to the horizontal is then :—

$$D = \frac{a}{\tan \varphi}$$

In this method the angle 2φ must be taken with fairly considerable accuracy; this is the process used for the measurement of bases in stereophotogrammetric operations; for example, with the Zeiss phototheodolite the micrometer screw permits 2φ to be determined with an approximation of 10'' and consequently a distance of 100 metres may be obtained within 5 ‰.

All the above processes are likewise used to determine differences of elevation. In this case the accuracy depends not only on the precision with which the measurement of the distance is made, but also on the accuracy with which the angle 2φ is measured.

In order to facilitate or even to eliminate the calculations required by the above formula or similar calculations for the evaluation of differences of elevation, reducing and auto-reducing instruments have been invented; these, when they are auto-reducing, have been devised to give directly the value of D and even the values of D and of the difference of elevation itself, when they are auto-reducing both for the distance and for the difference of elevation. At most it is but necessary to multiply by a factor such as 100, 50, 20, etc...

The SANGUET tacheometer, which is largely used in France, is an example of an auto-reducing instrument. The HAMMER-FENNEL tacheometer is another universal auto-reducing instrument, with variable parallax, the accuracy of which is sufficient in the majority of cases. The latter is fairly widely used in Germany (*See* G. CASSINIS "*Sulla precisione conseguibile nei rilevamenti di tracciati con il tacheometro autoriduttore* HAMMER-FENNEL" published by the Istituto di Geodesia della R. Scuola di Ingegneria di Roma, Nº 8, 1922).

In Italy the system of the invariable parallactic angle with vertical graduated pole is mostly used; it is moreover the most widely-used system. Ignazio PORRO greatly contributed to the diffusion of this method in Italy and other countries, and it is to him that we are indebted for the instruments of a practical type for topographical operations which are constructed by the firm of SALMOIRAGHI.

However, as has been pointed out, the accuracy of the PORRO method is not very great and this process can no longer be used in operations where a certain degree of accuracy is sought, for instance, for the measurement of traverses with an error not exceeding 1/2000 of the length of the sides.

Professor G. ORLANDI, to whom we are indebted for very practical tacheometrical tables, thought that it was possible to increase the accuracy by inverting (if it may be so expressed) the functions of the stadia and the graticule (*See* G. ORLANDI: "*Tacheometria di Precisione*", Firenze 1927). For this purpose he divided the interval H between the outer wires of the graticule into a certain number, N, of equal parts and he used a stadia in the shape of a vertical pole divided into various unitary intervals, each of which is adapted to different distances. The evaluation of the fraction h of the graticule corresponding to a given distance is made by setting the outermost wire selected as the origin onto the upper extremity of the interval chosen (the length of which is a), then reading the number, n, of graticule units and fractions thereof comprised between this wire and the other extremity of the unit a of the pole. Clearly, $h = n\,H/N$, and assuming that an anallatic telescope of focal length f is used, the multiplying factor to be employed will have the value :—

$$k = \frac{f}{h} = \frac{f}{H}\,\frac{N}{n} = K\,\frac{N}{n} \qquad \left(K = \frac{f}{H}\right)$$

and the distance is given by the formula :—

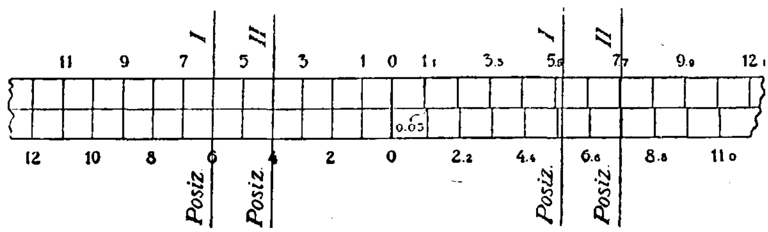
$$D = k\,a \cos^2 \alpha$$

As a rule $K = 100$, $N = 100$ and consequently $k = 10{,}000/n$.

In this way the PORRO telescope is transformed into an instrument suitable to the variable parallactic angle method. The advantage of this method consists in the rapidity with which field work may be carried out with a relatively small number of stadia. The same process may also be used with a horizontal pole, the graticule being rotated 90°.

The use of horizontal poles, in spite of the longer time required to fit them up, is tending to-day to become more general. The various processes are as follows :—

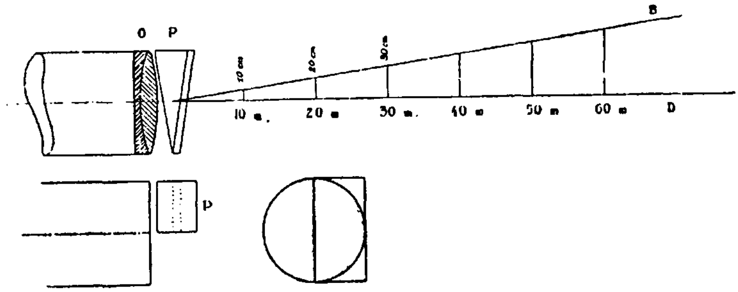
Fig. 1



In the first method, introduced by R. Werffeli, a constant parallactic angle telescope is used, the special feature of which is the graduation of the pole, shown in Figure 1 : on the left-hand side there is only a single graduation ; the right-hand portion shows two graduations superimposed and differing from each other by ½ m/m for a length of 11 m/m. After having set the central wire approximately onto the centre of the pole, the left vertical wire is made to coincide exactly with the division line nearest to it, the right wire then usually falls between two division lines of the other half of the pole (Position 1). The horizontal tangent screw is then turned to bring the left-hand wire into coincidence with one of the division lines of the left portion of the graduated scale and, at the same time, the right-hand wire into coincidence with one of the division lines of the upper or lower divisions of the right-hand portion of the pole or to lie between two of these division lines (Position 2). By this means the interval on the pole is directly read within ½ m/m : besides, for distances of less than 50 metres, tenths of millimetres may be estimated. By using a telescope of magnifying power 24, Werffeli obtained maximum errors of 5 c/m for distances up to 130 metres. This process may be adopted in practice with any tacheometer the graticule of which has been turned 90° ; it is but necessary to prepare horizontal poles as indicated above. Werffeli used metallic poles 1.40 metres in length and drew the divisions in the shape of wedges in order to facilitate accuracy in reading (*See "Die optische Distanzmessung und ihre Anwendung bei der Schweiz. Grundbuchvermessung"*, Sammlung von Referaten, etc. Zurich, 1925. J. Baltensperger "*Die Polarkoordinatenmethode mittelst optischer Distanzmessung als Aufnahmeverfahren für Katastervermessungen*", Zeitsch. f. Vermessungswesen, Bd. LV, 1926, pp. 449 to 481. E. Hammer "*Weitere neue schweizerische Präzisionsdistanzmesser*", Zeitsch. f. Instrumentenkunde, 1925, p. 546. R. H. Richard "*A new Prismatic Stadia*", Journal of the Assoc. of Engineering Societies, Vol. XIII, 1894, p. 43).

The Aregger, Wild and Bosshardt range-finders differ from the foregoing instrument in that the parallactic angle is obtained by optical means ; the principle of these instruments was derived from the idea of an American, Richard.

Fig. 2



If half of the objective of the telescope of any theodolite be covered by means of a wedge-shaped prism (Fig. 2), two images of any object are seen and, in particular, two images of a vertical stadia vertically separated from one another. For a given prism, the angular deflection is constant and the separation is in proportion to the distance : for instance, if a prism is chosen of angle equal to 1/100, *i.e.* 34'23", the separation equals 1/100 of the distance. Consequently, by reading this separation the distance may be deduced immediately and it may be reduced to the horizontal by the ordinary method. RICHARD used a vertical pole one part of which was graduated throughout its length, only a small fraction of the other part being graduated (Fig. 3); the latter part was used as a vernier for reading (*See* R. H. RICHARD "*A new prismatic stadia*", Journal of Assoc. of Engineering Societies, Vol. XIII, 1894, p. 43).

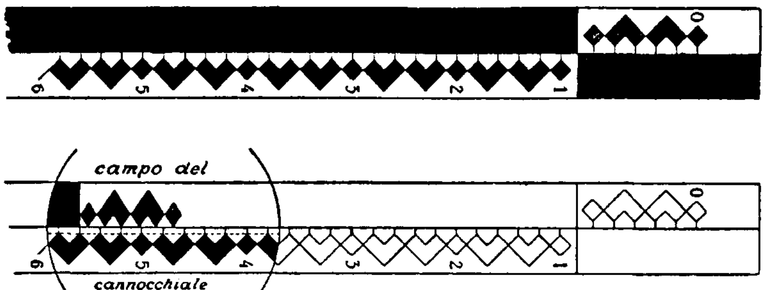


Fig. 3

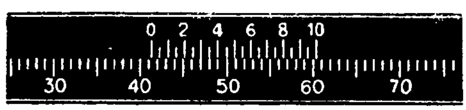
AREGGER, WILD and BOSSHARDT make use of a horizontal pole.

AREGGER (*See* A. AREGGER "*Der Doppelbild-Tachymeter Kern*", Zeitsch. f. Vermessungswesen, Bd. LVI, 1927, p. 134) placed the prism in the centre of the objective in order to avoid the chromatic aberrations resulting from an excentric position of the prism. The pole is fitted with two verniers, one for small and the other for great distances. Figure 4 illustrates the aspect of the images in the field of the telescope and also the method of reading. With a telescope of magnifying power 24, the mean error of measurement of a distance of 60 metres, by balanced offset reading, is about 2 %.

This range-finder, manufactured by Messrs KERN of Aarau, may be applied to any telescope provided that the diameter of the sun-screen of the objective is not larger than the diameter of the prism carrier-ring.

WILD uses a range-finder fitted with two fixed prisms, each covering half of the objective and the angle of which is equal to 1/200. The principle of the tacheometric device was given in *Hydrographic Review*, Vol. IV, N° 2, p. 210 (*See* also G. CASSINIS "*Il teodolito e il telemetro* WILD", l'Ingegnere, Vol. II, 1928, N° 4). The WILD device may be fitted to any theodolite on

Fig. 4



condition that its objective's diameter is not greater than that of the WILD objective and that the divisions on the pole are suitably drawn.

In order to measure the relative displacement of the two graduations on the pole correctly, the method of coincidence is adopted by rotating the images through equal angles in opposite directions by means of a micrometer drum which turns two parallel faced glass laminæ symetrically placed in front of the objective. The whole number of metres is read directly on the pole, (Fig. 5) and the centimetres are read from the micrometer drum.

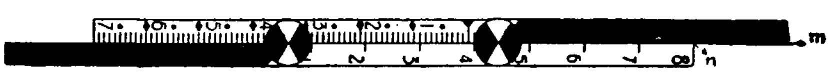


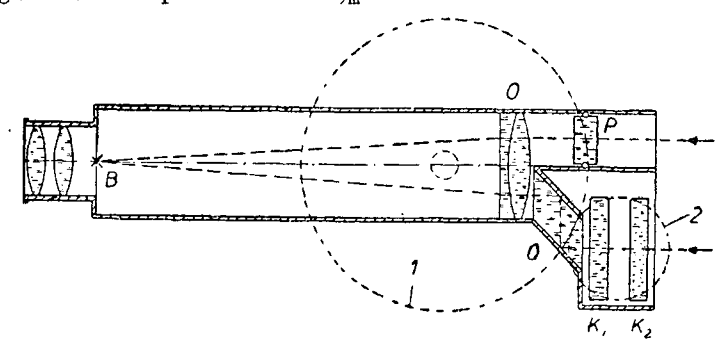
Fig. 5

The pole as seen through the WILD *range-finder when the coincidence setting has been effected. The whole number of metres is* 30 + 9 = 39, *or* 20 + 19 = 39, 10 + 29 = 39. *The number of centimetres is read from the micrometer drum of the telemeter.*

The respective distances between the division lines of the pole are determined by accurate calibration of each apparatus.

For distances less than 60 metres, the error does not exceed 3 % and for distances between 60 and 140 metres, errors not exceeding 6 % are obtained. When the atmosphere is calm and the images of the pole are very clearly defined, the error in determining distances of 80 metres is in the neighbourhood of plus or minus 2 c/m.

Fig. 6



The BOSSHARDT-ZEISS tacheometer is fitted with an auto-reducing device for measurement of distances which works as follows (Fig. 6): — The lower part of the objective is fitted with a deflector consisting of a rhombohedral prism, in front of which are placed two other prisms forming a diasporameter, *i.e.* capable of turning in opposite directions through equal angles around an axis parallel to the optical axis. As may be easily demonstrated, this rotation does not involve any displacement in height of the images but simply a variation of the parallactic angle introduced by the couple of prisms and, consequently, of the lateral displacement of the images of the horizontal pole seen through the prisms. This rotation is governed by a small conical pinion driven mechanically. Thus by multiplying the relative displacement of the lower and upper portions of the graduated pole by 100, the distance reduced to the horizontal is obtained directly. In order to reach a higher degree of accuracy, the method of coincidence may also be used by fitting a vernier onto the pole and placing before the upper half of the objective a parallel faced glass rotated by means of a divided drum. The pole is graduated in centimetres and is provided with a collimator for the accurate orientation of the pole and verification, from the stadia itself, of the absolute correctness of the orientation.

The result of the measurement is analogous to that obtained with the WILD range-finder. (*See* R. BOSSHARDT "*Das neue Reduktions-Tachymeter*", Schweiz. Zeit. f. Vermessungswesen u. Kulturtechnik, Nov. 1926. T. EDER "*Versuchsmessungen mit dem Bosshardt'schen Distanzmesser*", Zeitsch. f. Vermessungswesen, Bd. LV, 1926, p. 528. F. ACKERL "*Versuchsmessungen mit dem Selbstreduzierenden Distanzmesser von Bosshardt-Zeiss*", Zeitsch. f. Instrumentenkunde, Bd. 49, 1929, p. 64).

A very original method is that conceived by Dr. ENGI, who places the horizontal pole just above the tacheometer itself and on the spot to be located a mirror in front of which are two prisms. This mirror is fitted perpendicular to the axis of collimation of the telescope so that by looking into the telescope two images of the pole are seen and the distance is deduced in the ordinary way.

The HUGERSHOFF autotachygraph, manufactured by the Aerotopograph Company of Dresden, comes under the category of auto-reducing graphometers. The telescope is replaced by a stereoscopic range-finder of 70 c/m base which would give errors of 10 c/m in distances of 100 metres and of 2 metres in distances of 500 metres. The distance is automatically reduced to the horizontal by optical means and the planimetric position of the point is plotted directly on the plane-table by a thin needle held by a milled head. The differences of elevation are read on a vertical set-square as in the case of other reducing instruments such as the WAGNER-FENNEL tachygraphometer (*See Hydrographic Review*, Vol. VI., N° 1, page 223).

The above methods summarise the most recent improvements in connection with the optical measurement of topographical distances. These methods are already practical and convenient; by their means the topographical surveyor no longer has to measure the sides of traverses directly and, in the majority of cases, they make it possible for levelling operations to be performed simultaneously with the planimetric operations.