SLOPE CORRECTIONS FOR ECHO SOUNDINGS

by

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An interesting article on this subject by A. L. SHALOWITZ, Cartographic Engineer (United States Coast and Geodetic Survey) was published in the *Hydrographic Review,* Vol. VI, N° 2 of May 1930. Others have already studied this question — particularly in the case where the ocean bed may be assumed to be a more or less inclined plane. This problem in fact necessarily presented itself to the first operators who undertook to obtain soundings by acoustic methods. On page 164 of *Hydrographic Review,* Vol. IV, N° 2 (May 1928) a diagram of corrections for the slope of the bottom, drawn up by the Coast and Geodetic Survey, appears. Dr H. MAURER, in the *Annalen der Hydrographie* of September and October 1926 and later in November 1928, had also given tables and methods of correction which he had applied to the soundings obtained by the *Meteor.*

The hypothesis which forms the basis of all this work is that the sound emitted by the vessel sounding is propagated in straight lines and returns to it along the same line after reflection from the bottom, normal to the bottom surface (*) ; the only reflection to be taken into consideration being that which is produced at the point nearest the point of emission of the sound. It is not certain, in actual fact, that the sound travels to the bottom of the sea on a straight trajectory, since it passes through layers of different densities, but the differences in velocity in these various layers are so slight that the line of propagation of the wave will not diverge materially from a straight line. Further, it is not certain that sound is propagated with the same intensity in all directions, particularly when it is generated by vibrating diaphragms ; it is probable that outside the limits of a cone of a certain apex angle, the sound is attenuated to such an extent that an echo of the sonic beam is no longer perceived. Even though the angle at the apex of the cone is certainly considerable, exceptional cases may arise, which are precisely those in which correction of echo soundings will be of the greatest importance, where the echo is not returned from the point on the bottom surface nearest to the vessel. If this be the case the correction to be applied will be less than that indicated by theory. It appears further that a returning sound following the same trajectory as the emitted wave, may be produced even when the sound wave does not appear normal to the general surface of the bottom. In regions where the configuration is very uneven, the bottom surface is so irregular that, at some point, facets will be found which present a surface normal to the incident sonic beam. The echo will then follow the

(*) The correction due to the speed of the vessel is entirely negligible in practice.

shortest path compatible with the conditions of the experiment rather than a path strictly normal to the general surface of the bottom.

Appliances which give accurate indications of the direction of the sound emitted, and particularly those which allow the direction of the reflected wave to be determined with accuracy, would allow very useful experiments to be made (*). In the absence of such apparatus only very accurate comparative measurements of the depth, which however are very difficult to make, could be employed to aid in elucidating the question of slope corrections to be applied to echo soundings, in order to deduce the true depth and to be able to use them concurrently with soundings by wire — which latter will not disappear from charts for many years to come. So far comparisons made with soundings by wire have shown only very slight differences, of the order of magnitude of the possible experimental errors, and it does not appear that other conclusions could be reached than the following, viz. : soundings by acoustic methods give results which are comparable in accuracy with those obtained by wire.

We should first like to take up the general case of the slope corrections to be applied to echo soundings on the hypothesis of reflection from the nearest point, without assuming the bottom to be a plane surface, and then to examine some particular cases to direct attention to certain anomalies which may possibly suggest instructive experiments to verify the correctness of the theory. These will also demonstrate that, in certain cases, it is best to have recourse to soundings by wire.

a) The method to be employed in compiling a regular survey made with echo soundings appears to us to be as follows : — Locate all of the stations and enter all of the soundings on a provisional sheet without making corrections for slope ; then draw the depth lines, which will thus accurately define a surface which we shall call the *apparent bottom surface*. The problem then becomes, to convert this *apparent bottom surface* into the *true bottom surface*, from which it differs somewhat, even as the image of an object seen under refraction through water differs from the real object.

If we refer this surface to the three rectangular axes of which two, X and Y, are on the surface of the water and the third Z, directed towards the bottom, the equation will assume the form :—

$$Z = f\ (X,\ Y)$$

The beam of sound issuing from the vessel (X, Y, o) touched the true surface at a point whose co-ordinates are x, y, z, which is at a distance Z from the point of emission ; this distance is given by the echo sounding. We then have the equation :—

$$(X - x)^2 + (Y - y)^2 + z^2 = Z^2 = f^2\ (X,\ Y)$$

The envelope of the spheres thus defined, when the point of emission

(*) The submarine receiver of the Sonic Depth Finder is an apparatus of this kind. See the article by Dr. Harvey C. Hayes in the *Geographical Review*, New York 1924 and the *Hydrographic Review* of May 1925, Vol. II, N° 2, page 147.

moves everywhere on the surface of the water, will be the true surface sought; its equation will be obtained by deriving the above equation with respect to X and Y. Thus the co-ordinates of any one of these points will be given by the equations :—

$$(1) \qquad \begin{aligned} x &= X - Z\frac{\delta f}{\delta X} \\ y &= Y - Z\frac{\delta f}{\delta Y} \\ z &= Z\sqrt{1 - \frac{\delta f^2}{\delta X} - \frac{\delta f^2}{\delta Y}} \end{aligned}$$

The quantities $\frac{\delta f}{\delta X}, \frac{\delta f}{\delta Y}$ are known — these are the slopes of the plane tangent to the apparent bottom surface at the point X, Y, Z, measured parallel to the planes the co-ordinates of which are $X = 0, Y = 0$.

Let us take the axis of the X's in the plane perpendicular to the depth curve of the apparent bottom which passes through this point : $\frac{\delta f}{\delta Y}$ will be equal to zero at this point; $\frac{\delta f}{\delta X}$ will be the greatest slope (*) of the apparent bottom surface at the point X, Y, Z. Let us call this P. We then have :—

$$(1a) \qquad \begin{aligned} x &= X - PZ \\ y &= Y \\ z &= Z\sqrt{1 - P^2} \end{aligned}$$

The point on the bottom which was touched by our sounding is the point x, y, z; it is a specimen of the bottom at this point which we should have, provided our soundings furnished it (the recording apparatus of the supersonic sounding machines sometimes permits us to distinguish between a hard and a soft bottom). It is the depth at this point and the position of this point which we should mark on the chart.

If we use another procedure it is necessary to make an extrapolation, which is neither accurate nor always correct, on the hypothesis that the slope of the bottom at the point x, y, z continues the same to a point directly under X, Y. On the contrary, by the method which we are about to describe, it will be noted that above very steep ravines, where the echo never reaches the bottom but is only reflected from the steep sides closest to the vessel, no definite depth can be inscribed at that point and one is therefore warned that, in order to obtain the true depth, recourse must be had to

(*) The greatest slope of a surface at any point is that of its tangent plane — that is, the trigonometrical tangent of the angle the plane makes with the horizontal. In practice, it is generally calculated by taking the quotient of the difference in height of two successive depth lines on each side of the point, by the horizontal distance separating them.

Let two depth lines differ in height by h metres and be distant from each other on the sheet by e m/m on the scale $1{:}n$ — the slope P will equal $\frac{1000\ h.}{n.e.}$

sounding by wire (See Fig. 4 further on). Another advantage which follows from this procedure is that all corrections to be applied to the soundings are negative; the soundings are never increased. This is a necessary rule of prudence, particularly in cases like this where the value of the correction to be applied is the result of an hypothesis which is not yet sufficiently verified. Soundings obtained by echo are too great and not less than the true soundings as is suggested by certain methods of correction. We therefore propose the following rule :—

After having inscribed the soundings on a provisional sheet and drawn in the depth lines of the apparent bottom surface, move all of these soundings along a line normal to the depth line passing through them, towards the lesser depths. The amount of this displacement is equal to the observed depth multiplied by the apparent slope at each point ; then *diminish* all of these soundings by an amount equal to :—

$$Z\ (1 - \sqrt{1 - P^2})$$

This operation may be done on a sheet of tracing paper; it is very quickly done. The rule given above results in a strictly accurate correction being made to the soundings in conformity with the hypothesis; it is not a method of successive approximation. The slope P to be used in these calculations is not the true slope but the slope of the apparent bottom surface, as given by the uncorrected soundings. The corrections are easy to apply, two tables only being necessary, one giving the values of PZ for various values of Z and P (or the distance between the depth lines) the other giving the values of $Z\ (1 - \sqrt{1 - P^2})$ for the different values of P and Z.

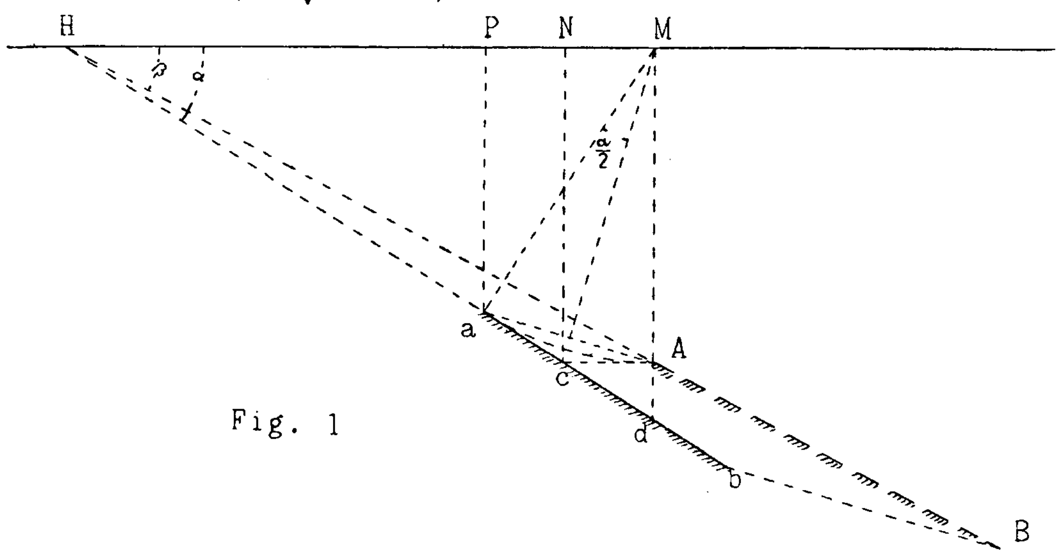

Fig. 1

In order to ascertain when the corrections may be neglected, principles analogous to those described by Mr. SHALOWITZ in the preceding Review may be applied. In the case of a regular survey one should not lose sight of the fact that it is the relative accuracy of the positions and of the depths of the soundings which must be taken into consideration rather than their absolute accuracy, which is considerably less, particularly in the case of soundings in great depths out of sight of land. If neither of the two corrections is made, this is equivalent to assuming, if the slope varies but little, that the error of position MN is negligible on the scale of the chart (See Fig. 1) since

$$MN = Z \frac{1 - \sqrt{1 - P^2})}{P},$$

or else that the error in depth, Ad, is negligible :

$$Ad = Z \frac{1 - \sqrt{1 - P^2}}{\sqrt{1 - P^2}}.$$

Tables may be drawn up giving these two quantities for the different values of Z and P. If we wish to make a correction to the sounding without correcting the position, it is necessary to increase the sounding by the quantity Ad; if we prefer to leave the sounding uncorrected and to shift position, the displacement MN must be made; but the last two methods of correction have not the accuracy of the double correction method which we have outlined and which appears to us preferable for application in all cases where the the quantities MN or Ad cannot be neglected.

If the slope P be gentle, the quantities MN and Ad may be expressed in the following simpler forms, which give a first approximation :—

$$MN = Z \frac{P}{2}$$

$$Ad = Z \frac{P^2}{2}$$

b) Let us examine a few special cases which will bring out the necessity for the corrections and the existence of certain anomalies.

In Figure 1 let the apparent bottom surface, which is a plane, be represented by its normal section through AB and let HPM represent the surface of the sea. The true surface will be the plane ab: the straight line common to the two planes is at the surface of the water at the point H. The true length ab will always be shorter than the apparent length AB. It is evident that if β is the angle made by the plane of the apparent bottom surface with the horizontal, and α is the angle made by the plane of the true bottom surface with the horizontal, we have :—

$$P = tan\ \beta = sin\ \alpha$$

The true slope p is always greater than the apparent slope P; the latter can never attain a value of 1 and β never reaches 45°.

We have :—

$$p = tan\ \alpha = \frac{P}{\sqrt{1 - P^2}}$$

The straight lines such as Aa and Bb are all parallel to the line bisecting the angle α.

If the plane AB is the plane tangent to the apparent bottom surface at the point A, the plane ab will be tangent to the true bottom surface at the point a, and the tangent at A to the depth line of the apparent bottom surface will be parallel to the tangent at a to the depth line (of different value) of the true bottom surface.

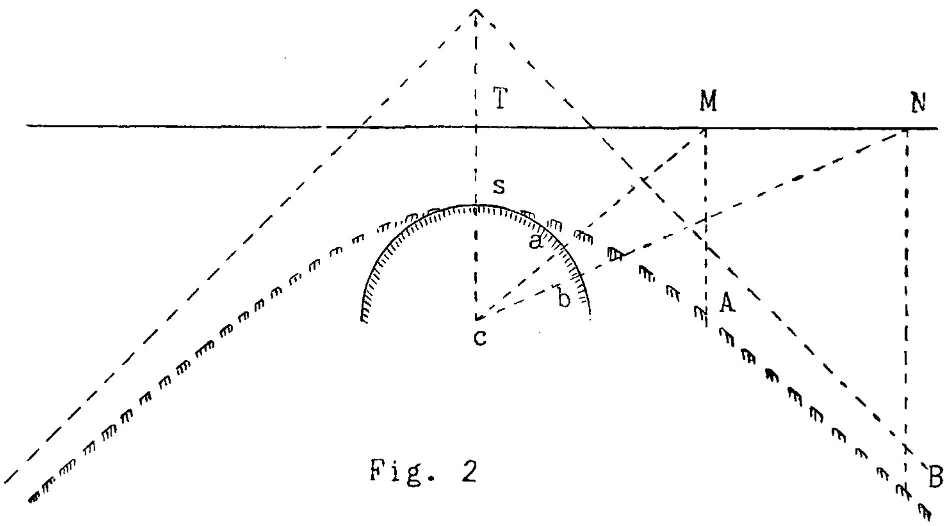

Fig. 2

Let Figure 2 represent a true bottom surface the section of which is a segment of a circle (the true surface being a portion of a sphere or of a cylinder of revolution).

If we assume, as in Fig. 2, that the surface is convex, the section of the apparent bottom surface will be represented by an equilateral hyperbola the centre of which is above the surface of the water at a distance equal to the radius of the circle; the arc AB of the apparent curve corresponding to the arc ab of the true profile and the arc AS corresponding to the arc aS. The vessel passing on the surface through the points NMT will observe the soundings NB, MA and TS by reflection from the points b, a and S.

Although the radius of the circle may be very small, the curve of the apparent surface may be a very large arc of an hyperbola (of which the centre would be very close to the surface of the water). The slopes of the apparent curve would still be less than those of the true curve and can never attain a value of 1.

The position of the centre of the hyperbola being independent of the depth of the convexity abS it follows that, the deeper this convexity the more the hyperbola is flattened out and the smaller the apparent slopes.

Figure 3 shows the case of a concavity of which the circular section has its centre below the surface of the water. The apparent curve will again become the arc of an equilateral hyperbola the centre of which, however, is below the surface of the water at a distance equal to the radius of the circle. The arc AB of the apparent curve corresponds to the arc ab of the true circle, which is on the opposite side with respect to the vertical through the centre. It will be seen later that in reality we shall never have an arc of an hyperbola such as that in Fig. 3.

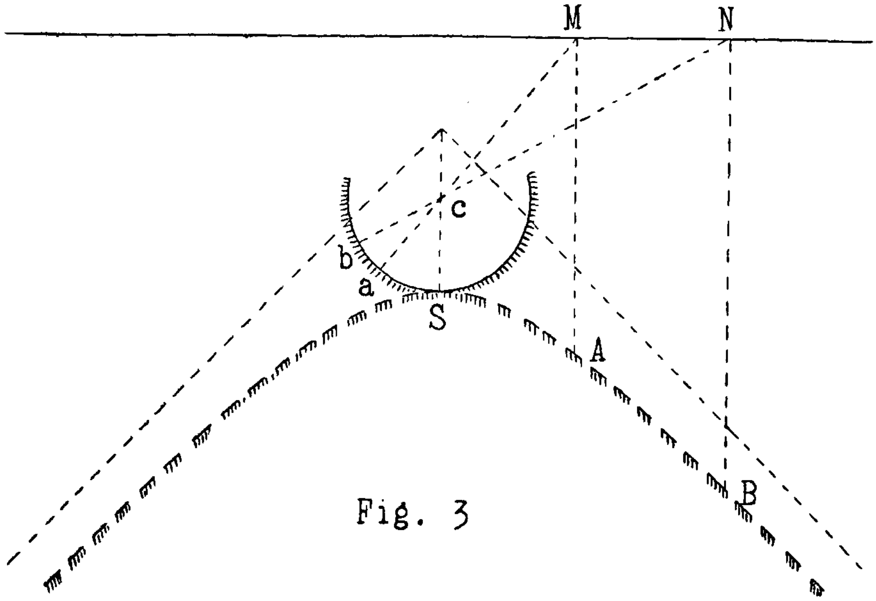

Fig. 3

Figure 4 represents the right section of a true surface formed by the planes *ab, de, fg, hi* joined by the circular cylinders *bd, ef* and *gh.*

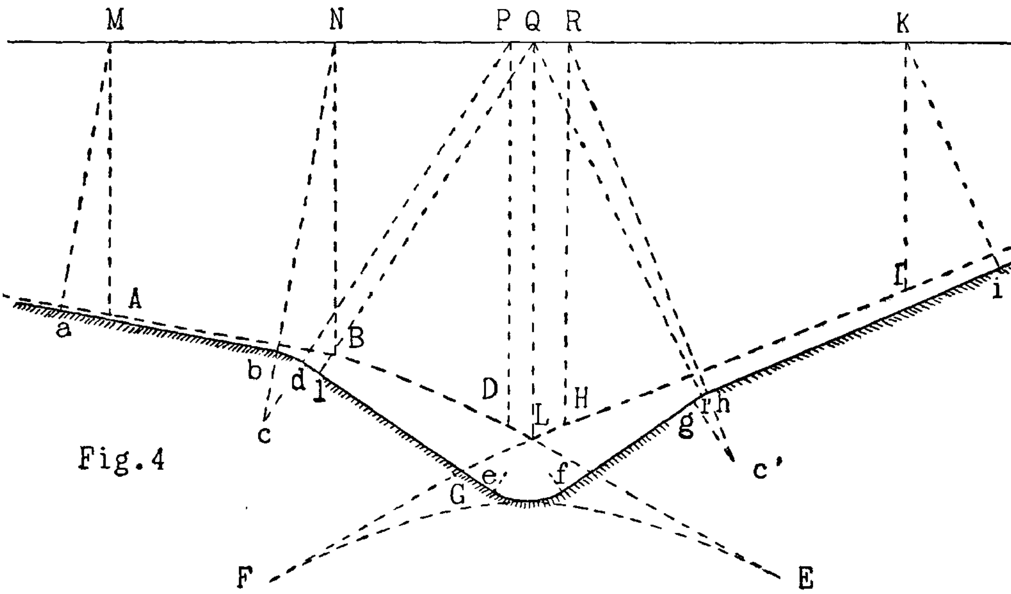

Fig.4

The section *ab* will be represented by the straight line *AB,* the arc of the circle *bd* will be the arc of an hyperbola *BD* (in accordance with the analysis of Figure 2), the representation of the section *de* would be *DE,* but beyond the point *e,* the distances from the vessel to the plane *de* will be greater than the distances to the portion *l'h* of the circle *gh.* It is therefore this portion which returns the echo and not the part *le.* Further, no echo will be received from the arc *ef,* which would be represented by *EF,* nor from the segment of the straight line *fg,* which would be portrayed by *FG.* The segment of the circle *gh* is represented by the element of an hyperbola *GH*, of which the portion *LH* only, corresponding to *l'h* need be considered; finally the segment of the straight line *hi* will be represented by *HI*.

In this case, which is very near to those which may be encountered in actual practice, we see that the apparent bottom surface is entirely above the true bottom surface; it can never be below it no matter what may be the form of the true bottom surface. The apparent bottom surface forms an angle at the point *L* while in the true bottom surface no such angle occurs. While the vessel is sounding and moving from *M* to *Q*, she will obtain good soundings corresponding to the section *al* and these soundings, corrected as previously indicated, will be inscribed on the verticals over the points comprised between *a* and *l*; and when passing from *Q* to *K*, soundings will be obtained corresponding to the points between *l'* and *i*, which should be inscribed on their verticals. But no soundings can be obtained in the region *lefgl'*, although the vessel continues sounding while passing over it; therefore no soundings should be inscribed over this region neither can they be inscribed if the corrections indicated have been applied.

We see therefore that soundings by acoustic methods are very imperfectly adapted for the study of trenches which are relatively narrow and deep, since they will give representations thereof which have both their depth and width greatly diminished; further, they fail absolutely to record their greatest depths. Thus to obtain the latter it is necessary to resort to sounding by wire.

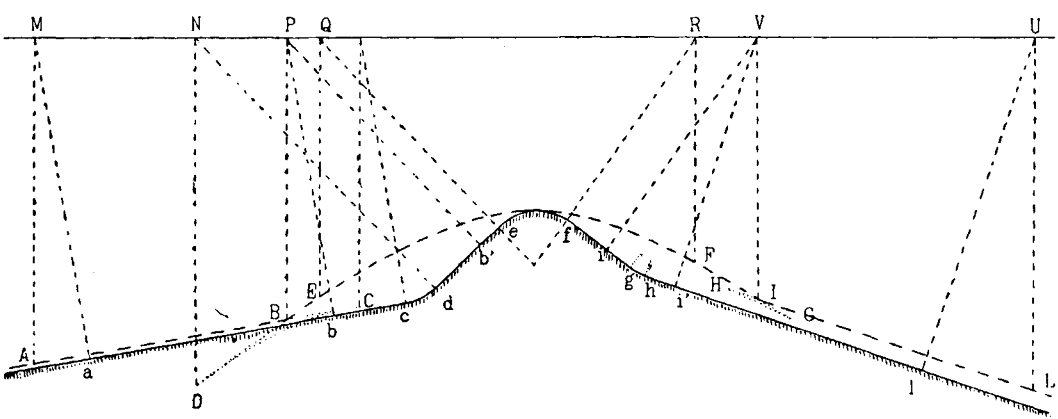

Fig. 5

Figure 5 represents the right section of a true profile of the bottom with characteristics which are opposite to those shown in the preceding figure. It presents a relief instead of a hollow. The straight line *ac* would be represented by *AC*; but after passing the point *b* the echo will be returned from the points in the region *b'e* of the plane *de*, which are closer to the vessel than the points lying in the region *bc*. Thereafter the arc of the circle *ef* will be represented by *EF*, and the element of the straight line *fi* by *FI*; while the elements of the straight lines *ig*, *hi'* as well as the concave arc *gh* will return no echo which can be utilised. Finally, the straight line *i'l* will be represented by *IL*. The apparent curve therefore shows two angular points at *B* and at *I* which are always indications of concave regions from which no echo has been received: in this case these are the portions *bcdb'* and *ighi'*. To compensate for this, the depth over the highest point has been exactly determined and the abrupt rise of the bottom in the region *b'efi* has been recorded throughout the entire length *PV* of the surface of the ocean. Thus the acoustic method of sounding is very favorable for the exploration of shoals and the search for wrecks. By making the corrections as we have indicated the soundings will accumulate in the vicinity of the summit of the shoal while no soundings will be inscribed above narrow concavities.

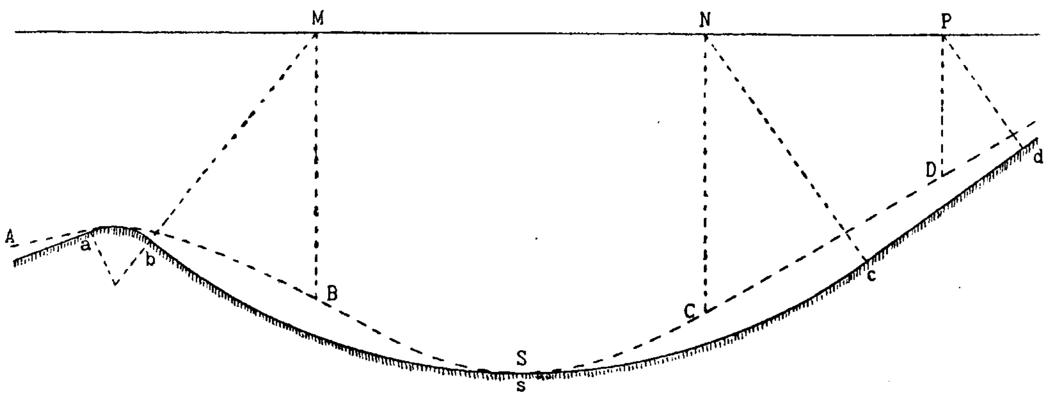

Fig. 6

Figure 6 represents a large concavity *bsc* of radius sufficiently great to have its centre above the surface of the water. It will be represented by the arc of an equilateral hyperbola, the centre of which will be below the lowest point *S* of the concavity. The arc of the circle *ab* will be represented by *AB*, an arc of the hyperbola which joins the arc *BSC* — the representation of the arc of the circle *bsc*. Then the straight line *cd* will be represented by the straight line *CD*. The depth *S* is obtained exactly and the soundings after correction will give an accurate representation of the bottom.

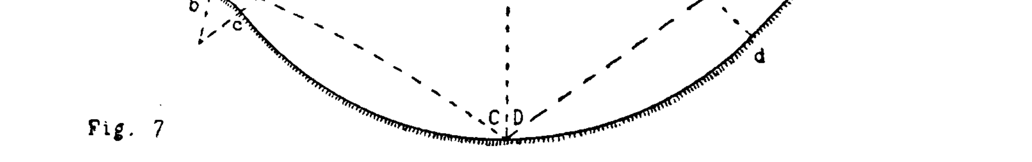

Fig. 7

Figure 7 represents a concavity in which the centre of curvature *P* is at the surface of the water. This is an intermediate case between those shown in figs. 4 and 6.

All of the echos returned from the various points of the concavity converge on point *P*, while the arc *bc* gives *BC* and the straight line *de* gives *DE* After correction no soundings can be inscribed above the hollow *cd*.

In all of these examples we have assumed that surfaces join up the various sections of different slope. This assumption appears necessary and proper, as it does not seem possible that there could be an angular point on the bottom of the sea incapable of reflecting sound in an infinite number of directions.

Figures 8, 9 and 10 are attempts to represent the true profile according to the apparent profile actually recorded with a super-sonic sounding machine off the coast of Algiers and published in the *Annales Hydrographiques*, Vol. 1927-1928, page 254. We have been forced to assume that the profiles were normal to the depth lines throughout their entire length. It is probable that this is not exactly the case and therefore these results are of interest only as representing examples which very closely approximate to actual conditions.

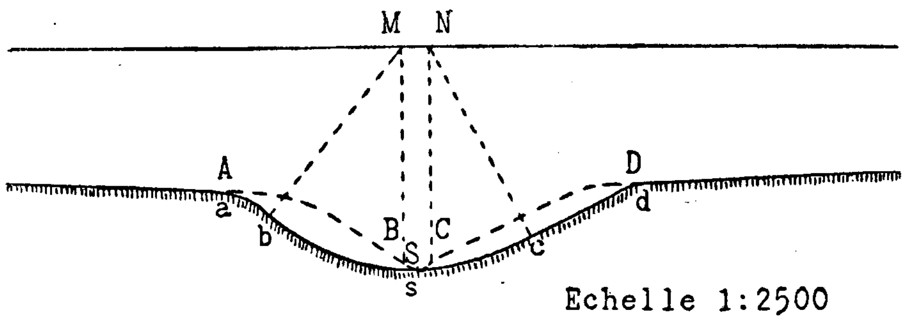

Fig. 8

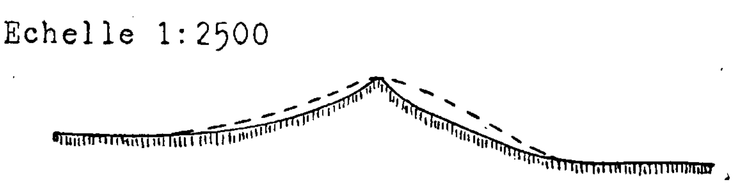

Fig. 9

Figure 8 represents a deep of a form intermediate between those shown in examples 6 and 7 ; while fig. 9 represents a peak analogous to that in fig. 5.

Fig. 10 represents a very abrupt change in slope and a slope which reaches a value of nearly 40/100. At point *D* there appears to be a sharp break, which we have smoothed out slightly because it could scarcely be pictured by an apparent bottom curve. The curve *ADB* appears to be the result of echos all of which are returned from point *a* (case of a circle of very small radius — see Fig. 2 in this connection). The slope *BC* results from echos from the slope *bc*. This profile gives no sounding whatever corresponding to the region *ab*, which, it can only be deduced, is outside a circumference

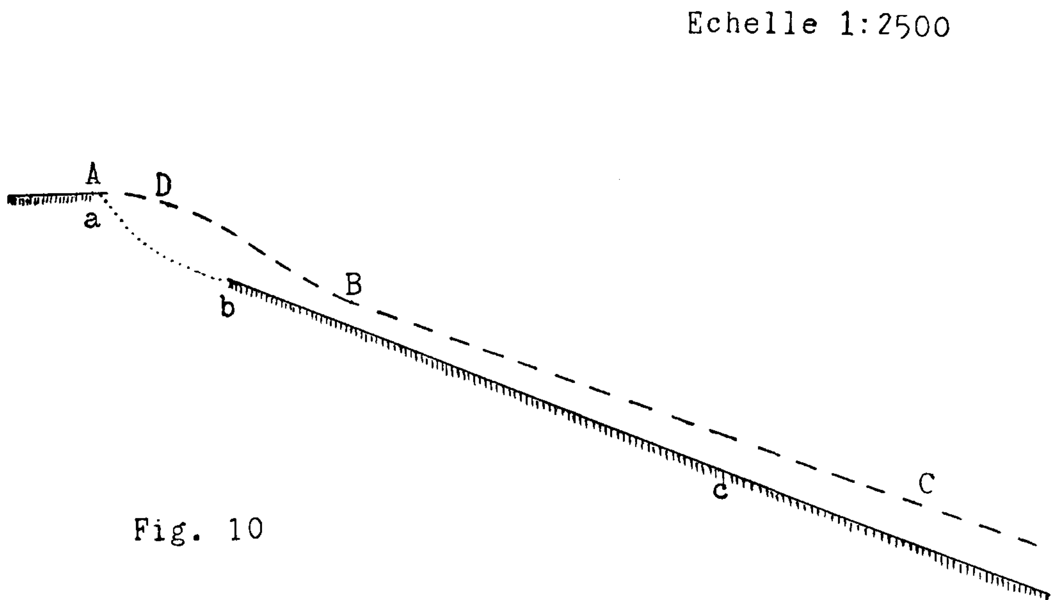

Fig. 10

having its centre at water level and which passes through points *a* and *b*, or coincides with such circumference.

c) The inverse of this problem, which has been dealt with in para. (*a*), may be treated in the same manner. It we have a chart giving details of the true depths and depth lines, it is easy to deduce from this representation of the true bottom surface an exact representation of the apparent bottom surface.

Here the quantities which we have called x, y, z are known ; and if p be the true slope at this point we have :—

$$X = x + pz$$
$$Z = z\sqrt{1 + p^2}$$

In order to obtain the apparent bottom surface it is necessary to (1) move each sounding perpendicularly to the depth curve passing through it, in the direction of increasing depth, a distance equal to the inscribed depth multiplied by the true slope at each point and (2) *increase* all the soundings by a quantity equal to $z\,(\sqrt{1 + p^2} - 1)$. But we have seen (Figs 4 and 5) that certain areas of the true bottom surface have no corresponding points in the apparent bottom surface. If the soundings on our chart were obtained by wire, these regions would be well defined by soundings. In applying the rule which we have just given for obtaining the apparent bottom surface we would be led to displace these soundings, to increase them and to inscribe them at points where other soundings of less depth have already been inscribed. These greater soundings define the portions of the apparent bottom surface corresponding to those in figs 4 and 5 which we have shown in dotted lines ; they must be suppressed because acoustic sounding cannot record them.

d) In paragraph (*a*) we started on the assumption that we had to correct a regular series of acoustic soundings, giving sufficient data to permit the various topographical details of the apparent surface to be clearly defined by depth lines. This is not always the case, however, and it frequently happens that one finds oneself in possession of an isolated profile on which closely spaced acoustic soundings have been taken.

Plotting these soundings again on a provisional sheet, without correction for slope, we note the points where the profile is intersected by the various depth lines of the apparent bottom surface. A few soundings taken earlier (regardless of the method employed in obtaining them) which may be in this region will give one a rough idea at least of the general orientation of the depth lines and consequently these may be sketched in, near the profile, with sufficient accuracy to permit the application of the method indicated in para. (*a*). In fact we have seen in para (*b*) that the tangents to the depth lines of the true bottom surface and those of the apparent bottom surface are parallel at corresponding points.

The corrections will not have the same guarantee of accuracy as in the case of a detailed system of acoustic soundings; but since they always tend to reduce the observed depths they are in accordance with the rules of prudence and it is seldom that examples will be found in which the errors made are of any great importance.

If, however, we consider that the orientation of the depth lines is entirely unknown, they would be assumed to be everywhere normal to the profile and as a result the maximum corrections would be applied.

e) We still have to investigate the case of a vessel which takes echo soundings for the purpose of fixing her position.

The strictly accurate procedure would be to transform the true soundings given on the chart, obtained either by acoustic methods or by wire, as indicated in (*c*), so as to obtain a chart giving the apparent bottom surface. The echo soundings thus obtained by the vessel may then be employed in the usual manner for determining the position of the vessel by soundings.

This procedure, which is perfectly accurate, would not take very long; but generally one could not ask the navigator to follow it at the cost of much time which would be balanced by an entirely illusive accuracy.

It will probably always suffice if the soundings obtained be inscribed on a sheet of paper, to the scale of the chart, without correction for slope and the navigator then makes the corrections as indicated in (*d*), either by estimating the approximate orientation of the depth lines from the chart or by assuming all depth lines to run at right angles to the course of the vessel. With these soundings corrected he can then locate his position on the chart in the usual manner.

We believe, however, that, in cases where it is not a question of drawing up a chart but of fixing the position of a vessel for the purposes of navigation, the slope corrections are entirely negligible. In practice if the slope is gentle, the corrections are inexistent and, further, the position of the vessel would be very inaccurately determined by soundings; if the slope is steep it will never extend over a very large horizontal area and the fact that the existence of this slope is recognised by the navigator determines his position rather than the soundings themselves, which will differ considerably depending on whether they are taken from forward or aft.

In making a landfall, one is primarily interested in whether the vessel has reached the continental shelf, which is generally indicated by differences of several hundred metres in the soundings. In greater depths, which are ordinarily out of sight of land, the soundings on the chart are rarely sufficiently numerous or located with such accuracy that the navigator can determine quantities of the order of the slope corrections. On the other hand, in shoal water the corrections for slope, which are proportional to the depth, are entirely negligible for navigational purposes. A true slope of 50/100 which, it may be said, is never encountered, gives as the value of MN (standard indicated under (*a*)) a length equal to $Z \times 0.24$. This length is entirely without interest to a vessel under way and is much less than the errors inevitably associated with the determination of the ship's position by soundings.

In order that the length MN in the open sea should reach a value of 2000 metres, *i.e.* a value approximately equal to the accuracy of an astronomical position, Z would have to exceed 8000 metres (if the slope did not exceed 50/100) — a depth which is very rarely encountered.

f) An interesting example of the application of slope corrections may be found in the bathymetric chart of the west coast of the United States between San Francisco and Descanso Point, published in 1922 by the U. S. Hydrographic Office on a scale of about 1:861,400, from a series of acoustic soundings. The depth lines, which are inserted at each 100 fathoms in depths less than 2000 fathoms, give the impression of a very irregular bottom surface. The greatest slope is found in about Lat. 33° Long. 120°35', between 1500 and 2000 fathoms. If we assume that no corrections have been made for slope, the surface represented on the chart by the depth lines is that which we have called the apparent bottom surface. The apparent slope in the area indicated above is $P = 0.53$; the true slope would be $p = 0.625$.

Applying the rule given under (*a*), the position in which the echo indicates a depth of 2000 fathoms should be moved towards the lesser depths 1938.5 m. (2120 yards), *i.e.* 2.25 m/m on the scale of the chart. The depth which should be shown at this point is 1696 fathoms. The position where the echo gave 1500 fathoms should be moved in the same direction a distance of 1453.9 m. (1590 yards), which will equal 1.69 m/m on the scale of the chart. The depth to be inscribed at this point should then be 1272 fathoms.

If we assume that the slope continues unchanged beyond the 1696 fathoms (the actual depth when the apparatus indicated 2000 fathoms) we may say that, retaining this figure of 2000 fathoms depth, it should be moved 1049 m. (1147 yards) *i.e.* 1.22 m/m, on the chart. If, on the contrary, we retain the position of the 2000 fathoms sounding, according to same hypothesis it should be increased by 358 fathoms and a depth of 2358 fathoms recorded instead of 2000.

If it be considered that a length of ½ m/m is negligible on this chart, the length we have called MN will not attain this value if the depth given by echo is less than 1502 m. (821 fathoms) even though the slope reaches the maximum value we have noted, namely, 0.53. Within the 800 fathoms line, therefore, we may dispense with slope corrections even though the bottom surface be very uneven.

Mai 1930.