ANSERMET'S SOLUTION OF THE PROBLEM OF SNELLIUS

by

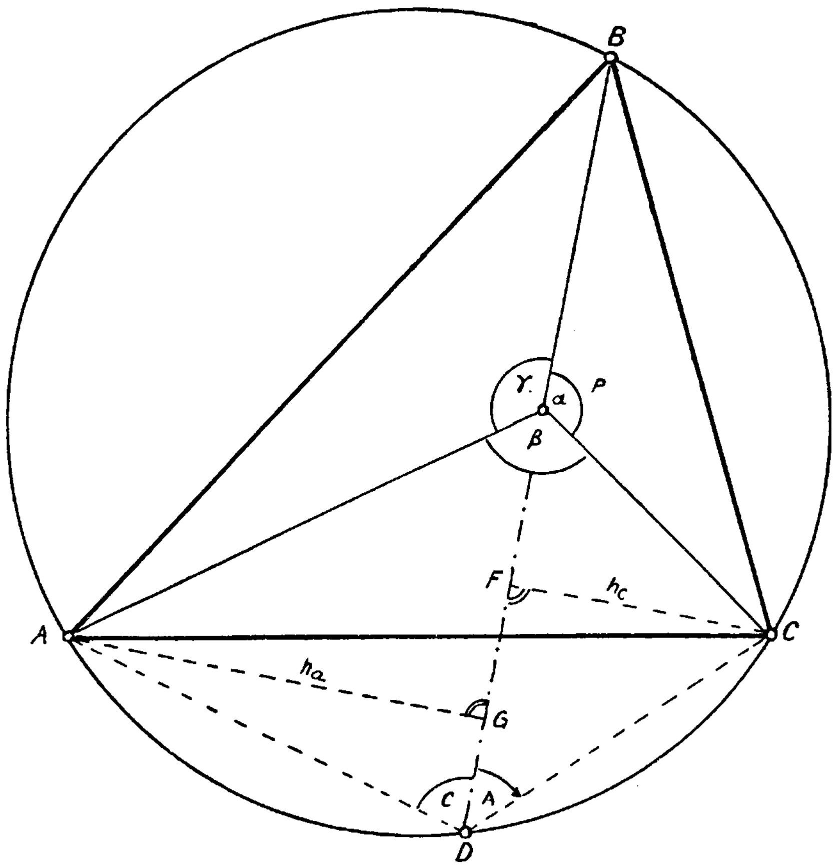
INGÉNIEUR HYDROGRAPHE GÉNÉRAL P. DE VANSSAY DE BLAVOUS, DIRECTOR.

Dr. KERL published in the *Allgemeine Vermessungs Nachrichten*, N° 8, of 22nd February, 1933, a study of the solution of the SNELLIUS Problem as propounded by the Swiss ANSERMET in 1912. This solution appears to us sufficiently curious and little known to be of interest to readers of the Review.

Let A, B, C be control points between which the angles α, β, γ have been observed from a point P. It is possible to determine the weights p_a, p_b, p_c which, applied to A, B, C, will have their resultant passing through the point P. In fact, if the straight line CP meets AB at a point E, it is possible to find weights p_a and p_b the resultant of which passes through E; p_c can then be determined in such a way that by combining it with this resultant the total resultant will pass through P. The weights are only fixed to within one common factor. It is obvious on the other hand that this determination is impossible if the three points A, B, C are in a straight line, for the point of application of the resultant will perforce be on this straight line.

We can easily find these weights by the following procedure :

Let us consider the circle circumscribed about the triangle, and let D be the point of intersection of this circle and the straight line BP; the angles PDC and PDA are equal respectively to BAC and BCA. From A and C drop perpendiculars AG and CF to BD, and let h_a and h_c be their lengths.



We shall have:

$$\frac{p_a}{p_c} = \frac{h_c}{h_a}$$

$\frac{h_c}{h_a}$ may be written:

$$\frac{\frac{h_c}{DP}}{\frac{h_a}{DP}} = \frac{\frac{h_c}{DF + FP}}{\frac{h_a}{DG + GP}} = \frac{\frac{1}{cot A - cot \alpha}}{\frac{1}{cot C - cot \gamma}}$$

This and circular permutation leads us to the equations:

$$(1) \qquad \frac{p_a}{\frac{1}{cot A - cot \alpha}} = \frac{p_b}{\frac{1}{cot B - cot \beta}} = \frac{p_c}{\frac{1}{cot C - cot \gamma}}$$

We can take the denominators of these equations as values of p_a, p_b, p_c, and the well-known formula for the centre of gravity will give the coordinates x, y of the point P as a function of the coordinates of the points A, B, C:

$$x = \frac{p_a\, x_a + p_b\, x_b + p_c\, x_c}{p_a + p_b + p_c}$$

$$y = \frac{p_a\, y_a + p_b\, y_b + p_c\, y_c}{p_a + p_b + p_c}$$

If the points A, B, C are in a straight line, these equations take the form $\frac{0}{0}$, and cannot be used, as we have already seen.

If the point P is on the circumference of the circumscribed circle, the denominators of formulae (1) become infinite, and thus these formulae no longer provide a solution.

NOTE. — This same property is utilised in an article published in the *Empire Survey Review* for October 1932, N° 6, Vol. I, page 275. It appears to have been indicated for the first time by NEUBERG in 1889 and reproduced in 1891 in the treatise on geometry by ROUCHÉ and COMBEROUSSE (6th edition).