MEAN SEA LEVEL AND HALF-TIDE LEVEL

by

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As reference planes for elevations, mean sea level and half-tide level belong to the general class of tidal datum planes, in that they are derived from tide observations. And in common with all tidal datum they possess the advantages of simplicity of definition, accuracy of determination, readiness of correlation, and certainty of recovery even though all bench-mark connection be lost.

Of all tidal datum planes, mean sea level possesses in most marked degree the advantage of readiness of correlation. Charts or maps, whether of the same region or of different regions, are directly comparable in regard to elevations, if the datum used is that of mean sea level; but if some other datum is used, direct comparison is generally not possible, and correlation must be made through the relation of that datum to mean sea level.

In regard to accuracy of determination, too, mean sea level heads the list of tidal datum planes; for from a given series of observations mean sea level is the most readily and most accurately determined datum plane. In fact, all other tidal datum planes are best determined by reference to mean sea level. It is the possession of all these advantages that makes mean sea level the basic and most universally used datum plane for elevations.

Since it falls to the lot of the engineer — specifically, the tidal engineer — to determine and establish the various tidal datum planes, it is important that engineering usage in regard to terminology relative to these datums be precise. A clear understanding of the relation of mean sea level to half-tide level, which datum is frequently taken as synonymous with mean sea level, is thus of importance.

In regard to both definition and determination, mean sea level and half-tide level differ. The former is defined as the mean level of the sea and is determined by averaging the height of the sea as measured at frequent intervals. Half-tide level is defined as the plane that lies exactly half-way between the planes of mean high water and mean low water, and is determined by averaging the heights of the high and low waters.

It is now customary in tidal work to make two different tabulations of the tidal record. The first, known as the hourly height tabulation, gives for each day the height of the tide at the beginning of each hour. The second, called the high and low water tabulation, gives for each day the times and heights of the high and low waters. From the former tabulation, therefore, the average height of sea level for any period may be readily determined by averaging the hourly heights of the tide during that period. From the high and low water tabulation, the average heights of high and low water are readily determined and thus the datum of half-tide level.

Prior to the invention of the automatic tide gauge, however, the reading of the height of the tide throughout the 24 hours of the day necessitated the services of relays of observers and thus involved very considerable expense. It was therefore customary to observe the tide only near the times of high and low water. This permitted a tabulation of the high and low waters of the tide but not of the hourly heights. Half-tide level could be determined from such tabulations, but not mean sea level. Hence, almost without exception, the earlier determinations were those of the plane of half-tide level.

It is clear that, since high and low water represent, respectively, the maximum and minimum points of the tidal oscillation, the datum of half-tide level will not differ much from that of mean sea level, which is determined by averaging the hourly heights of the tide. Indeed, if the curve representing the rise and fall of the tide were that of a simple sine curve, the planes of mean sea level and of half-tide level would coincide exactly. The movement of the tide, however, is compounded of the movements of a number of simple sine curves, some of which have fixed phase relations with respect to each other. The rise of high water above sea level is therefore, as a rule, not exactly the same as the fall of low water below sea level, and hence mean sea level and half-tide level generally differ.

VARIATIONS IN SEA LEVEL AND HALF-TIDE LEVEL.

Both sea level and half-tide level vary from day to day, from month to month, and from year to year, so that, for a direct determination of either of these planes at any point a number of years of observations is necessary. However, the tidal engineer has developed methods for correcting the results from short series of observations to mean values, so that the datums of mean sea level and half-tide level at any point may be derived with considerable precision from short series of observations.

On investigation, it is found that from one day to the next both sea level and half-tide level may vary by as much as half a foot or more, while within a month the difference between two days may be as much as 2 feet. This is illustrated in Fig. 1 which shows also that despite the relatively large changes in sea level and in half-tide level at Boston throughout the month of April, 1923, the relation between the two remained very nearly constant. To be sure, this relation changes somewhat from day to day, but these changes are relatively small, and specially so as compared to the change in sea level or half-tide level from day to day. Almost without exception, sea level is seen to have been above half-tide level — on the average by 0.1 foot. The greatest difference between the two was for the twenty-fifth when this difference amounted to 0.3 foot.

It is not difficult to see why the relation of sea level to half-tide level is not constant from day to day, for changes in wind and weather must obviously bring about changes in this relation. As an example, suppose at a given place we take two days during which the high waters, and likewise the low waters, were exactly similar. Half-tide level for the two days would therefore be exactly the same. And if the weather conditions during the two days were similar, sea level likewise would be the same.

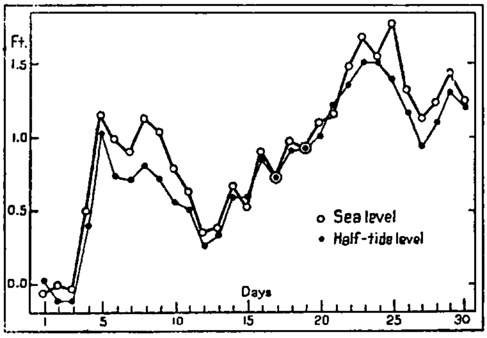

Fig. 1, - *Daily levels at Boston, April 1923*

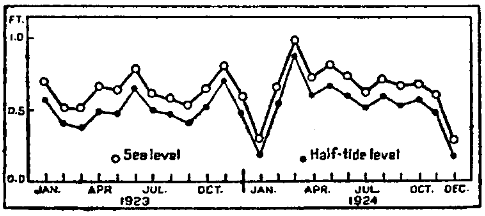

Fig. 2. - *Monthly levels at Boston, 1923-1924*

Suppose, however, that the weather conditions on the second of the two days were the same as on the first day only until the occurrence of the last high or low water of the day (which, for the sake of illustration, we may assume to have occurred about 6 p. m.). Suppose that from that time to the end of the day the direction or velocity of the wind was different. Obviously, the half-tide level for that day would not be changed, since the last high or low water used in deriving it has already occurred. But the hourly heights of the tide for the remainder of the day would differ from the corresponding heights on the first day and hence, although half-tide level for the two days would still be the same, the sea levels would differ.

If monthly heights of sea level and half-tide level are investigated, it is found that the variations are much smaller than in the case of the daily levels. From month to month it is seen from Figure 2 that both sea level and half-tide level vary from one to three tenths of a foot, while within a year, two different months may vary by more than half a foot.

The diagrams in Figure 2 bring out clearly the fact that the relation between monthly values of sea level and of half-tide is very nearly constant. Without exception, sea level for each month is seen to have been above half-tide level. For the 2-year period shown in the figure, sea level was 0.12 foot above half-tide level. The closeness with which the individual monthly values approximated this mean value is indicated by the fact that the least difference between corresponding monthly values of sea level and half-tide level was 0.10 foot, while the greatest difference was 0.17 foot.

For continuing the investigation on the variations of sea level and half-tide level at Boston by comparing yearly values of these levels, there are at hand observations covering the period 1922 to 1930. These observations prove that in general, from year to year both sea level and half-tide level at Boston vary by several hundredths of a foot, although at times this may amount to a tenth of a foot or even more. But as regards the relation of sea level to half-tide level, this remains practically constant. For this 9-year period, sea level was 0.12 foot above half-tide level. Of the individual years, seven gave this same difference of 0.12 foot between the yearly values of sea level and half-tide level, while two years gave a difference of 0.13 foot.

The features characterizing sea level and half-tide level at Boston are found to be the characteristic features all along the Atlantic coast of the United States. With but rare exceptions, sea level, from Maine to Florida, lies above half-tide level; and, while the difference between the two varies somewhat from place to place, it is generally about a tenth of a foot. Furthermore, from one year to another this relation is practically constant, differences of more than 0.02 foot from a mean value being rare.

CONDITIONS ALONG THE PACIFIC COAST.

A study of sea level and half-tide level along the Pacific coast of the United States brings out the fact that, in so far as the variations in sea level or half-tide level are concerned, these variations from day to day, month to month, and year to year are of the same character as on the Atlantic coast. In regard to the relation between the two levels, however, conditions on the Pacific coast are different. In the first place, half-tide level lies above sea level on the Pacific coast, instead of below as is the case on the Atlantic coast. And in the second place, the difference between sea level and half-tide level on the Pacific coast is not constant from year to year, but exhibits a distinct periodicity as is evident from Figure 3, which represents the difference between the yearly heights of sea level and half-tide level at San Francisco, California, for the 33-year period 1898 to 1930.

In the diagram the distance from the lower horizontal line to the open circles represents the difference between sea level and half-tide level for the given years, while the upper horizontal line represents the average height of half-tide level above mean sea level for the 33 years of observations shown. For this period of time the average difference was 0.054 foot, but during this period it varied from 0.085 foot in 1914 to 0.028 foot in 1924. Despite irregularities in this variation from year to year, which are to be expected from the nature of tide observations, there is unmistakable evidence of the existence of a periodic variation in the relation of half-tide level to sea level with a period of about 19 years. The range of this variation is seen to be about 0.03 foot each side of its mean value.

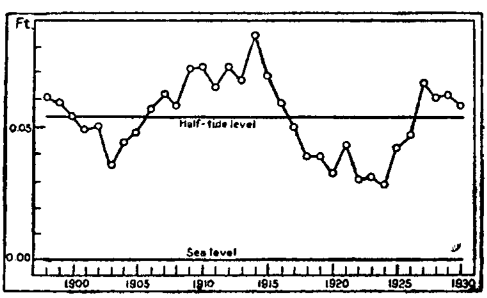

Fig. 3. - *The yearly levels at San Francisco*

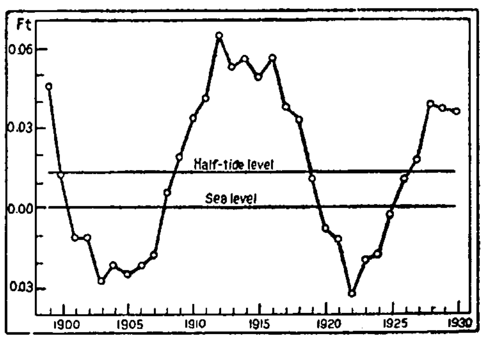

Fig. 4. - *The yearly levels at Seattle*

At Seattle, Washington, the range of the variation in the relation of yearly half-tide level to sea level is just a little larger than at San Francisco. However, the mean difference between sea level and half-tide level at Seattle is only 0.013 foot. At Seattle, therefore, for certain years of the 19-year cycle, half-tide level is below sea level, while for other years it is above sea level. This is shown in Figure 4, in which the periodic character of the variation in the relation of sea level to half-tide level at Seattle is exhibited more unmistakably even than at San Francisco. But since, on the average, half-tide level is but 0.013 foot above sea level, while the range of the variation is a little over 0.03 foot each side its mean value, it follows that part of the time half-tide level is below sea level, while part of the time it is above.

THEORETICAL RELATION OF HALF-TIDE LEVEL TO SEA LEVEL.

In seeking for an explanation of the difference between the Atlantic and Pacific coasts as regards the relation of half-tide level to sea level, it is necessary to note that the tide actually observed at any place is made up of a number of simple constituent tide waves of different periods. By means of the harmonic analysis of the tide, the amplitude and phases of these simple constituent tides can be determined.

Now it is the relation between the amplitudes and phases of these constituent tides that determines the relation of half-tide level to sea level. In the theory of tides it is shown that this relation may be expressed by the following formula:

$$HTL = SL + M_4 \cos\,(2M^\circ_2 - M^\circ_4) - \left[0.04\,(K_1 + O_1)^2/M_2\right] \cos\,(M^\circ_2 - K^\circ_1 - O^\circ_1).$$

In this formula *HTL* stands for half-tide level, *SL* for sea level, and the other terms have their usual significance in the harmonic notation of the tide. Since the amplitudes of the various components vary somewhat from year to year, it follows that the relation between sea level and half-tide level may differ from year to year. Furthermore, the *cosine* of $(2M^\circ_2 - M^\circ_4)$ and also of $(M^\circ_2 - K^\circ_1 - O^\circ_1)$ may be either positive or negative. Hence sea level may be either above or below half-tide level, depending upon the phase and amplitude relations.

Along the Atlantic coast of the United States $\cos\,(2M^\circ_2 - M^\circ_4)$ is generally negative, while $\cos\,(M^\circ_2 - K^\circ_1 - O^\circ_1)$ is generally positive. Hence along this coast half-tide level is below sea level, with but few exceptions. Along the Gulf coast both *cosine* terms in the formula are generally positive, so that here half-tide level may be either above or below sea level, depending upon which term has the greater value. On the Pacific coast the first *cosine* term is positive at some places, while at others it is negative; the second *cosine* term, however, is generally negative. Here, however, $(K_1 + O_1)^2/M_2$ is, as a rule, more than 25 times as great as M_4, and therefore at most places along this coast half-tide level is above sea level.

The periodic variation in K_1 and O_1 from year to year is much greater than in M_4. Hence, where the ratio of $(K_1 + O_1)^2$ to M_2 is large, appreciable variations in the relation of half-tide level to sea level may be expected from year to year. On the Atlantic coast this ratio is small, being at most places about 0.1, but on the Pacific coast it is relatively large, being at most places greater than unity. It is therefore to be expected that the relation of half-tide level to sea level will differ but little from year to year on the Atlantic coast, while on the Pacific coast larger differences will appear.

The theoretical relation as expressed in the above formula also explains why the relation between sea level and half-tide level varies through a period of about 19 years. For the amplitudes of the two constituent tides denoted by K_1 and O_1 in the formula depend on the declination of the moon, and this goes through a period of 18.6 years. Hence, the variation in the relation of sea level to half-tide level goes through a like period of 18.6 years.