NOTE ON COMPUTATION OF DENSITY OF SEA-WATER AND ON CORRECTIONS FOR DEEP-SEA REVERSING-THERMOMETERS

by

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In the reductions of the oceanographic observations made on board the *Carnegie* during her seventh cruise, it was found quite necessary to devise methods by which the great amount of computational work involved might be simplified and reduced.

A considerable portion of this work was the determination of the density of sea-water from its values of salinity and temperature, for which purpose special tables were prepared in the Department of Terrestrial Magnetism.

Table 1 is a specimen sheet of the table prepared for computing the density σ_t, being based on the formula :

$$\sigma_t = \Sigma_t + (\sigma_o + 0.1324)\left[(1 - A_t + B_t(\sigma_o - 0.1324)\right] \qquad (1)$$

together with the values of the involved constants as given in KNUDSEN'S *Hydrographical Tables*.

Experience has proved the table more satisfactory than graphs because of the more or less unwieldy graphs resulting from the scale requirements imposed by the requisite degree of refinement.

Table 2 gives the corrections for depth and temperature and for depth and salinity necessary to reduce the values of density, σ_t, to those *in situ*, σ_{td}. It is a modification of the tables of HESSELBERG and SVERDRUP to the extent that the separate corrections for depth and for temperature of the latter tables have been combined, thus reducing the number of entries from three to two.

A similar modification was made of the HESSELBERG and SVERDRUP correction-tables for computing specific volume and dynamic depth.

The accompanying graph was devised for determining the corrections for unprotected deep-sea reversing-thermometers. It is based on the formula for correction :

$$\Delta_t = \frac{(T_w + V_o)(T' - t)}{K} \qquad (2)$$

in which T_w is the recorded temperature of the unprotected thermometer, T' the recorded temperature of main thermometer, t is the recorded temperature (corrected) of auxiliary thermometer, V_o is the volume of broken-off column of mercury at 0°, and K the coefficient of expansion of the glass (Jena 59^{III} for the thermometers used on the *Carnegie*, for which $K = 6100$).

TABLE 1. — Specimen sheet Table for determining density of sea-water for various values of salinity and of temperature.

(Tabular values give excess of density over unity in units of fifth decimal: thus for $S = 34.2$ 0/00 and $t = 4°55$ C, density is 1.02711).

TABLE 1. — *Spécimen de la Table pour la détermination de la densité de l'eau de mer pour diverses valeurs de la salinité et de la température.*

(Les valeurs inscrites dans la Table donnent le supplément de densité à ajouter à l'unité, exprimé en unités de la cinquième décimale: par exemple pour S = 34,2 0/00 *et pour* t = 4°,55 C, *la densité sera* 1,02711).

Temperature, t	Salinity, S, in 0/00										
	34.0	34.1	34.2	34.3	34.4	34.5	34.6	34.7	34.8	34.9	35.0
°											
4.00	2701	2709	2717	2725	2733	2741	2749	2757	2765	2773	2781
05	01	09	17	25	33	40	48	56	64	72	80
10	00	08	16	24	32	40	48	56	64	72	80
15	2700	08	16	23	31	39	47	55	63	71	79
20	2699	07	15	23	31	39	47	55	63	71	79
25	99	07	14	22	30	38	46	54	62	70	78
30	98	06	14	22	30	38	46	54	62	69	77
35	97	05	13	21	29	37	45	53	61	69	77
40	97	05	13	21	29	37	45	52	60	68	76
45	96	04	12	20	28	36	44	52	60	68	76
4.50	2696	2704	2712	2720	2728	2736	2743	2751	2759	2767	2775
55	95	03	11	19	27	35	43	51	59	67	75
60	95	03	11	19	26	34	42	50	58	66	74
65	94	02	10	18	26	34	42	50	58	66	74
70	94	02	10	17	25	33	41	49	57	65	73
75	93	01	09	17	25	33	41	49	57	64	72
80	93	01	08	16	24	32	40	48	56	64	72
85	92	2700	08	16	24	32	40	48	55	63	71
90	92	2699	07	15	23	31	39	47	55	63	71
95	91	99	07	15	23	31	38	46	54	62	70
5.00	2690	2698	2706	2714	2722	2730	2738	2746	2754	2762	2770
05	90	98	06	14	22	29	37	45	53	61	69
10	89	97	05	13	21	29	37	45	53	60	68
15	89	97	04	12	20	28	36	44	52	60	68
20	88	96	04	12	20	28	36	43	51	59	67
25	87	95	03	11	19	27	35	43	51	59	67
30	87	95	03	11	18	26	34	42	50	58	66
35	86	94	02	10	18	26	34	42	50	57	65
40	86	94	01	09	17	25	33	41	49	57	65
45	85	93	01	09	17	25	32	40	48	56	64
5.50	2684	2692	2700	2708	2716	2724	2732	2740	2748	2756	2763
55	84	92	2700	08	15	23	31	39	47	55	63
60	83	91	2699	07	15	23	31	39	46	54	62
65	83	91	98	06	14	22	30	38	46	54	62
70	82	90	98	06	14	22	29	37	45	53	61
75	81	89	97	05	13	21	29	37	45	53	60
80	81	89	97	05	12	20	28	36	44	52	60
85	80	88	96	04	12	20	28	35	43	51	59
90	80	87	95	03	11	19	27	35	43	51	59
95	79	87	95	03	11	18	26	34	42	50	58
6.00	2678	2686	2694	2702	2710	2718	2726	2734	2742	2749	2757
05	78	86	94	01	09	17	25	33	41	49	57
10	77	85	93	01	09	17	24	32	40	48	56
15	76	84	92	2700	08	16	24	32	40	47	55
20	76	84	92	2699	07	15	23	31	39	47	55
25	75	83	91	99	07	15	22	30	38	46	54
30	74	82	90	98	06	14	22	30	38	45	53
35	74	82	90	97	05	13	21	29	37	45	53
40	73	81	89	97	05	13	20	28	36	44	52
45	72	80	88	96	04	12	20	28	36	43	51
6.50	2672	2680	2688	2695	2703	2711	2719	2727	2735	2743	2751
55	71	79	87	95	03	11	18	26	34	42	50
60	70	78	86	94	02	10	18	26	34	41	49
65	70	78	86	93	01	09	17	25	33	41	49
70	69	77	85	93	01	09	16	24	32	40	48
75	69	76	84	92	2700	08	16	24	32	39	47
80	68	76	84	91	2699	07	15	23	31	39	47
85	67	75	83	91	99	07	14	22	30	38	46
90	67	74	82	90	98	06	14	22	30	37	45
95	66	74	82	89	97	05	13	21	29	37	45
7.00	2665	2673	2681	2689	2697	2705	2712	2720	2728	2736	2744

TABLE 2. — Corrections for depth and temperature and for depth and salinity to obtain density of sea-water *in situ* (σ_{td}).
(Tabular values are in units of fifth decimal).

TABLE 2. — *Corrections « profondeur - température » et corrections « profondeur - salinité » pour l'obtention de la densité de l'eau de mer* in situ (σ_{td}).
(Les valeurs inscrites dans la Table sont exprimées en unités de la cinquième décimale).

Depth dynamic meters.	Temperature, *t*, in degrees centigrade.																	Salinity, *S*, in 0/00											Depth dynamic meters.
	−2	−1	0	1	2	3	4	5	6	7	8	9	10	15	20	25	30	30	31	32	33	34	35	36	37	38	39	40	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	5
25	12	12	12	12	12	12	12	12	12	12	12	12	12	11	11	11	11	0	0	0	0	0	0	0	0	0	0	0	25
50	25	25	25	25	25	25	25	25	24	24	24	24	24	23	23	23	23	0	0	0	0	0	0	0	0	0	0	0	50
75	38	38	37	36	36	36	36	36	35	35	35	35	35	35	34	34	34	0	0	0	0	0	0	0	0	0	0	0	75
100	50	49	49	49	48	48	48	48	47	47	47	47	47	46	45	45	45	1	1	0	0	0	0	0	0	0	−1	−1	100
150	75	74	74	73	73	73	72	72	71	71	71	71	70	69	68	67	66	1	1	1	0	0	0	0	0	−1	−1	−1	150
200	99	99	98	97	97	96	96	95	95	94	94	94	93	92	90	89	87	2	1	1	1	0	0	0	−1	−1	−1	−2	200
250	124	124	123	122	121	121	120	120	119	118	118	117	117	115	113	111		2	2	1	1	0	0	0	−1	−1	−1	−2	250
300	149	148	147	146	145	144	144	143	142	142	141	140	140	137	136	134		2	2	1	1	0	0	0	−1	−1	−2	−2	300
400	198	197	196	195	194	193	192	191	190	189	188	187	187	183	181	179		3	3	2	1	1	0	−1	−1	−2	−3	−3	400
500	248	246	245	244	242	241	240	238	237	236	235	234	233	229	226			4	3	3	2	1	0	−1	−2	−3	−3	−4	500
700	346	344	342	340	338	336	334	333	331	330	328	327	326	320	316			6	5	3	2	1	0	−1	−2	−3	−5	−6	700
1000	493	490	487	484	482	479	476	474	472	470	468	466	464	456				8	7	5	3	2	0	−2	−3	−5	−7	−8	1000
1500	736	732	728	724	719	716	712	709	705	702	699	696	694	682				12	10	8	4	2	0	−2	−4	−8	−10	−12	1500
2000	977	971	965	960	954	949	945	940	936	931	927	924	920	905				16	13	10	6	3	0	−3	−6	−10	−13	−16	2000
2500	1214	1207	1200	1193	1187	1181	1175	1169	1164	1159	1154	1149	1145	1126				20	16	12	8	4	0	−4	−8	−12	−16	−20	2500
3000	1451	1442	1434	1426	1419	1411	1404	1398	1391	1385	1379	1374	1369	1347				23	19	14	9	5	0	−5	−9	−14	−19	−23	3000
3500	1684	1674	1665	1656	1647	1639	1631	1623	1616	1609	1602	1596	1590	1565				27	22	16	11	5	0	−5	−11	−16	−22	−27	3500
4000	1915	1903	1893	1883	1873	1864	1855	1846	1838	1830	1823	1816	1809	1780				30	24	18	12	6	0	−6	−12	−18	−24	−30	4000
4500	2144	2132	2120	2109	2098	2088	2078	2068										34	27	20	14	7	0	−7	−14	−20	−27	−34	4500
5000	2371	2358	2345	2333	2321	2309	2299	2288										37	30	22	15	8	0	−8	−15	−22	−30	−37	5000
5500	2596	2581	2567	2553	2541																16	8	0	−8	−16				5500
6000	2819	2803	2788	2773	2760																17	9	0	−9	−17				6000

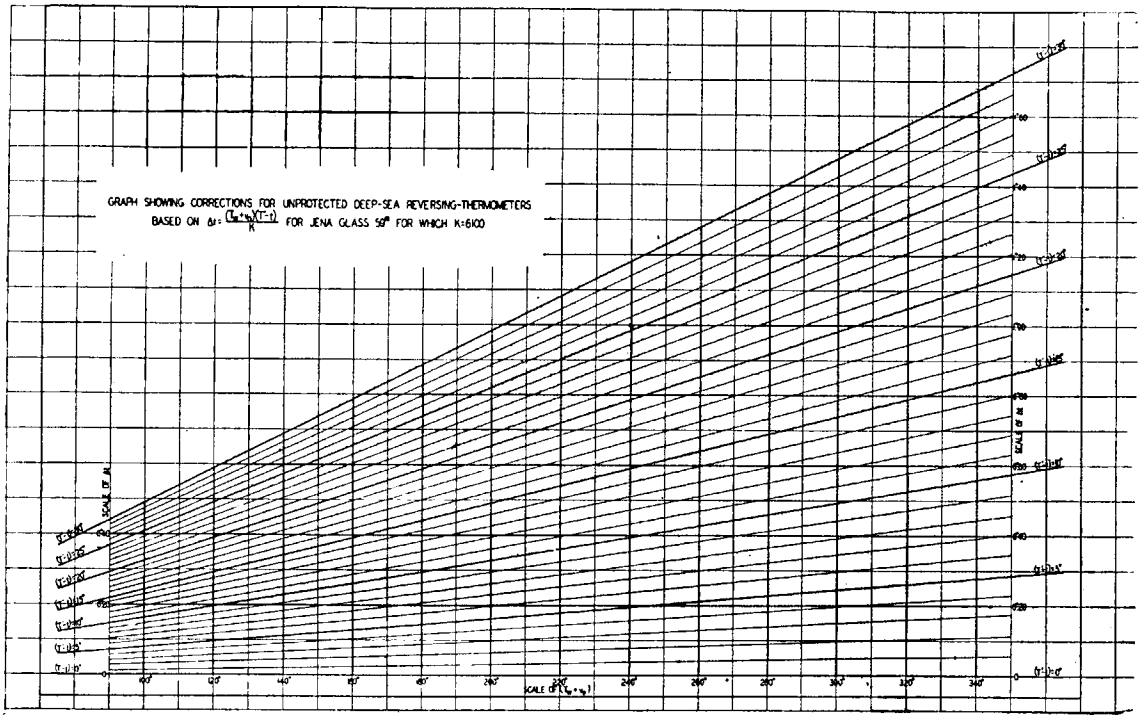
TABLE 3. — Corrections for protected deep-sea reversing-thermometer because of differences between observed reading T' and reading t of auxiliary attached thermometer; total correction Δ_t is sum of tabular value (negative for negative values of $T' - t$) and index-correction I*.

TABLE 3. — *Corrections à apporter aux indications des thermomètres plongeurs à renversement par suite de la différence entre la lecture observée* T' *et la lecture* t *du thermomètre auxiliaire. La correction totale* Δ_t *est la somme de la valeur donnée par la Table (négative pour des valeurs négatives de* T' — t) *et de la correction d'index* I*.

Obs'd temp.-diff. ($T' - t$)	($T' + V_0$) in degrees centigrade.									
	91	92	93	94	95	96	97	98	99	100
0	0	0	0	0	0	0	0	0	0	0
1	0.015	0.015	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.017
2	0.030	0.031	0.031	0.031	0.032	0.032	0.032	0.033	0.033	0.033
3	0.045	0.046	0.046	0.047	0.047	0.048	0.048	0.049	0.049	0.050
4	0.061	0.061	0.062	0.063	0.063	0.064	0.065	0.065	0.066	0.067
5	0.076	0.077	0.077	0.078	0.079	0.080	0.081	0.082	0.082	0.083
6	0.091	0.092	0.093	0.094	0.095	0.096	0.097	0.098	0.099	0.100
7	0.106	0.107	0.108	0.110	0.111	0.112	0.113	0.114	0.115	0.117
8	0.121	0.122	0.124	0.125	0.127	0.128	0.129	0.131	0.132	0.133
9	0.136	0.138	0.139	0.141	0.142	0.144	0.145	0.147	0.148	0.150
10	0.151	0.153	0.155	0.156	0.158	0.160	0.162	0.163	0.165	0.167
11	0.167	0.168	0.170	0.172	0.174	0.176	0.178	0.180	0.181	0.183
12	0.182	0.184	0.186	0.188	0.190	0.192	0.194	0.196	0.198	0.200
13	0.197	0.199	0.201	0.203	0.206	0.208	0.210	0.212	0.214	0.217
14	0.212	0.214	0.217	0.219	0.221	0.224	0.226	0.229	0.231	0.233
15	0.227	0.230	0.232	0.235	0.237	0.240	0.242	0.245	0.247	0.250
16	0.242	0.245	0.247	0.250	0.253	0.256	0.258	0.261	0.264	0.267
17	0.257	0.260	0.263	0.266	0.269	0.272	0.275	0.277	0.280	0.283
18	0.273	0.276	0.279	0.282	0.285	0.288	0.291	0.294	0.297	0.300
19	0.288	0.291	0.294	0.297	0.301	0.304	0.307	0.310	0.315	0.317
20	0.303	0.306	0.310	0.313	0.316	0.320	0.323	0.326	0.330	0.333
21	0.318	0.321	0.325	0.329	0.332	0.336	0.339	0.343	0.346	0.350
22	0.333	0.337	0.340	0.344	0.348	0.352	0.355	0.359	0.363	0.367
23	0.348	0.352	0.356	0.360	0.364	0.368	0.372	0.375	0.379	0.383
24	0.363	0.367	0.371	0.376	0.380	0.384	0.388	0.392	0.396	0.400
25	0.379	0.383	0.387	0.391	0.395	0.400	0.404	0.408	0.412	0.417
26	0.394	0.398	0.402	0.407	0.411	0.416	0.420	0.424	0.429	0.433
27	0.409	0.413	0.418	0.422	0.427	0.432	0.436	0.441	0.445	0.450
28	0.424	0.429	0.433	0.438	0.443	0.448	0.452	0.457	0.462	0.467
29	0.439	0.444	0.449	0.454	0.459	0.464	0.468	0.473	0.478	0.483
30	0.454	0.459	0.464	0.469	0.474	0.480	0.485	0.490	0.495	0.500
31	0.469	0.475	0.480	0.485	0.490	0.496	0.501	0.506	0.511	0.517
32	0.485	0.490	0.495	0.501	0.506	0.512	0.517	0.522	0.528	0.533
33	0.500	0.505	0.511	0.516	0.522	0.528	0.533	0.539	0.544	0.550
34	0.515	0.521	0.526	0.532	0.538	0.543	0.549	0.555	0.561	0.567
35	0.530	0.536	0.542	0.548	0.554	0.559	0.565	0.571	0.577	0.583
36	0.545	0.551	0.557	0.563	0.569	0.575	0.582	0.588	0.594	0.600
37	0.560	0.566	0.573	0.579	0.585	0.591	0.598	0.604	0.610	0.616
38	0.575	0.582	0.588	0.595	0.601	0.607	0.614	0.620	0.627	0.633
39	0.590	0.597	0.604	0.610	0.617	0.623	0.630	0.637	0.643	0.650
40	0.606	0.612	0.619	0.626	0.633	0.639	0.646	0.653	0.660	0.666
41	0.621	0.628	0.635	0.641	0.648	0.655	0.662	0.669	0.676	0.683
42	0.636	0.643	0.650	0.657	0.664	0.671	0.678	0.686	0.693	0.700
43	0.651	0.658	0.666	0.673	0.680	0.687	0.695	0.702	0.709	0.716
44	0.666	0.674	0.681	0.688	0.696	0.703	0.711	0.718	0.726	0.733
45	0.681	0.689	0.696	0.704	0.712	0.719	0.727	0.735	0.742	0.750
46	0.696	0.704	0.712	0.720	0.728	0.735	0.743	0.751	0.759	0.766
47	0.712	0.720	0.727	0.735	0.743	0.751	0.759	0.767	0.775	0.783
48	0.727	0.735	0.743	0.751	0.759	0.767	0.775	0.784	0.792	0.800
49	0.742	0.750	0.758	0.767	0.775	0.783	0.792	0.800	0.808	0.816
50	0.757	0.765	0.774	0.782	0.791	0.799	0.808	0.816	0.825	0.833
51	0.772	0.781	0.789	0.798	0.807	0.815	0.824	0.832	0.841	0.850
52	0.787	0.796	0.805	0.814	0.822	0.831	0.840	0.849	0.858	0.866
53	0.802	0.811	0.820	0.829	0.838	0.847	0.856	0.865	0.874	0.883
54	0.818	0.827	0.836	0.845	0.854	0.863	0.872	0.881	0.891	0.900
55	0.833	0.842	0.851	0.861	0.870	0.879	0.888	0.898	0.907	0.916
56	0.848	0.857	0.867	0.876	0.886	0.895	0.905	0.914	0.924	0.933
57	0.863	0.873	0.882	0.892	0.902	0.911	0.921	0.930	0.940	0.950
58	0.878	0.888	0.898	0.907	0.917	0.927	0.937	0.947	0.957	0.966
59	0.893	0.903	0.913	0.923	0.933	0.943	0.953	0.963	0.973	0.983
60	0.908	0.919	0.929	0.939	0.949	0.959	0.969	0.979	0.990	1.000

(*) Strictly speaking, Δ_t = tabular value $+ I + 0.000164\ (T' + V_0)\ I$, but the term $0.000164\ (T' + V_0)\ I$ may be neglected for well-made thermometers for which I does not exceed 0°1.

(*) *A proprement parler,* Δ_t = *valeur tabulaire* + I + 0,000164 (T' + V_0) I. *Toutefois le terme* 0,000164 (T' + V_0) I *pourra être négligé pour les thermomètres de bonne construction pour lesquels* I *ne dépasse pas* 0°,1.



On account of the large number of thermometers used in the *Carnegie* observations, it was not deemed expedient to use graphs for obtaining the corrections for the protected thermometers, since, because of the different values of V_0, it would have been necessary to construct a graph for each thermometer.

Instead a table, of which *Table* 3 is a specimen sheet, was prepared covering all the *Carnegie* values of the tabular arguments and based on the formula for correction:

$$\Delta_t = \frac{(T' + V_0)\ (T' - t)}{K} + I + \frac{T' + V_0}{K}\left[\frac{(T' + V_0)(T' - t)}{K} + I\right] \quad (3)$$

T' and t denoting, respectively, the recorded temperatures of the main and auxiliary thermometers, I denoting the index-correction of the main thermometer, and V_0 and K having the same significance as in equation (2). Making $K = 6100$, equation (3) reduces to (4):

$$\Delta_t = 0.000164\,(T' + V_0)\,(T' - t)\left[1 + 0.000164\,(T' + V_0)\right] + I + 0.000164\,I\,(T' + V_0)$$

The first term of the right-hand member of (4) is represented by the tabular values in Table 3, hence

$$\Delta_t = \text{tabular value} + I + 0.000164\,I\ (T' + V_0).$$

The term $0.000164\,I\,(T' + V_0)$ may be considered negligible for well-made thermometers for which I does not exceed $0^{\circ}10$.

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