NOTE ON THE PRACTICAL CORRECTION OF DEEP-SEA REVERSING THERMOMETERS AND THE DETERMINATION OF THE DEPTH OF REVERSAL FROM PROTECTED AND UNPROTECTED THERMOMETERS

by

FLOYD M. SOULE, PHYSICIST AND OCEANOGRAPHER,
Department of Terrestrial Magnetism, Carnegie Institution of Washington.

Because of its simplicity and its elementary character, little has been published regarding the actual steps involved in the practical reduction of the readings of deep-sea reversing thermometers, protected and unprotected, to obtain temperatures and depths. Yet, judging from the number of requests for such information, there seems to be a need for its publication. The aim of this article is to supply that need and no claim of originality is made for the following.

In a reversing thermometer there are two corrections which must be applied. One is the index- or scale-correction, I, which arises from irregularities in the cross section of the capillary-tube, and the other is a temperature-difference correction arising from the fact that the temperature at which the thermometer is read is usally different from the temperature at which it was reversed. The index-correction is determined by calibration and is dependent only on the reading of the thermometer. Hovewer, as the temperature-difference correction is a correction for expansion, it depends on both the reading of the thermometer and the temperature at which it is read. Since the exact temperature-difference correction involves the temperature of reversal, which is unknown, the practical formula used is an approximation which may take various forms. In the Russian Oceanographical Tables, 1931, compiled by N. N. SUBOW, S. W. BOUJEWICZ, and Was. W. SHOULEJKIN, the correction for protected thermometers has the form

$$\Delta T=\left[\frac{(T'-t)\,(T'+V_o)}{K}\right]\left[1+\frac{(T'+V_o)}{K}\right]+I$$

where ΔT is the total correction, T' is the reading of the main thermometer, t is the reading of the auxiliary thermometer (the temperature at which the reversing thermometer is read), V_o is the volume of mercury in the thermometer after reversal at 0° C. expressed as degrees, K is a constant depending on the relative thermal coefficient of expansion of mercury and the glass of which the thermometer is made, and I is the index-correction.

In the *Memoirs of the Imperial Marine Observatory*, Kobe, Japan (page 11, vol. V, N° 1, 1932), Koji HIDAKA gives the correction for protected thermometers as

$$\Delta T=\frac{(T'-t)\,(T'+V_o)}{K\left[1-\frac{(T'+V_o-t)}{K}\right]}+I$$

where the symbols all have the significance described above.

The correction given by A. SCHUMACHER in *Annalen der Hydrographie und Maritimen Meteorologie* (p. 273, v. 51, 1923) is, using the same symbols,

$$\Delta T = \left[\frac{(T'-t)(T'+V_o)}{K}\right]\left[1+\frac{(T'-t)+(T'+V_o)}{K}\right]+I$$

As an unprotected thermometer is used in conjunction with a protected thermometer, the temperature of reversal is known from the protected thermometer. The temperature-difference correction, in the case of an unprotected thermometer, is therefore simpler, and the total correction is

$$\Delta T = \frac{(T_w-t)\,(T'+V_o)}{K}+I$$

where T_w is the temperature of reversal as determined by the protected thermometer and where the other symbols have the same significance as before.

The constant K is determined by the quality of the glass, and is 6100 for Jena 59^{iii} and 6300 for Jena 16^{iii}. As most deep-sea reversing thermometers are made from either one or the other of these kinds of glass, it is possible to prepare a table, based on one or the other of these values of K, giving the value of the temperature-difference correction for different values of $(T' - t)$ and $(T' + V_o)$. If two tables are prepared, one for $K = 6100$ and one for $K = 6300$, it is then possible by their use to correct any protected thermometer whose index-correction has been determined. Similar tables may also be prepared for unprotected thermometers, but such tables should give the correction for different values of $(T_w - t)$ and $(T' + V_o)$. Such tables may be converted into graphical form.

However, the time required at sea for reducing observations is greatly lessened by the preparation ashore of complete correction-graphs for individual thermometers. Such graphs may be constructed as follows. If C represents the temperature-difference correction, we have from SCHUMACHER'S formula for protected thermometers given above

$$C = \frac{(T'-t)\,(T'+V_o)}{K} + \frac{(T'-t)\,(T'+V_o)^2+(T'-t)^2(T'+V_o)}{K^2}$$

or rearranging

$$(T'-t)^2\,\frac{(T'+V_o)}{K^2}+(T'-t)\left[\frac{(T'+V_o)^2+K(T'+V_o)}{K^2}\right]-C=0$$

whence

$$(T'-t) = -\frac{(T'+V_o+K)}{2}+\sqrt{\frac{(T'+V_o+K)^2}{4}+\frac{K^2}{(T'+V_o)}C}$$

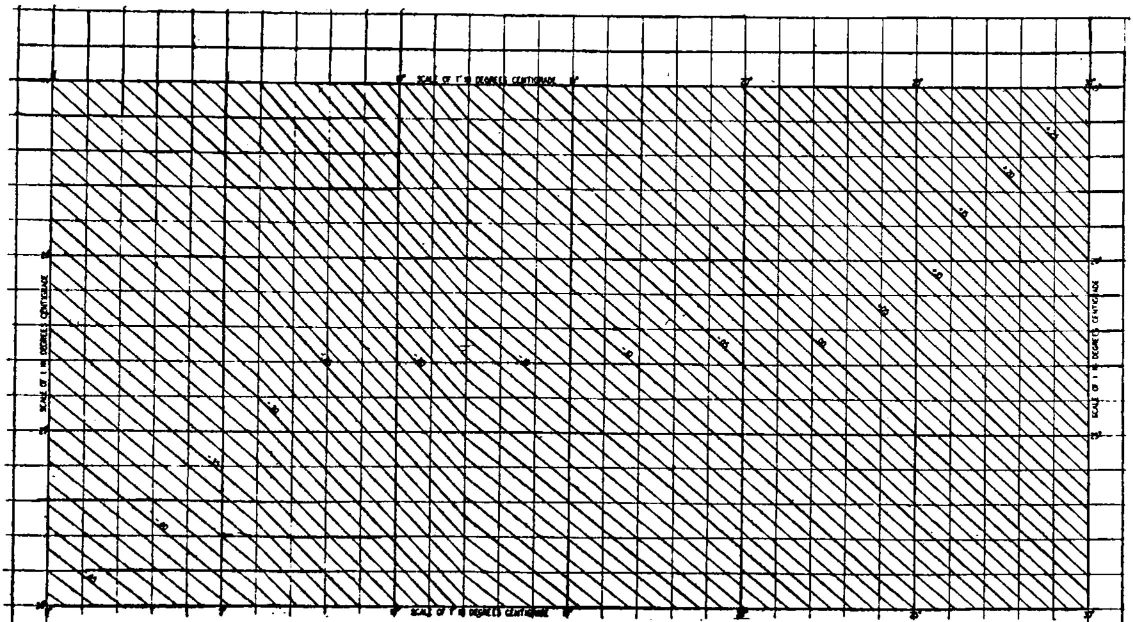
Now if the radical of the right-hand member of the above equation is expanded by the binomial theorem, we have

$$(T'-t) = \frac{K^2}{(T'+V_o+K)(T'+V_o)}C - \frac{K^4}{(T'+V_o+K)^3(T'+V_o)^2}C^2 + \frac{2K^6}{(T'+V_o+K)^5(T'+V_o)^3}C^3\ldots$$

Now T' is assigned a selected value near one extreme of the range of the thermometer and $(T' - t)$ is evaluated as C is assigned different values in steps of 0.01 from 0.00 to such a figure as will give the temperature-difference $(T' - t)$ as large a value as is necessary to cover the anticipated conditions. Except in restricted environments (such as polar summers) this value of $(T' - t)$ will probably be about 30° since water-temperatures as low as about 0° may be expected, and reading-temperatures as high as 30° are common. The process is then repeated with T' assigned an even-degree value near the other extreme of the range of the thermometer. For most thermometers, the first two terms on the right-hand side of the above equation determine the value of $(T' - t)$ with sufficient accuracy.

The correction-graph may now be constructed, on cross-section paper with the readings of the reversing thermometer (T') as ordinates and the corrected readings of the auxiliary thermometer (t) as abscissae. A convenient scale is 0°,1 to the millimeter. The length of the plotting sheet should be somewhat longer than three times the length of the finished graph which will occupy approximately the middle third of the original plotting sheet. On this graph the line of zero-correction will be a 45° line through all points of $T' - t$. This line is drawn in lightly through those values of T' for which the index-correction is known.

The values of $(T' - t)$ computed as mentioned above, are then laid off as points measured from the zero-correction line along the appropriate T' lines, one near the upper edge and one near the lower edge of the graph. These points are laid off in both directions from the zero-correction line since the correction may have either sign. Straight lines approximately parallel to



GRAPH SHOWING CORRECTIONS FOR PROTECTED DEEP-SEA REVERSING-THERMOMETERS BASED ON $(T'-t) = \frac{CK^{\circ}}{(T'+t_0+K)(T'+t_0)} - \frac{C^2K^{\circ}}{(T'+t_0+K)(T'+t_0)^2}$ FOR $t_0 = 95°$ AND JENA GLASS 59III FOR WHICH K=6100

ASSUMING VALUES OF READINGS MAIN THERMOMETERS, T' AND OF INDEX-CORRECTIONS, I, AS FOLLOWS:

T'	0°	6°	12°	18°	24°	30°
I	0.00	0.00	+0.02	+0.01	0.00	-0.01

the zero-correction line representing lines of equal temperature-difference correction are then drawn in lightly through those values of T' for which the index-correction is known. The graph would now be complete if there were no index-corrections, but the lines must be shifted either to the right or to the left at all values of T' where the index-correction is not zero. Thus if at 0° the index-correction is $+ 0^\circ.01$, the zero-correction line as well as all the other correction-lines at $T' = 0^\circ$ are shifted one line (or $0^\circ.01$ correction) to the right. When these shifts have been made to accommodate all known index-corrections, the resulting graph consists of a number of zigzag lines all approximately parallel and having an approximate 45° trend. The correction-lines exterior to the required range of T' and t may now be cut off and the graph is ready for use. A specimen correction-graph is shown above.

As described above, the lines of equal correction for temperature-difference between reversal and reading are assumed to be straight. As this assumption is not exactly true, an error is introduced. This error is greater, the greater the interval between the two values of T' for which the points are computed, and is greater, the greater the numerical value of $(T' - t)$. As an example of the magnitude of this error, let us take a graph for a thermometer whose range is 0° to $+ 20^\circ$ C. and prepared for a maximum value of $t = 30^\circ$ C. In this case the maximum error in the graph will occur in the neighborhood of $T' = 10^\circ$ and $t = 30^\circ$ where the error will be approximately 0.003° C. Such an error is not usually significant, but if greater accuracy is desired the values of $(T' - t)$ can be computed for intervening values of T', thus breaking the single straight lines into two or more parts. Because of the increased labor required in this procedure and the small magnitude of the error involved, the refinement is not recommended.

In the case of unprotected thermometers, where C is again the temperature-difference correction

$$(T_w - t) = \frac{CK}{(T' + V_o)}$$

As with the protected thermometers, the temperature-difference $(T_w - t)$ is evaluated for a series of C which is varied in steps of $0^\circ.01$ and the computations carried through for two extreme values of T'. Now, however, a plot of $(T_w - t)$ against T' is to be prepared but it is carried out in much the same manner as the previously described plot of T' against t, the index-correction shifts being made as before.

Having determined the corrected readings of a protected thermometer and its accompanying unprotected thermometer, the depth at which they were reversed can be computed from the foımula

$$D = \frac{(T_u - T)}{Q\, \rho_m}$$

where D is the depth in meters, T_u is the corrected reading of the unprotected thermometer, T is the true temperature given by the corrected reading of the protected thermometer, Q is the pressure-constant of the unprotected thermometer or the change in number of degrees in the corrected reading of the unprotected thermometer produced by a change in pressure of one-tenth kilogram per square centimeter, and ρ_m is the mean specific gravity of the water-column above the thermometers when they were reversed. The constant Q is of the order of magnitude of 0.01 and is given in the thermometer certificate, usually in the form of the degrees change in reading per kilogram per square centimeter change in pressure.

The approximate depth of the various water-bottles and thermometers will be known from the wire-angle and the readings of the meter-wheel. From the corrected temperatures and the salinity-measurements, the density (σ_t) of the water-samples can be determined from KNUDSEN'S "Hydrographical Tables". Knowing these values, the values of density *in situ* (σ_{tD}) are determined by applying three corrections, each of which is given in tabular form in HESSELBERG and SVERDRUP'S paper in *Bergens Museums Aarbok*, 1914-1915. The most important of these corrections is a function of depth, and since the exact depth of the samples is unknown the resulting values of density *in situ* will be only approximate. These values are then plotted against their approximate depths, a curve drawn, and a value of the mean density scaled from the curve at half the approximate depth. It is to be remembered that this density-depth chart is constructed solely for the purpose of determining a mean density which is to be used as a factor in the reduction of thermometer-depths. It is only necessary to determine this density to the nearest unit in the third decimal place; for example, to know that the mean density is 1.034 rather than 1.033 or 1.035. In terms of σ_{tD} this would mean the nearest unit. As the order of magnitude of depth-variation of σ_{tD} is about one unit per 200 meters, it is easily seen that the density-depth curve need not be very accurate. After the adjusted depths of the samples have been determined in this manner, and the vertical distribution-curves of salinity and temperature have been drawn, these may be scaled for salinity and temperature at selected depths and values of σ_{tD} computed for these depths. The values of σ_{tD} so derived may then be used to construct a more accurate density-depth curve which can be used to check the values of mean density used in the reduction of the thermometer-depths. If the values do not check within the limits mentioned above, a second approximation must be made, but this will rarely be necessary.

From the foregoing it will be seen that one meter in depth corresponds to a difference of about $0^\circ.01$ C. between the corrected readings of the protected and unprotected thermometers. Experience has shown that unprotected thermometers having a range of about 60° C. divided into $1/5^\circ$ can be read with an accuracy of better than $0^\circ.01$. Comparisons of thermometer-depths with depths determined by wire-length when the wire-angle was small indicate that the method gives depths reliable to within about $\pm$ 10 meters. The use of unprotected thermometers at intervals along the length of a wire to which a number of water-bottles is attached, in conjunction with meter-wheel readings, thus provides a satisfactory method of determining the depths of all the water-bottles on the wire.