A STUDY OF THE THEORY OF TIDES

by

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Reviewed by Ingénieur Hydrographe Général E. FICHOT,
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In this study Mr. Shimesu Miyahara has undertaken an examination of the general case of oceanic tides by introducing the results of tidal observations on the coast limits of the ocean in the equation, as a first approximation.

Ingénieur Hydrographe Général E. Fichot has been kind enough to communicate a review of Mr. Shimesu Miyahara's work to the International Hydrographic Bureau.

His review is as follows :

A complete tidal theory should enable us, by calculation alone and without recourse to observations, to determine the periodic oscillation engendered in an oceanic system of given configuration by a particular term of the perturbing potential. Poincaré tried to provide a solution of the problem thus expressed in the most general terms, by the theory of integral equations. He actually showed that by the preliminary formation of a generalised Green's function corresponding to a particular condition over the whole boundary of the area in question, the problem of the tides is reduced to the determination of a certain function φ satisfying an integral equation which can be treated by Fredholm's method. At the same time this equation takes a very complicated form, for it includes both a linear and a surface integral, so that in spite of the rapid convergencies of the series, one would be brought to a standstill by extremely lengthy calculations, even if the study were confined to a basin of constant depth, bounded by a geometrically simple outline and which is not crossed by any critical parallel of latitude.

This quasi-impossibility of a practical nature hitherto has baffled investigators, who have usually been content to complete and occasionally rectify, in various delicate points, the learned analysis outlined by Poincaré.

Quite recently, it occurred to Mr. Shimesu Miyahara to consider the problem from a geophysical rather than a purely mathematical point of view, and to use certain of the data furnished by observation to facilitate the determination of the function φ, the knowledge of which would entail that of the solution covering the whole field of investigation. This point of view, to which pure logicians would subscribe with diffidence, may nevertheless be justified by the necessity which, in any case, is imposed on us, of schematizing the actual shape of the seas to excess when we wish to deal with them theoretically.

By borrowing certain elements from the solution with which Nature herself has provided us, we are assured that the conditions of the problem included in our equations remain unchanged, on the express condition that the

hypotheses which have served to establish them are perfectly legitimate, i.e. consistent with natural conditions.

Putting this reservation to one side, we will show briefly how Mr. MIYA-HARA has turned his conception to account in the important memorandum published by the *Japanese Journal of Astronomy and Geophysics* (Vol. X, N⁰ 1, 1932).

Assuming the integral equation to have been solved, the components of the horizontal displacement of the liquid molecules in a given place will be yielded by linear combinations of the partial derivatives of φ in relation to the local coordinates, and the expression of the tidal elevation above the mean sea level will at once be deduced from the condition of constant pressure on the free surface, merely as a function of φ. Conversely, if this elevation were known to us at certain points, we would, by the same token, know the values which the function φ must assume at these points. Now all along the coasts, tide-gauges are installed at longer or shorter intervals according to the variations manifested by the tidal regime, and it will always be possible to add to the number of observation stations in such a way that the values assumed by the function φ along the boundary of the oceanic basin can be considered as data.

It must be clearly noted that this is no new and arbitrary condition that we are imposing on φ, without which the problem would be impossible ; the problem is in fact perfectly determined from the fact that we have compelled the normal component of the displacement to remain at zero along the whole length of the limiting contour which we assume to consist of vertical cliffs. But we know that, if our equating has been correct, φ cannot assume any other values along this contour than those which our observations have revealed. Thus it is legitimate to use these values to facilitate the determination of those which φ must assume within the contour, over the whole extent of the system.

This reasoning supposes that our equations are perfectly adapted to the shape of the basin in question. In practice, we have had to make certain hypotheses tending towards simplicity : the banks are not everywhere vertical walls and do not form a definite geometrical contour, also the law of depth which we have had to adopt does not accurately represent the surface of the ocean bed. From all this it follows that the values φ_0 deduced from coastal observations do not exactly agree with the function φ as defined by our integral equation. We must admit therefore that, on the boundary, the function φ to be determined assumes, not the value φ_0, but the value $\varphi_0 + \varphi_1$ in which φ_1 always remains a small quantity which may be neglected in a first approximation.

In short, subject to this approximation, a problem of the NEUMANN type is reduced to a problem of the DIRICHLET type, the solution of which is theoretically simpler.

It is to this solution that Mr. MIYAHARA devotes the greater part of his memorandum. Without following the author in all his mathematical developments which are of standard type, it is sufficient to say that, by virtue of the artifice used, the integral equation, of the most general elliptic type esta-

blished by POINCARÉ, is reduced to an integral equation of the first order the kernel of which presents no singularity. We know that, by a convenient application of the method mainly due to HILBERT and SCHMIDT, the solution of such an equation can be put in the form of an absolutely and uniformly convergent series of fundamental functions, corresponding to the characteristic constants of the kernel.

Having thus determined the function φ which fulfills the conditions of a first approximation, the difference $\varphi - \varphi_0$ along the coast will become a known function, and to obtain a closer value of φ we shall then have to solve another integral equation of the same type as the former one, the kernel of which will be the same, and this will shorten the new calculations considerably.

The author further points out judiciously that, as neither the kernel nor the known function which enter into the integral equation are real, it will be convenient to consider the real and the imaginary parts separately, but that, as the latter, which are due to the influence of rotation, are comparatively small, we need only take them into consideration in the second approximation.

This, from the fact that we are now dealing with a real kernel, allows us to calculate the characteristic constants with greater ease.

Be that as it may, once φ is determined, the expression for the tidal elevation with reference to mean level over the whole extent of the basin can easily be deduced from it by the condition at the free surface.

We may add incidentally that, in discussing the formation of the series which give the expression of φ, Mr. MIYAHARA has called attention to the possibility of various already well-known peculiarities : on the one hand, the existence of static tides of the so-called second kind, characterised by the presence of continuous currents which do not alter the surface figure of the sea ; on the other hand, the absence of change of level under the action of diurnal tide in LAPLACE's ideal ocean of constant depth covering the whole surface of the Earth.

After this attempt to glean what appear to be the essentials from Mr. MIYAHARA's memorandum, we will venture to make a few comments.

We are led at once to wonder what can be the practical utility of the new method he recommends. This method, as we have seen, depends upon a previous knowledge of the tide along the coasts of the ocean under examination.

We must acknowledge that once the hydrographic surveyor — and even more the seaman — has got this information, there is practically nothing else for him to want. This holds good for navigating or for making a chart ; but if, as always happens, the surveyor combines his functions with that of an oceanographer — rather let us say a geophysicist — this purely coastal conception of the phenomenon will not satisfy him. He will want to extend his outlook, to know how the cotidal lines, of which he can only nibble the bare extremities, agree from one shore to the other, around what amphidromic points this system of lines must pivot, in a word to visualise the immense palpitation of the mighty ocean in its entirety. And this desire is so natural that there is no lack of attempts at synthesis !

The hybrid method of Mr. MIYAHARA, more easily no doubt than any of the others which owe nothing to observations, is capable of bringing a solid foundation to these hitherto over-hypothetical attempts. Moreover, we venture to express a regret. Why has not the author illustrated his method by applying it to a concrete case ? It is the Pacific, he explicitly states, that he has had in mind — that ocean, still so little known, whose shape could seemingly be translated with comparative ease into a relatively simple formula. What a chance to prove the fecundity of a study which, without such a criterion, runs the risk of looking like a mere succession of brilliant mathematical variations on already known themes !

Mr. MIYAHARA in fact gives us no new conclusion. This is perhaps due to the fact that he has directed his investigation solely towards the simplest case. He deliberately puts aside, from the very beginning, the essential difficulties which arise, be it from zero depths at the edges, or from the existence of critical latitudes. He also neglects the influence of the attraction due to the tidal deformation of the Earth. True, this latter simplification is not very serious, for the method of successive approximations used by the author would enable it to be allowed for, though not without appreciable complications in the calculations. But there is plenty to cavil at in the first two.

Let us agree with Mr. MIYAHARA that the Pacific, for example, can be considered as an ideal ocean of constant depth, limited by vertical cliffs. When a semi-diurnal wave is in question, the critical latitudes, those for which the expression $\sigma^2 - 4\,\omega^2\,cos^2\,\theta$ (where σ represents the angular velocity of the tidal wave, ω that of the rotation of the Earth and θ the co-latitude) becomes zero, are banished to the polar regions; physically speaking, it is thus permissible to neglect them. But for diurnal tides, these latitudes are of the order of $\pm\,30^o$. We now find ourselves faced by a fatal difficulty, which is worth serious examination.

POINCARÉ gave this matter much attention, and in his "Theory of Tides" he showed two methods tending to surmount it. Unfortunately, the exposition of the first and more developed one is vitiated by a miscalculation affecting equation (7) on page 276, where the derivatives of Ω are inadvertently left out, with the result that the question is not entirely elucidated and remains a fine and difficult subject of research for mathematicians.

We think that we have shown sufficiently what may be expected from Mr. MIYAHARA'S ingenious method, subject to the limits within which it is capable of employment. It is to be hoped that its author will soon be able to give us a theoretical chart of the co-tidal lines and amphidromic points of the Pacific. This result would be all the more desirable in that Mr. BRILLOUIN hopes shortly to give us a solution of the same problem from a completely different line of approach; and a comparison between two theses arising from radically different conceptions could not fail to be of the greatest interest.