EXPERIMENTS IN THE STABLE COMPENSATION OF THE QUADRANTAL DEVIATION OF MAGNETIC COMPASSES, OBTAINED BY MEANS OF CYLINDRICAL COMPENSATORS

by

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(RESULTS OF EXPERIMENTS CONDUCTED ON BOARD THE *Città di Milano* OF THE ROYAL ITALIAN NAVY DURING THE ARCTIC EXPEDITION OF 1928 — UNDER THE COMMAND OF COMMANDER ROMAGNA MANOIA).

(*Extract from the forthcoming volume of the* Annali Idrografici — *published as a continuation of the article entitled :- "On the Question of the Correction of Quadrantal Errors in Compasses of large Magnetic Moments".* — Hydrographic Review, Vol. V, N° 1, p. 173).

1. INTRODUCTION : In order to take advantage of the great difference of latitude covered by the H. I. M. S. *Città di Milano,* during the Arctic Expedition of 1928, the commanding officer decided to make certain decisive experiments designed to check the *stability* of the compensation for quadrantal deviation of the compass accomplished by means of cylindrical compensators, which have been in use for several years in the Royal Italian Navy.

The above-mentioned cylindrical compensators were worked out experimentally at the *Istituto Idrografico* by Captain MODENA in 1912 (*). Captain TONTA in his memorandum of 1911, and again in the *Annali Idrografici* (**), had already given the theoretical proof of the possibility of obtaining a stable compensation with ellipsoidal compensators.

By means of ingenious experiments, Captain MODENA sought the appropriate dimensions to be given to cylindrical compensators capable of giving stable compensation. Captain MODENA checked the stability both by testing the compensators on compasses of various magnetic moments and also by modifying the strength of the magnetic field acting on the compensators and on the compass. But it is evident that the difficulties of producing a uniform magnetic field, such as that of the earth, and modifying this field by laboratory means, are so great that an opportunity of checking the data by other means is manifestly of great value.

The compensators of elongated shape as used by the Royal Italian Navy, which are the subject of this discussion, are of the following dimensions :-

390×90 m/m.	Ratio	$\left\{\frac{\text{length}}{\text{diameter}}\right\}$	= 4.335
300×70 "	"	"	= 4.285
230×55 "	"	"	= 4.185
160×40 "	"	"	= 4.000.

They are of solid silicic iron. (***)

The largest size compensators were not available on board the *Citta di Milano,* the largest on board measuring 300×70 and most of the experiments were conducted with this size. Further experiments were simultaneously conducted with a pair of spherical compensators of 15 c/m diameter.

2. EXPERIMENTAL CONDITIONS : The object of the experiments was to verify the following properties :- when the terrestrial magnetic field varies, the *spherical compensators* should naturally

(*) *Rivista Marittima,* May 1912.

(**) *Rivista Marittima,* October 1911 and *Annali Idrografici,* published by the *Istituto Idrografico della Regia Marina,* Vol. 8 (1911-12), p. 433.

(***) *Alloy of* 4,5 % *silicium, not over* 3 % *carbon, the rest iron. This may be obtained from the firm of* Rubinetterie Italiane, *via Solari, Milan.*

act as stable compensators for the quadrantal error, in the case where they are acting upon a compass of weak magnetic moment, and they would thus be unaffected by the inductive effect of the needles ; they should on the other hand show their instability for variations in the terrestrial field in cases where they act on a compass of strong magnetic moment. The cylindrical compensators, on the contrary, if they are suitable for producing a stable quadrantal compensation, should produce a constant quadrantal effect in any latitude (for every value of the terrestrial field) on a compass of any type.

However that may be, the tests were conducted with various compensators on the different kinds of compasses with which the vessel was equipped, as follows :-

One THOMSON, with a card of low magnetic moment of about 200 C. G. S. units (compass made by SALMOIRAGHI N° 34762).

One ship Compass MAGNAGHI — original model (N° 102), with a card of magnetic moment of 2882 C. G. S. units.

One torpedo-boat compass (N° 25), with magnetic moment of 721 C. G. S. units.

The last two compasses were constructed by the *R. Istituto Idrografico* at Genoa.

The voyage of the vessel allowed experiments to be conducted in very varied magnetic fields — the horizontal components H, expressed in C. G. S. (GAUSS) units were :-

30th AUGUST 1928 :
$H = 0.08$ at Kings Bay (*). (Lat. 78.9° N. Long. 12.1° E.).
20th SEPTEMBER 1928 :
$H = 0.12$ at Tromsö. (Lat. 69.5° N. Long. 19.0° E.).
25th SEPTEMBER 1928 :
$H = 0.13$ at Bergen. (Lat. 60.5° N. Long. 5.4° E.).
14th OCTOBER 1928 :
$H = 0.25$ at Malaga. (Lat. 36.7° N. Long. 4.4° E.).

During the course of the experiments conducted at Kings Bay, continuous observations were made with a portable magnetometer at about 30 metres from the compass under test, to make certain that the experiments in progress were not being disturbed by appreciable changes in variation.

The compasses were mounted in turn with their gymbals on the revolving platform of a solid tripod ; the torpedo compass N° 25 was fitted with a Voigtländer azimuth sight-vane, the MAGNAGHI N° 102 with a telescopic sight-vane, and the THOMSON with its alidade sight-vane. The telescope or sight-vane was laid on a fixed point and the bearing was then read on the compass-card. After each reading the sight-vane was turned through 15° by means of the azimuth circle of the compass ; the entire apparatus together with its compensators was then turned until the fixed point was brought into the line of sight and a new compass bearing was taken. Care was taken to allow an interval of about four minutes between laying the vane and reading the bearing to allow the card to settle down.

The cylindrical and spherical compensators were adjusted on a diametrical horizontal support, perpendicular to the vertical at the lubbers' point and the centre of the card, and turning with the compass. On such a support they could be adjusted closer to or further away from the centre of the card. Appropriate slots permitted their centres to be adjusted in the horizontal plane passing through the needles.

In conducting these experiments the author was ably assisted by Midshipman DE MORATTI.

The readings thus obtained were employed in constructing the deviation curves, and thus to determine the deviations corresponding to the elements P_m and P_b.

By using a FOURIER series development of the deviations obtained, instead of calculating as usual according to the form :

$$\delta = A + B \sin P_b + D \sin 2P_b + F \sin 3P_b + H \sin 4P_b + \dots$$
$$+ C \cos P_b + E \cos 2P_b + G \cos 3P_b + K \cos 4P_b +$$

the coefficients of the series were calculated directly :-

$$\delta = A + B_1 \cos(P_b - b_1) + D_1 \cos(2P_b - d_1) + F_1 \cos(3P_b - f_1) + H_1 \cos(4P_b - h_1)$$

(where $B_1 = \sqrt{B^2 + C^2}$; $D_1 = \sqrt{D^2 + E^2}$; $F_1 = \sqrt{F^2 + G^2}$; $H_1 = \sqrt{H^2 + K^2}$ and b_1 , d_1 , f_1 and h_1 are the

(*) *At Ny Aalesund, near the grave of Dr Otto Stoll.*

arguments of the component elements when $P_b = O$) in such a manner as to obtain directly the values of the amplitude of the semi-circular, quadrantal, sextantal and octantal deviations produced by the compensators.

3. RESULTS. — *Experiments with the Cylinders.* Let us examine separately the amplitudes for each compass taken individually, for a fixed position of the compensators with respect to the compass.

I. — *"Magnaghi" Compass, N° 102 ; cylinder* 300×70 *at* 23.5 % *from the centre (minimum distance allowed by construction) ; magnetic moment of the card* 2882 *C. G. S. units.*

	H C. G. S.	B_1	D_1	F_1	H_1
KINGS BAY, August 1928...	0.08	1.8°	6.3°	0.4	0.1
BERGEN, September 1928...	0.15	0.5°	6.1°	0.1	0.2
MALAGA, October 1928......	0.25	0.2°	5.8°	0.0	0.3

The semi-circular deviation B_1 caused by the compensators, evidently due to a slight permanent magnetisation on their part (in spite of the fact that they had been annealed at Kings Bay) naturally produced an effect which was smaller the stronger the terrestrial field ; we see therefore that B_1 diminishes in proportion to the increase in the intensity of the field. One would not have been able to verify this fact if the semi-circular deviation had been compensated at the same time as that of the ship, as is done on board. The diminution is so great, however, that it can only be explained by a progressive demagnetisation of the compensators as a result of the unavoidable handling necessitated by their transport and mounting.

The quadrantal D_1 remained practically constant in spite of the proximity of the compensators and the magnetic moment of the card.

II. — *Torpedo-boat Compass N° 25; cylinder* 300×70 *at* 26.9 % *(minimum distance allowed by construction), magnetic moment of card* 721 *C. G. S. units.*

	H C. G. S.	B_1	D_1	F_1	H_1
KINGS BAY, August 1928...	0.08	0.7°	4,7°	0.2	0.2
TROMSÖ, September 1928...	0.12	0,9°	4.7°	0.3	0.3
BERGEN, September 1928...	0.15	1.0°	4.4°	0.1	0.1
MALAGA, October 1928.....	0.25	0.5°	4.5°	0.1	0.1

During the experiments at Kings Bay and Malaga, one of the two compensators was inverted with respect to the other, whence the smaller semi-circular deviation. In this case also the quadantal D_1 remained practically constant.

III. — *"Thomson" Compass N° 34762; cylinder* 300×70 *at* 23.5 % *(same distance as that in Table I); magnetic moment of card* 200 *C. G. S. units.*

	H C. G. S.	B_1	D_1	F_1	H_1
KINGS BAY, August 1928...	0.08	0.8°	6.2°	0.0	0.8
BERGEN, September 1928...	0.15	0.8°	6.2°	0.1	0.6
MALAGA, October 1928.....	0.25	1.1°	5.8°	0.5	0.4

Here also the stability of the quadrantal effect is evident and very satisfactory, as was to be expected.

4. RESULTS. *Experiments with the Spheres and Verification of the Accuracy of the Data.*

The action of the spheres was quite different; they were tested both with the THOMSON compass and the torpedo-boat one N° 25. The induction of the needles would certainly have had a more apparent effect with the MAGNAGHI Compass N° 102; but the magnetic moment of the torpedo-boat compass alone sufficed to show the effect of induction.

IV. — *Torpedo-boat Compass N° 25 ; spheres* 15 c/m *diameter at* 29.4 c/m *(minimum structural distance); magnetic moment of card* 721 *C. G. S. units.*

	H C. G. S.	B_1	D_1	F_1	H_1
KINGS BAY, August 1928...	0.08	0.4°	5.3°	0.1	0.1
TROMSÖ, September 1928...	0.12	0.6°	3.9°	0.1	0.0
BERGEN, September 1928...	0.15	0.5°	3.8°	0.1	0.0
MALAGA, October 1928......	0.25	0.4°	3.4°	0.1	0.0

It is apparent that the quadrantal effect D_1 varied considerably and in a sense easily foreseen, when we consider the influence of the induction of the needles. Finally, neglecting the effect of the other terms: $P_b = 45°$, we have:

$$tg\,\delta_{45} = tg\,D_1 = \frac{H\mathfrak{D}' + i}{H}$$

$H\,\mathfrak{D}'$ is the quadrantal force induced by the terrestrial field H_1, and i is the quadrantal force produced by the induction of the needles.

From the expression of *tan* δ, we see that the greater the value of H, the smaller the effect $\frac{i}{H}$ of the force due to induction.

By taking for δ_{45} the extreme values observed at Kings Bay (5.3°) and at Malaga (3.4°) where H has the values of 0.08 and 0.25 respectively, we readily obtain:-

$$i = 0.00377 \qquad \mathfrak{D}' = 0.0442$$

from which we see that in the absence of the inductive effect of the needles, the spheres placed at the distance mentioned would have produced a quadrantal of

$$D'_1 = 2.5°$$

whereas by the induction of the needles alone these gave a quadrantal effect of $\frac{i}{0.08} = 0.047$ radians $= 2.8°$ at Kings Bay and of $\frac{i}{0.25} = 0.01508 = 0.9°$ at Malaga.

Since for small arcs, the tangent of the sum of two arcs is very nearly equal to the sum of their tangents, the observed quadrantal error coincides exactly with the sum of the two separate quadrantals which have just been calculated.

If we compare the IVth experiment with the IInd, we see that the same card with the cylinders produced a quadrantal force i, due to the induction of the needles, of 0.0004 C. G. S. units, while with the spheres, although further off, it produced a quadrantal force i, due to the induction of the needles, of 0.0037, or about ten times as great, while the complex quadrantal effects *i. e.* those of the cylinders, due almost exclusively to the fixed field in space and those of the spheres, due about half to the result of the induction of the needles (at Kings Bay) and consequently unstable, are very nearly equal. This already demonstrates the practical superiority of the cylinders over the spheres.

The calculations made show also that on a compass of weak magnetic moments the spheres would have simply produced a quadrantal of 2.5°. In reality the observations made with the THOMSON gave :-

V. — *"Thomson" Compass N° 34762; spheres of* 15 ${}^{c}/_{m}$ *diameter at* 29.4 ${}^{c}/_{m}$ *(same distance as in experiment IV); magnetic moment of card about* 200 *C. G. S. units.*

	H C. G. S.	B_1	D_1	F_1	H_1
KINGS BAY, August 1928...	0.08	0.8°	3.4°	0.5	0.2
BERGEN, September 1928...	0.15	0.6°	3.0°	0.2	0.5
MALAGA, October 1928......	0.25	0.4°	2.8°	0.1	0.8

If, as above, neglecting the higher terms of these series, we assume that for δ_{45}:-

$$\operatorname{tg} \delta_{45} = \frac{H\mathfrak{D}' + i}{H}$$

where as usual $H\mathfrak{D}'$ = the quadrantal force due to field H, and i is the quadrantal force due to the induction of the needles, by substituting respectively for δ_{45} the value D_1 obtained at Kings Bay where $H = 0.08$, and the value D_1 obtained at Malaga, where $H = 0.25$, we have:- $\mathfrak{D}' = 0.0442$, as above with compass N° 25; and $i = 0.00094$.

The force produced by the induction of the needles on the spheres is thus about 1/4 of that which we found with compass N° 25, just as the magnetic moment of the inductor card is about 1/4 that of the card itself — the above mentioned force has therefore exactly the value which is to be expected.

It is evident, however, that even with the THOMSON compass the spherical compensators are not entirely free from an appreciable induction action from the needles. They therefore always produce a quadrantal greater than that which would be produced if there were no induction, a fact which makes the compensation unstable with such changes of field, even with the THOMSON compass.

Small as the variations with this compass may be, they are still of the same order of magnitude as those of the cylinders, which may, however, be due to the fact that the compensators in question were investigated and experimented with other compasses of very different magnetic moments and probably having also slightly different distributions of the field.

It is not impossible that, with well-designed cylindrical compensators, even more perfect results might be obtained with the THOMSON card. The determinations made above, that the quadrantal $\mathfrak{D}'$, due to the terrestrial field for the same couple of spheres, located at a constant distance from the centres of the two compasses, is identical in the two cases, is a check on the accuracy of the measurements.

5. DEDUCTIONS. In order to be able to appreciate the range covered by the experiments thus described, it appears advisable to insert a few theoretical considerations to show, more simply than has been done so far, the action of elongated compensators, in order to eliminate certain faults which up to now have prevented their general adoption. In particular we will show the muchfeared variations in the directive force of the compensated card when the ship's head changes are totally inexistant, which variations even the inventors of this ingenious method thought it necessary at one time to admit, and further which no one thought of checking, even, quoting from the inventors. (See MELDAU, *Ann. der Hydrog.*, etc., 1919, page 277).

6. FUNCTION AND OBJECT OF QUADRANTAL COMPENSATORS OF ELONGATED SHAPE (CYLINDRICAL). As may be readily verified, the Faraday lines of a compass card (which may most easily be obtained by sprinkling iron filings on a thin piece of card-board placed over the compass card) indicate that, beyond a certain distance from the centre of the card, the lines of magnetic force which it produces coincide very nearly with circles tangent to the axis of the magnet.

This being the case, at any point Q located beyond this limiting distance (*i. e.* at the distance r, fig. 1), the magnetic field QT produced by the magnetic system ns, makes an angle K with the perpendicular QN to the straight line joining QC, equal to the angle P between the axis ns and the normal CL. Besides the sides of the angle K are perpendicular to the straight lines CQ and QO and those of the angle P are perpendicular to the straight lines QC and CO which contain the two equal angles (*).

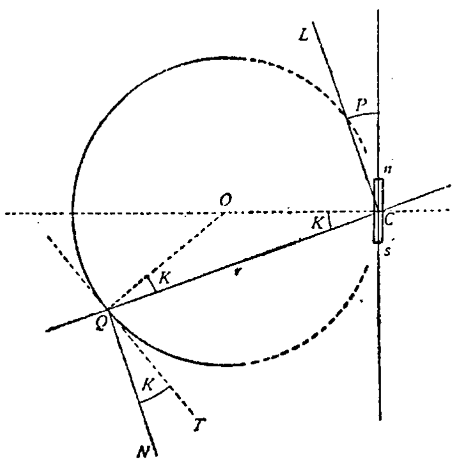

***Fig.* 1.**

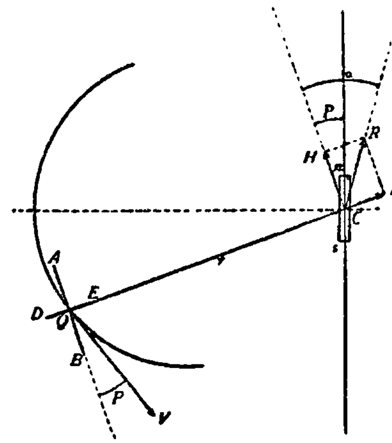

***Fig.* 2.**

Let us imagine that at the point Q (Fig. 2) at a distance r from the centre, there is placed the centre of a small slender bar of soft iron, AB, with its axis normal to the straight line joining CQ, and that this small bar, remaining normal to it, turns with the straight line CQ about the centre of the magnet. If V indicates the intensity of the field, along the tangent to the line of force produced by the magnet, ns in the centre of the bar, the latter will be subjected to an inductive moment of intensity $V \cos P$, in the direction of its axis. The bar will then have a magnetic moment of intensity $kV \cos P$, where k is a constant ratio dependent on the dimensions and the magnetic permeability of the bar. The other component, $V \sin P$, will have no effect.

The bar, thus magnetized, produces in the centre of the inducting magnet ns, a magnetic field H, proportional to the above-mentioned moment, the direction of which (when the magnet lies in the equator of the temporary magnet formed by the bar) is parallel to the bar itself, *i. e.* oriented as CL in Figure 1. The coefficient of proportionality, p, between the moment of the bar and the field H will be $p = \frac{1}{r^3}$ (as we should have by neglecting the demagnetizing action of the poles).

Let us now suppose that at the same point Q, in which the bar under consideration intersects the straight line r from the centre of the magnet, there is placed the centre of a second

(*) *If the magnetic system of the card be taken to be represented by a couple of poles* m *at a distance* 2 l, *it will produce, as we know, at point* Q *at the distance* r, *a radial force of* $\frac{2\,ml}{r^3} \sin P$, *and a tangential force perpendicular to* r *equal to* $\frac{ml \cos P}{r^3}$. *The line of force passing through the point* Q *will thus lie at an angle* K *to the normal* QN *to* r (Fig. 1) *such that*

$$\tan K = \frac{2\,ml}{r^3} \sin P \Big/ \frac{ml}{r^3} \cos P = 2 \tan P.$$

The angle K *therefore differs in this case from the angle* P, *but it is easy to demonstrate that the difference between* K *and* P *will not reach* 20°. *Such a difference will become zero when* P = 0°, 180°, 90° *and* 270°. *It will have its maximum value when* P = 35°3. *The distribution of the lines of force found at a certain distance from the centre of the card is certainly due, in this case, to the particular arrangement of the needles of the cards under consideration. The experiments with cylinders reported above, show that all that happens is in complete conformity with the hypothesis that the lines of force are practically circular.*

small slender bar DE, of soft iron, with its axis along the straight line r. Let us further assume that this last bar turns, at the same time as the bar previously considered, about the centre of the magnet. It will always be subject to the influence of the same field V, of which the component $V \sin P$, lies in the direction of the axis of this new bar. It will assume a magnetic moment $k' V \sin P$, where, as usual, k' is dependent on the dimensions and the magnetic permeability of the bar.

The new bar thus magnetized produces a new field H'' in the centre of the magnet, proportional to the moment mentioned above according to a factor q ($q = \frac{2}{r^3} = 2\,p$ approximately). This field H'' will be oriented along the axis of the new bar, *i. e.* normal to the field H. The two fields H and H'' normal to each other, merge into a field R, which lies at an angle α from the direction of H:

$$\alpha = \operatorname{arc\,tg} \frac{H''}{H} = \operatorname{arc\,tg} \frac{pk \sin P}{qk' \cos P}$$

We see here that α will be equal to P when $pk = qk'$, *i. e.* when the bars are of adequate dimensions. *In such case the resultant field R, due to the induction of the magnet, will always be oriented in the direction of the inductor magnet and does not tend to produce any deviation effect on the latter, regardless of the value of* P.

If the inductor magnet is a needle of the compass card, it is no less true that, with appropriate dimensions, the two bars will not produce any deviation under the induction influence of the needle. Their effect, in so far as concerns deviation, is reduced to that of a soft iron compensator which acts by simple induction of the field influencing the card.

It is evident that if q differs from p, k' should be different from k, which means that the two bars should have different dimensions (the first dimension greater than the second).

It is easy therefore to appreciate that to obtain an analogous effect, we may consider in place of a single bar, such as the first, a bundle of soft iron bars arranged parallel to each other and, in place of the second bar, another group of parallel bars. *The whole will be equivalent to a solid of elongated shape (paralellopiped, cylindrical or ellipsoid) which, given appropriate dimensions, will not produce any deviation effect under the inductive influence of the needles.* There remains, as the active element, only the effect due to the induction of the field acting on the card, as though the card exerted no inductive effect on the compensators. Therefore a stable compensation results, *i. e.* independent of the exterior field, as though the card had such a weak magnetic moment that its induction on the compensators may be neglected. The experiments made prove the validity of this hypothesis within the limits of practical application.

The compensator, having no deviation effect on the card as a result of the induction of the needles, for this reason tends to increase the horizontal force acting on the card. This effect will reach a maximum on the E and W compass courses and a minimum on the N and S courses. It is easy to demonstrate that this augmentation has a value proportional to $\sqrt{1 + 3\cos^2 P_b}$ where P_b is the compass course. *But as such force always has nil moment with respect to the axis of rotation of the card, it will produce no alteration in the directive force,* in so far as concerns the displacements of the card about its vertical axis.

7. Behaviour of the elongated Compensators on board and on shore. Mean directive force after compensation and constancy of the directive force of the card on all courses. We shall confine ourselves to the case where the compensator is located in a uniform field, such as is the case where the needles, the compensators and their distance may be considered to be small in comparison with the distance of the iron mass of the ship. Therefore, *on board*, all the conditions for the validity of the Poisson equation are satisfied. In this case the correctors influence solely the value of the force $\lambda\, H\mathfrak{D}$, because they may be included with all the other soft iron of the ship which produce a deviation effect solely by the induction of forces fixed in space, expressed precisely by $\mathfrak{D}$.

A) *The compensators on shore.* On shore identical considerations hold good. Let H be the intensity of the horizontal component of the terrestrial magnetic field and δ' the deviation which the compensator produces on shore, also, let a' and e' be the coefficients of the soft iron of the compensators on shore analogous to the known coefficients a and e of the Poisson equation for the ship.

Let X and Y be the components of H in the plane of the lubber-line of the compass on shore (X positive with respect to the lubber-line), and in a plane normal to the above (Y positive to the right). Let P_m be the angle between the vertical to the lubber-line and the North magnetic meridian (reckoned positively in the direction N, E, S.). If the dimensions of the compensator have been chosen in such a manner as not to exert a deviating effect on the card by the induction effect of the needles, there remain to be considered, on shore, two forces:-

$$a'X = a'H \cos P_m \quad \text{and} \quad e'Y = -e'H \sin P_m$$

due solely to the induction of H: the first acting in the direction of the lubber-line and the other in a direction normal to it.

By analogy with what is known of the forces on board, and adopting the usual notations, we have, designating by $\lambda' H$ the mean directive force of the earth and by $\mathfrak{D}'$ the quadrantal force produced by the compensator on shore in units of $\lambda' H$:-

$$\lambda' = 1 + \frac{a' + e'}{2} \qquad \lambda'\mathfrak{D}' = \frac{a' - e'}{2}$$

The compass is then subjected to the two forces:-

$\lambda' H$, fixed, towards the magnetic north

$\lambda' H\mathfrak{D}'$, capable of turning through double the angle through which the lubber-line turns.

By resolving these forces along N_m and E_m, we have a force towards North$_m$ of intensity:

$$\lambda' H + \lambda'\mathfrak{D}' H \cos 2P_m$$

and a force directed towards East$_m$ of intensity

$$\lambda' H\mathfrak{D}' \sin 2P_m$$

The resultant of these two forces makes an angle with N_m equal to:-

$$\delta' = \operatorname{arc\,tg} \frac{\mathfrak{D}' \sin 2P_m}{1 + \mathfrak{D}' \cos 2P_m}$$

For $P_m = 45^0$ (as well as for the other intercardinal magnetic courses) we have the equation which we have already employed:-

$$\mathfrak{D}' = \operatorname{tg} \delta'_{45}$$

Generally, by developing the above equation for δ', we have in radians:-

$$\delta' = -\mathfrak{D}' \sin 2P_m - \frac{1}{2}\mathfrak{D}'^2 \sin 4P_m$$

and in degrees:- $$\delta^\circ = 57^\circ 3 \times \left(-\mathfrak{D}' \sin 2P_m - \frac{1}{2}\mathfrak{D}'^2 \sin 4P_m\right)$$

Therefore it occurs that, in all swinging of the compass on shore, in the development of δ^o, there should be, besides a quadrantal term, an octantal term the coefficient of which is $-\frac{1}{2}\mathfrak{D}'^2$. (*)

B) *The compensators on board: constancy of the directive force on all courses.* Let us now see how the compasses behave on board. The compass is acted upon by two inductive forces:-

$$X' = (1+a)X = (1+a)H \cos P_m \quad \text{(positive towards the stem).}$$
$$Y' = (1+e)Y = -(1+e)H \sin P_m \quad \text{(positive towards the right).}$$

(*) *The octantals observed in these experiments are too small to be used in verifying the above equation. Further, the question of the suitability to the individual compasses of the experimental compensators does not form part of this work.*

When inducing the compensators, such forces produce the following forces on the compass:

$$a'X' = a'(1+a)H\cos P_m \quad \text{(positive forward).}$$
$$e'Y' = -e'(1+e)H\sin P_m \quad \text{(positive towards the right).}$$

In projecting these on $W_m - E_m$, we obtain the deviating force produced by the induction of the field of the ship on the compensator:-

$$\frac{a'(1+a) - e'(1+e)}{2} H\sin 2P_m$$

The deviating force due to the soft iron aboard being:-

$$\frac{a-e}{2} H\sin 2P_m$$

the compensator will annul the effect of such force when:-

$$a'(1+a) - e'(1+e) = -(a-e)$$
or
$$(1+a)(1+a') - (1+e)(1+e') = 0$$

Thus there will result in addition equal and contrary action of the components towards the N_m of the quadrantal field of the ship and of that of the compensators, *and the directive force of the compass will have a constant value.*

c) *Mean directive force of the compensated compass.* But, as it is easy to verify by starting with the expressions for λ and $\mathfrak{D}$, we have:-

$$\lambda(1+\mathfrak{D}) = (1+a) \quad \text{and} \quad \lambda(1-\mathfrak{D}) = (1+e)$$
$$\text{and} \quad \lambda'(1+\mathfrak{D}') = (1+a') \quad \text{and} \quad \lambda'(1-\mathfrak{D}') = (1+e')$$

in which we have by substitution, after having compensated for the quadrantal effect:-

$$\lambda\lambda'(1+\mathfrak{D})(1+\mathfrak{D}') - \lambda\lambda'(1-\mathfrak{D})(1-\mathfrak{D}') = \lambda\lambda'(\mathfrak{D}+\mathfrak{D}') = 0$$

$$\text{or} \quad -\mathfrak{D} = \mathfrak{D}'$$

This being determined we see that after placing the compensators there will be a new λ'' on board:-

$$\lambda'' = (1 + \frac{a+e}{2} + \frac{a'(1+a) + e'(1+e)}{2}) = \lambda + \lambda \frac{a'(1+\mathfrak{D}) + e'(1-\mathfrak{D})}{2};$$
$$= \lambda(1 + \frac{a'+e'}{2} + \frac{a'-e'}{2}\mathfrak{D}) = \lambda\lambda'(1+\mathfrak{D}.\mathfrak{D}') = \lambda\lambda'(1-\mathfrak{D}^2)$$

8. Application to cylindrical compensators. Let us apply the case to our compensators. We observe, first of all, that for $P_b = 45°$, the elongated compensator is subjected, *by the effect of the needles,* to an inductive field V which, as has been stated, makes an angle of 45° with the axis of the compensator, and which has equal components, both in the direction of the axis of the compensator and in the transverse direction. If, as may be assumed, the compensator satisfies the condition of producing on the card a force oriented in the direction of the needle, it follows that the force produced by the longitudinal component and that produced by the component transversal to the compensator (which as we have seen are normal to each other and both of which make an angle of 45° with the magnetic axis of the inductive card) will be equal to each other. In this case, for the approximation required by the above considerations, we should have:-

$$-a' = e'.$$

The contrary sign of the two coefficients is due to the fact that the longitudinal component, with respect to the compensator, produces an effect on the card which is in the opposite direction to the inductive field V, while the transverse component produces an effect in the same direction as the inductive field V.

This established, we then have for the compensators in which we are interested:-

$$\lambda'\mathfrak{D}' = \frac{a'-e'}{2} = a' \qquad \lambda' = 1 + \frac{a'+e'}{2} = 1$$

By substituting these values in the preceding expressions for λ", and having made the compensation, we then have, on board, a new λ given by :-

$$\lambda'' = \lambda(1-\mathfrak{D}^2)$$

Owing to the presence of the compensators and since these act by induction of the field of the ship, the λ on board then tends to diminish by $\lambda\,\mathfrak{D}^2$.

We shall not concern ourselves further with the other horizontal forces due to the induction of the compensators because we have already shown (and the experiments described have verified the fact) that these have a moment equal to zero with respect to the vertical axis, and for this reason do not enter into the calculation of the directive force expressed by λ". A casual examination shows, however, that in all these cases where such forces may intervene (rotation of the card about horizontal axes) we always have in practice a stabilizing couple greater than that inherent in the non-compensated compass, in spite of the slight reduction in the directive force.

We should state rather, as is simple to calculate by starting from the expression obtained for λ "; that the reduction of λ is :-

less than	0.01	for compensated quadrantals		up to	5°7
	0.025	”	”	”	9°0
	0.05	”	”	”	12°8
	0.075	”	”	”	15°7
	0.10	”	”	”	18°1
and	0.125	”	”	”	20°20
	0.15	”	”	”	22°2
	0.20	”	”	”	25°6.

We see therefore that the mean directive force, in practice, *i. e.* up to 18°, is subject to perfectly tolerable reductions. (*)

9. Conclusions. The cylindrical compensators studied produce a stable compensation of the quadrantal. (Experiments in Para. 3).

In the compensated compass on board, the directive force which results is constant on all courses, contrary to the notion accepted up to now. (Considerations in Para. 6).

Cylindrical compensators produce a slight reduction in the mean directive force of the compensated compass — which reduction, for quadrantals less than 18°, is practically negligible. (See para. 8 and note on this subject).

(*) *If the magnetic system of the card could be taken to be represented by a couple of poles* m, *at a distance* 2 l, *for* $P_b = 45°$, *we would have on the compensator a field making an angle* K *with its axis, such that* tan K = 2 tan 25°, *or* K = 63°5. *Following the line of reasoning given in the text (the radial field being in this case exactly double the longitudinal field) we should have :-*

$$-a' = 2e' \quad \textit{and not} \quad -a' = e'$$

It follows therefore that $\lambda' = 1 - a'/4$, $\lambda'\mathfrak{D}' = -\frac{3}{4}a'$ *and consequently* $\lambda'' = \lambda\frac{3}{3+\mathfrak{D}}(1-\mathfrak{D}^2)$

The λ" *is therefore slightly different from that obtained above but only differs by the factor* 3/ (3+𝔇) *which, being always less than* 1, *does not differ much from unity in practice. The reduction in* λ *will therefore be less than* 0.15 λ *up to* 18°.

The experiments described at the beginning seem to indicate that the reduction in directive force which we might expect on board for a compensated compass are those given in the text and not those deduced thus on the simplifying hypothesis in which the system of the card is reduced to a couple of point-like poles.