TIDAL INFORMATION FOR MIXED TIDES AND A COMPARISON OF TWO METHODS FOR A ROUGH PREDICTION OF DIURNAL TIDES

by

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The greater part of the tides of the Netherlands East Indies are mixed and, accordingly, the tidal phenomena show for the majority of places a character varying from a preponderant semidiurnal type to a preponderant diurnal one, with important irregularities both in time and in range.

Although the progress of the tide for a given place can be computed by means of the tidal constituents, it has been considered useful to insert in the Netherlands Pilot for the East Indian Archipelago descriptions of the tide (1), which allow of making a rough but rapid estimate of its course. Calculation takes much time and is not necessary if only a general prediction is required.

The concerned tides being mixed, it is unavoidable to give a separate description of the semidiurnal and of the diurnal groups, mentioning only the most essential data because minor details would complicate the use and overshoot the mark: a rough but rapid estimation. Whenever more accurate knowledge is wanted, recourse must be had to calculation on the base of the harmonic constants. As a rule the officers of the Netherlands merchant navy are expected to be able to do so.

The descriptions include :

1) Mentioning whether the character of the tide is: semidiurnal, diurnal or mixed. In the last case whether the semi-diurnal or the diurnal group is preponderant or whether neither of them dominates ($\frac{H_{K_1} + H_{O_1}}{H_{M_2} + H_{S_2}}$ being about 1).

2) Height of mean sea level above chart datum : A_0.

3) *Semidiurnal group.* — Age of the tide; mean range at springs and neaps; time of HW springs and LW springs and the mean daily inequality in time of HW and LW during a certain number of days before and after springs (2).

4) *Diurnal group.* — Age; mean range at springs (3) and neaps; time of HW and LW at 1 January and 1 July with a note that the HW's and LW's accelerate every week half an hour.

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(1) These descriptions are given in the text of the Pilot; a list of Harmonic Constants is given in the preliminary part. Moreover Tide Tables for some nine places, giving the height of the tide for each hour, are published yearly.

(2) Computed from the ratio $\frac{H_{S_2}}{H_{M_2}}$

(3) With diurnal springs is meant the coincidence of the constituents K_1 and O_1.

5) *Particulars.* — Whether the HW springs or the LW springs of both groups can coincide or whether they cannot. In case of coincidence, at what day and with what height; no coincidence being possible, when the highest or lowest waters will occur and with what height.

The average time of HW and LW of K_1 for 1 January and 1 July is given as the time of diurnal HW and LW at these days. This is of course an approximated time, on an average exact only for the springs and neaps of these days and more or less incorrect for other phases, depending from the ratio $\frac{H_{O_1}}{H_{K_2}}$. Further on these errors will be discussed.

As an example the description of the tide of *Zwaantjes droogte* in Strait Madura (7°,5 S and 113°,1 E), is given according to the Pilot. The harmonic constants are:

S_2 :	25 cm	342°	K_1 :	46 cm	298°
M_2 :	43 cm	319°	O_1 :	22 cm	276°
K_2 :	8 cm	82°	P_1 :	9 cm	280°
N_2 :	8 cm	300°			

Description of the tide of *Zwaantjes droogte*:

Character. — Mixed with equal ranges: A_0 13 dm.

SEMIDIURNAL GROUP OF THE TIDE.	$\frac{\text{Springs}}{\text{Neaps}}$ 24^h after $\frac{\text{F M and N M (1)}}{\text{F Q and L Q}}$; mean range $\frac{13\ ½\ \text{dm}}{3\ ½\ \text{dm}}$. $\frac{\text{HW}}{\text{LW}}$ springs at $\frac{11^h\,30^m \text{ and } 23^h\,30^m}{5^h\,30^m \text{ and } 17^h\,30^m}$ From 6 days before till 6 days after springs: each day 40 min. later. At neaps 6^h later than at springs.
DIURNAL GROUP OF THE TIDE.	$\frac{\text{Springs}}{\text{Neaps}}$ 24^h after ☾ $\frac{\text{max. decl.}}{\text{decl.} = 0}$; mean range $\frac{13\ ½\ \text{dm.}}{4\ ½\ \text{dm.}}$ $\frac{\text{HW}}{\text{LW}}$ at 1 Jan. at $\frac{19^h}{7^h}$; 1 July at $\frac{7^h}{19^h}$; each week later, half an hour earlier.

Particulars. — Neither the HW springs, nor the LW springs can coincide. The highest HW's may be expected at springs of the semidiurnal group about May/June at $11^h\,30^m$, Nov./Dec. at $23^h\,30^m$, height about 25 dm above chart datum; the lowest LW's are to be expected at springs of the semidiurnal group about Dec./Jan. at $5^h\,30^m$, June/July at $17^h\,30^m$, height about equal to chart datum.

(1) The abbreviations FM, NM, FQ, and LQ stand for: Full Moon, New Moon, First and Last Quarter.

The date of 27 June 1927 is chosen as an example how the seaman has to use these data, in order to get a rough estimate of the tide for that day.

New Moon and maximum moon's declination coincide according to the Nautical Almanac on the 29th June 1927 and from the description given above may be deduced :

Semidiurnal group :

27 June was two days before NM and three days before springs ; $3 \times 40^m = 2^h$. So HW occurs at $9^h 30^m$ and $21^h 30^m$; LW at $3^h 30^m$ and $15^h 30^m$. Three days before springs the estimated mean half range is about 5 dm. (1)

Diurnal group :

27 June approximately HW at $7^h 15^m$, LW at $19^h 15^m$, or for a rough estimation at 7^h and 19^h. Three days before springs the estimated mean half range is 5 ½ dm. (2)

Both groups combined :

	$3^h 30^m$	7^h	$9^h 30^m$	13^h	$15^h 30^m$	19^h	$21^h 30^m$
semidiurnal..	− 5	—	+ 5	—	− 5	—	+ 5
diurnal	—	+ 5 ½	—	0	—	− 5 ½	—

Result :

Only one genuine HW, in the morning between 7^h and $9^h 30^m$ about 10 dm above mean sea level, or 23 dm above chart datum ; LW between $15^h 30^m$ and 19^h, approximately 10 dm below mean sea level, or 3 dm above chart datum.

Calculation by means of the tidal constants gives HW at 9^h with 11 dm above mean sea level, or 24 dm above chart datum ; LW at 16^h with 11 ½ dm below mean sea level, or 1 ½ dm above chart datum.

The discrepancies between estimation and calculation are due to rise of the HW and a fall of the LW by the combined action of N_2, K_2 and P_1, an accidental and rare coincidence.

(1) See for a closer estimation: "Investigation of Harmonic Constants, etc...", Special Publication No 12 of the I. H. B., page 35. The ratio $\frac{HS_2}{HM_2}$ being about 0,5, the half range is about 1.2 HM_2.

(2) See for a closer estimation Special Publication No 12 of I. H. B., page 35. The ratio $\frac{HO_1}{HK_1}$ being about 0.5, the half range is about 1.2 HK_1.

The author's attention was drawn to the French Tide Tables 1933 for the colonies bordering the China Sea. Page XI of these Tables states as to the Tides of the Gulf of Tonkin which are diurnal :

"Les heures des pleines mers suivent d'une quantité sensiblement constante le passage de la Lune au méridien supérieur lorsque la déclinaison est boréale, et suivent de la même quantité le passage au méridien inférieur lorsque la déclinaison est australe ; les heures des basses mers présentent la même particularité".

The reader will at once see the difference between the two systems of predicting the time of diurnal high water. Whereas the Dutch method accepts for every day of the year a for each day fixed hour of diurnal HW, the French method gives a fixed hour angle between that HW and the moon's transit.

It was thought worthwhile to investigate the degree of accuracy of both methods.

a) The Dutch method finds a fixed hour for HW on the first of January and July by adding *kappa* K_1 to the mean value of the astronomical argument at these dates. Now this argument may vary 10°, or 40^m in time, from the mean value. This is the first inaccuracy.

The second inaccuracy results from the assertion that HW of the diurnal group (K_1 and O_1) is synchronous with HW of K_1. The errors arising from this statement depend from the ratio $\frac{H_{O_1}}{H_{K_1}}$. By means of the angular velocities of these tides the differences in time between HW springs and the HW's at various dates before and after springs can be computed. The differences in time between the HW of K_1 and HW of the diurnal group are given in the following Table I in which the time of HW springs is assumed to be 0^h.

TABLE I

Day before/after Springs — Jour avant/après la Vive Eau	Hour HW — Heure PM — K_1	$\frac{H_{O_1}}{H_{K_1}} = 0.3$ Hour HW — Heure PM — Diurnal group	Error — Erreur	$\frac{H_{O_1}}{H_{K_1}} = 0.5$ Hour HW — Heure PM — Diurnal Group	Error — Erreur	$\frac{H_{O_1}}{H_{K_1}} = 0.7$ Hour HW — Heure PM — Diurnal group	Error — Erreur	$\frac{H_{O_1}}{H_{K_1}} = 1$ Hour HW — Heure PM — Diurnal group	Error — Erreur
4	0^h16^m	23^h12^m	$-1^h\ 4^m$	22^h28^m	-1^h48^m	21^h48^m	-2^h28^m	20^h58^m	-3^h18^m
3	0 12	23 14	-0 58	22 42	-1 30	22 14	-1 58	21 42	-2 30
2	0 8	23 25	-0 41	23 4	-1 4	22 47	-1 21	22 26	-1 42
1 bef. - *av.*	0 4	23 42	-0 22	23 31	-0 33	23 23	-0 41	23 12	-0 52
0 Spr. - *V.E.*	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1 aft. - *apr.*	23 56	0 18	+0 22	0 29	+0 33	0 37	+0 41	0 48	+0 52
2	23 52	0 35	+0 41	0 56	+1 4	1 13	+1 21	1 34	+1 42
3	23 48	0 46	+0 58	1 18	+1 30	1 46	+1 58	2 18	+2 30
4	23 44	0 48	+1 4	1 32	+1 48	2 12	+2 28	3 2	+3 18

This Table shows that the error increases with the ratio $\frac{H_{O_1}}{H_{K_1}}$, which is only quite natural. As, however, the diurnal tide has a long period, the changes in height near the culmination points are slow and the influence of a time error rather small. Accepting an error of time of about 2^h, of which 40^m must be reserved for the first cause of inaccuracy mentioned above, it appears that the method gives acceptable errors for four days before and after springs with a ratio of 0.3, diminishing to 2 days at each side of springs with a ratio 0.7 and only one day with the ratio 1. However, for the larger ratio's the influence of the diurnal group decreases rapidly with the increase of the number of days after springs. For the ratio 1, neaptide causes no changes in level and about four days after springs the range has already decreased to half the amount of that of springs. For the lesser ratio's 0.7 and 0.5 half range is reached five days after springs, whereas for the ratio 0.3 it is reached at neaps (1).

As the ratio $\frac{H_{O_1}}{H_{K_1}}$ is smaller than 0.7 for the greater majority of the diurnal groups in the Netherlands Indian Archipelago the use of the method is acceptable. It should be examined whether the French method gives more accurate results.

b) In order to investigate the degree of precision of the French method and to be able to compare it with the Netherlands method, it was necessary to scrutinise the theoretical foundation.

According to my opinion the French method is based upon the fact that on or near 21 March and 21 September the quarter phases of the moon coincide approximately with maximum declination, whereas on 21 June and 21 December maximum declination occurs with F M and N M.

Assuming the orbit of the moon to lie as an average in the ecliptica at these days this fact is easily acknowledged. Supposing further that it is N M on the 21st of March, it is clear the moon must be that day in the equator and therefore have 0^o declination. About 7 days earlier it was Last Quarter, with nearly maximum S declination; 7 days later First Quarter coinciding with nearly maximum N declination. Following a similar reasoning for the other dates mentioned, it may be asserted that:

Max. dec. N coincides with	F Q	N M	L Q	F M
	± 21 March	± 21 June	± 21 Sept.	± 21 Dec.
with max. S declination.	± 21 Sept.	± 21 Dec.	± 21 March	± 21 June

while as an average:

First Quarter has upper transit at	18^h, lower transit at	6^h
N M	12^h	0^h
L Q	6^h	18^h
F M	0^h	12^h

(1) See Special Publication N° 12 of the I. H. B., page 35.

Assuming the age of the tide to be zero, diurnal springs occurs on the day of maximum declination at the time of H W of K_1 and assuming further that the moon's phase on the 21th of March is first quarter, H W of K_1 on that day will occur as an average at the time corresponding with: mean of term table VIII for K_1 + term Table XI*a* for K_1 + *kappa* K_1 (1), or :

$$350^\circ + 282^\circ + kappa\ K_1 = 272^\circ + kappa\ K_1.$$

This amount must be added to 0^h and since the moon's upper transit occurs on that day at 18^h or 270° it is clear that we get time of H W diurnal springs by adding *kappa* K_1 to moon's upper transit. The "quantité sensiblement constante" is therefore, in this case, the *kappa* number of K_1 in time.

The same reasoning will show this to be true also for the other phases which coincide with maximum N declination, and for S declination if that *kappa* number is added to the moon's lower transit.

This rule is also acceptable for other data than the four phases considered although, generally speaking, maximum declination will not coincide with either quarter moon or N or F moon on those other data.

Suppose F Q of the moon occurs on the 21st of March and coincides with maximum N declination. After an anomalistic revolution (27 d 13^h,3) around the earth this celestial body has once again maximum N declination, but it takes still about 2 days before the synodical revolution (29 d 12^h,7) is achieved. Consequently the moon's upper transit occurs on the day of maximum declination about 2×50^m or $1^h\,40^m$ earlier than on the 21st of March. But H W of the K_1 tide taking place also about $1^h\,40^m$ earlier, the rule is therefore applicable as well. It may therefore be accepted that, if the age of the diurnal group is zero and if F Q coincides with maximum declination on the 21st of March, the addition of *kappa* K_1 to the time of moon's upper transit on the day of maximum N declination (to the time of lower transit on the day of maximum S declination) gives the time of diurnal springs.

However the supposition that the orbit of the moon lies in the ecliptica is only true as an average and moreover the principal phases of the moon (F Q, L Q, N M and F M) will only occur occasionally on the principal dates (21 March, etc.) as was accepted in the given reasoning. Therefore a difference of time between the days and terms of occurrence of the principal phases and those of maximum declination can be met with even on or near these principal days. An investigation over half a moon cyclus (9 ½ year) has shown that this difference may reach 49^h or two days, which divergency can cause an error in the predicted time of high water of 2×50^m or $1^h\,40^m$.

Neglecting for the moment this divergency, the Table II given below shows, the age of the diurnal tide being zero, the errors which appliance of the rule gives for other days than the one of maximum declination. For

(1) See for the numbering of the Tables: Supplement to Special Publication N° 12 of the I. H. B., "Tables for the calculation of the tides by means of harmonic constants".

simplicity's sake, it is supposed that *kappa* K_1 as well as the time of maximum declination are zero and that the hours of H W of the diurnal group for the different ratio's are the same as in Table I.

TABLE II

Day before / after Springs & Max. Decl. *Jour avant / après la Vive Eau et la Decl. Max.*	Transit of moon & predicted time of HW *Passage lune meridien et Heure prédite de la PM* (1)	$\frac{HO_1}{HK_1} = 0.3$ Hour HW *Heure PM* Diurnal group (2)	Error in time of prediction *Erreur sur prédiction d'heure* (2)-(1)	$\frac{HO_1}{HK_1} = 0.5$ Hour HW *Heure PM* Diurnal group (3)	Error in time of prediction *Erreur sur prédiction d'heure* (3)-(1)	$\frac{HO_1}{HK_1} = 0.7$ Hour HW *Heure PM* Diurnal group (4)	Error in time of prediction *Erreur sur prédiction d'heure* (4)-(1)	$\frac{HO_1}{HK_1} = 1$ Hour HW *Heure PM* Diurnal group (5)	Error in time of prediction *Erreur sur prédiction d'heure* (5)-(1)
4	$20^h 38^m$	$23^h 12^m$	$+2^h 34^m$	$22^h 28^m$	$+1^h 50^m$	$21^h 48^m$	$+1^h 10^m$	$20^h 58^m$	$+0^h 20^m$
3	21 29	23 14	+1 45	22 42	+1 13	22 14	+0 45	21 42	+0 13
2	22 19	23 25	+1 6	23 4	+0 45	22 47	+0 28	22 26	+0 7
1 bef.-*av.*-	23 10	23 42	+0 32	23 31	+0 21	23 23	+0 13	23 12	+0 2
0 Spr. / *V.E.* & Max. Dec.	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1 aft.-*apr.*-	0 50	0 18	-0 32	0 29	-0 21	0 37	-0 13	0 48	-0 2
2	1 41	0 35	-1 6	0 56	-0 45	1 13	-0 28	1 34	-0 7
3	2 31	0 46	-1 45	1 18	-1 13	1 46	-0 45	2 18	-0 13
4	3 22	0 48	-2 34	1 32	-1 50	2 12	-1 10	3 22	-0 20

However, the age of the diurnal group is only exceptionally zero. In the Netherlands East Indies this age is less than 12^h in 65 places; from 12 to 36^h in 129 places; from 36 to 60^h in 52 places and larger than 60^h in 21 places.

If the age is not zero, the symmetry of errors observed in Table II disappears because the values in column (1) remain unaltered whereas, springs occurring later, the values in the columns (2), (3) and (4) have to be moved downwards over a number of days corresponding to the age. Furthermore, H W of K_1 occurring every next day four minutes earlier, the values of these columns must be diminished with 4 minutes per diem of the age.

In so doing the symmetry of errors disappears but, by taking their algebraic mean and applying this amount to *kappa* K_1, a column of amended errors may be reached that will be the smallest obtainable. The algebraic sum of *kappa* K_1 and the mean mentioned above will form the "quantité sensiblement constante" of the French tide table, if the diurnal tide has an age of some amount. In Table III this amendment has been introduced, the age of the tide having been supposed to be 2 days and the time of moon's transit at the day of maximum declination $0^h\,0^m$.

Example. — According to page XIII of the "Tables des Marées des Mers de Chine" 1933, the tide at *Do Son* is practically diurnal; half range $(M_2+S_2) = 7$ cm, id. $(K_1 + O_1) = 144$ cm. K_1 ampl. 72 cm., *kappa* 91^o, in time $6^h\,6^m$; O_1 ampl. 70 cm, *kappa* 35^o. Age $(91^o - 35^o) - 10\,\% = 2$ days; Ratio $\frac{HO_1}{HK_1} = 1$.

TABLE III

Day before / after Max: Dec. *Jour avant / après la Dec. Max.*	Time of moon's transit *Heure passage lune meridien*	$\frac{HO_1}{HK_1} = 0.3$			$\frac{HO_1}{HK_1} = 0.5$			$\frac{HO_1}{HK_1} = 0.7$			$\frac{HO_1}{HK_1} = 1$		
	(1)	Time HW *Heure PM* Diurnal group (2)	Errors *Erreurs* (2)-(1)	Amended Errors *Erreurs amendees* (3)	Time HW *Heure PM* Diurnal group (4)	Errors *Erreurs* (4)-(1)	Amended errors *Erreurs amendées* (5)	Time HW *Heure PM* Diurnal group (6)	Errors *Erreurs* (6)-(1)	Amended errors *Erreurs amendées* (7)	Time HW *Heure PM* Diurnal group (8)	Errors *Erreurs* (8)-(1)	Amended errors *Erreurs amendees* (9)
4	$20^h\ 38^m$	$23^h\ 44^m$	$+3^h\ 6^m$	$+3^h\ 38^m$	$23^h\ 7^m$	$+2^h\ 29^m$	$+3^h\ 22^m$	$22^h\ 9^m$	$+1^h\ 31^m$	$+2^h\ 44^m$	$19^h\ 46^m$	$-0^h\ 52^m$	$+0^h\ 47^m$
3	21 29	21 14	+1 45	-2 17	22 25	+0 56	+1 49	21 30	+0 1	+1 14	20 7	-1 22	+0 17
2	22 19	23 4	+0 45	+1 17	22 20	+0 1	+0 54	21 40	-0 39	+0 34	20 50	-1 29	+0 10
1 bef.- av.-	23 10	23 6	-0 4	+0 28	22 34	-0 36	+0 17	22 6	-1 4	+0 10	21 34	-1 36	+0 3
0 Max: Decl.	0 0	23 17	-0 43	-0 11	22 56	-1 4	-0 11	22 39	-1 21	-0 8	22 18	-1 42	-0 3
			a			b			c			d	
1 aft:- apr.-	0 50	23 34	-1 16	-0 44	23 23	-1 27	-0 34	23 15	-1 35	-0 22	23 4	-1 46	-0 7
2 Spr.- V.E.-	1 41	23 52	-1 49	-1 17	23 52	-1 49	-0 56	23 52	-1 49	-0 36	23 52	-1 49	-0 10
3	2 31	0 10	-2 21	-1 49	0 21	-2 10	-1 17	0 29	-2 2	-0 49	0 40	-1 51	-0 12
4	3 22	0 27	-2 55	-2 23	0 48	-2 34	-1 41	1 5	-2 17	-1 4	1 26	-1 56	-0 17

a) Algebraic mean - 32^m. Applied to (1) gives the rectified errors in (3). Applying this amount to the *kappa* K_1 the "quantité sensiblement constante" is obtained.
b) Algebraic mean. — 53^m.
c) id. id. — $1^h\ 13^m$.
d) id. id. — $1^h\ 39^m$.

Thus the "quantité sensiblement constante" is $6^h\ 6^m$ — $1^h\ 39^m$ or $4^h\ 27^m$. The legend of the French chart N° 3519 mentions "environ cinq heures". This discrepancy may be caused by a somewhat different method of calculation, especially as regards the deduction of the algebraic mean.

As to the precision of this method, it has already been stated that a general maximum error of $1^h\ 40^m$ may be expected (page 34). Comparing the amended errors of Table III with the errors of Table I it appears that the former are larger for the ratio 0.3, about the same for the ratio 0.5, smaller for 0.7 and much smaller for the ratio 1.

Accepting for the Dutch method an error of time of about 2 hours as has been done on page 33, it follows that the French method gives acceptable errors if the age is two days: from one day before till one day after moon's maximum declination with ratios up to 0.5; for two days at each side of maximum declination with the ratio 0.7 and for 3 days before until four days after maximum declination with the ratio 1.

As a general rule these limits shrink according to the increase of the age of the tide.

Comparing the two methods, it will be seen that the precision is about the same for the ratio 0.7 whereas the French method gives better results for the larger ratio's and the Dutch method for the smaller one's.

Now the ratio's of the diurnal groups of the tides of the places mentioned in the French tide tables for the China sea are, with the exception of Hatien (0.5), equal to or larger than 0.7. Consequently the French method is to be preferred to the Dutch method for these places.

The greater majority of the diurnal groups in the Netherlands Indian Archipelago having a ratio smaller than 0.7, as has already been stated above (page 33), the use of the Dutch method is to be preferred for this part of the world.