# THE AZIMUTH METER.

## (AN INSTRUMENT FOR FINDING AN AZIMUTH).

by

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*DESCRIPTION OF APPARATUS* (Fig. 1).

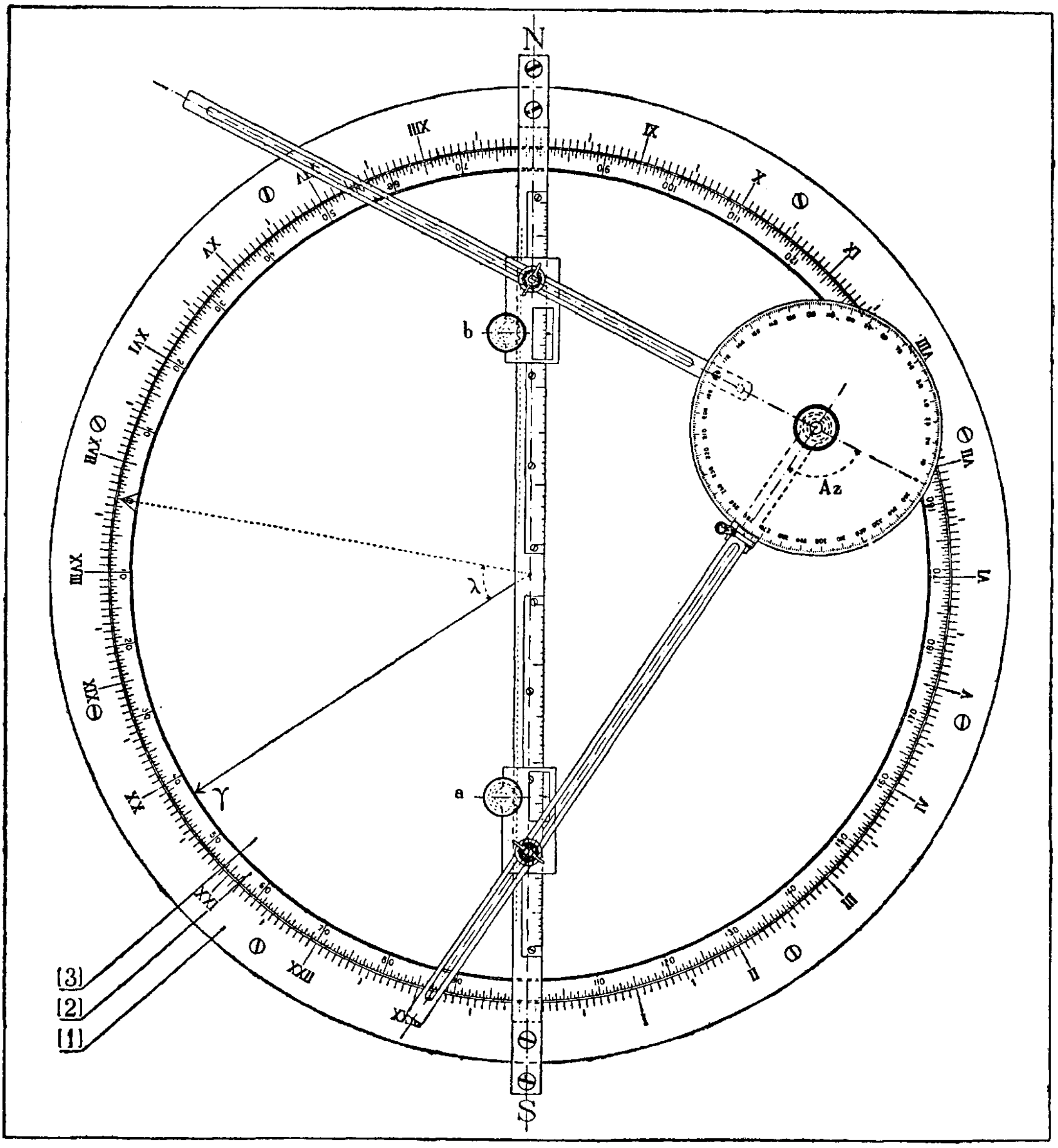


***Fig. 1***

The instrument consists of :–

1. A fixed outer circle (1) divided into twenty-four hours, marked every four minutes, anti-clockwise.

The diameter consists of a raised bar, on a line joining 0 h. to 12 h ; two index slides marked with arrows *a* and *b* can be moved along the divisions of each of the two halves of the raised diameter.

2. A dial (2) divided into 360° (0° to 180° in each direction) which is turned (anti-clockwise) mechanically.

The zero end of the line 0° to 180° (which is marked by an arrow) can be set to indicate sidereal time of Greenwich on the fixed outer circle.

3. A stellar chart on a stereographic projection (3) (similar to that of the Greek Nautical Almanack) and concentric with the other two dials, which can be turned independently. The ecliptic is divided into months and days, which serves to fix the position of the sun in the course of the year.

Through the division for the 21st of March, a radius $\gamma$ is inscribed which shows the meridian of the ascending node.

If the stellar chart disc be turned so that the radius indicates the geographical longitude on the moveable dial, the radius will indicate local sidereal time on the fixed outer circle.

4. A separate small circle divided into degrees 0° - 360° (anti-clockwise) carrying two grooved arms one of which is fixed and one free to revolve.

The azimuth is read off on this circle.

*DIRECTIONS FOR USE.*

1. It is assumed that the clockwork mechanism has been started and that the zero arrow of the turning dial is set to indicate on the outer circle the sidereal time at Greenwich.

2. Turn the stellar chart until the inscribed radius points to the longitude of the place $\lambda$ on the turning dial (East = left of 0° and West = right). Thus the same radius $\gamma$ will indicate the local sidereal time on the outer circle.

3. Place the two index arrows *a* and *b* of the slides so that they indicate on the divisions of the fixed bar the latitude of the place (North = towards the N. and South = towards the S.).

4. Place the two grooved arms of the moveable circle over the buttons of the sliding indexes. Then place the centre of the circle over the body whose azimuth is required, or onto the date on the ecliptic if the sun's azimuth is required.

5. The azimuth may be read off opposite the arrow of the moving arm of the separate circle.

*PRINCIPLE OF THE INSTRUMENT.* (Fig. 2).

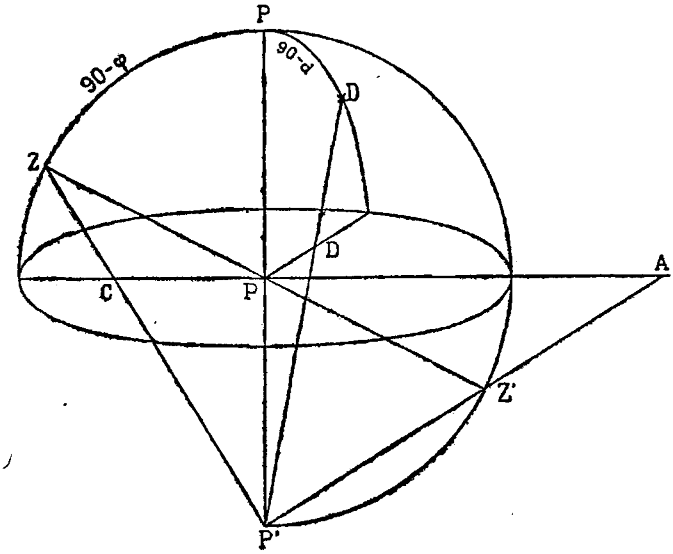


*Fig. 2*

Let $P$ be the pole or its stereographic projection on the Equator.
Let $D$ be the celestial body or its stereographic projection onto the Equator.
Let $d$ be the celestial body's declination, $\omega$ its hour angle for the place, $Az$ its azimuth.
Let $Z$ be the zenith of the place on the celestial sphere, $C$ its stereographic projection onto the Equator.
Let $Z'$ be the nadir and $A$ its projection onto the Equator.

From Figure 2 we have :-

$$\text{arc } PZ = 90 - \varphi \qquad \text{arc } PZ' = 90 + \varphi \qquad \text{arc } PD = 90 - d$$

$$PP'Z = \frac{90-\varphi}{2} \qquad PP'Z' = \frac{90+\varphi}{2} \qquad PP'D = \frac{90-d}{2}$$

Taking the radius of the sphere as unit, we have onto the Equator :-

$$PC = \tan \frac{90-\varphi}{2} = a \qquad PA = \tan \frac{90+\varphi}{2} = b \qquad PD = \tan \frac{90-d}{2} = c$$

$$CPD = \omega$$

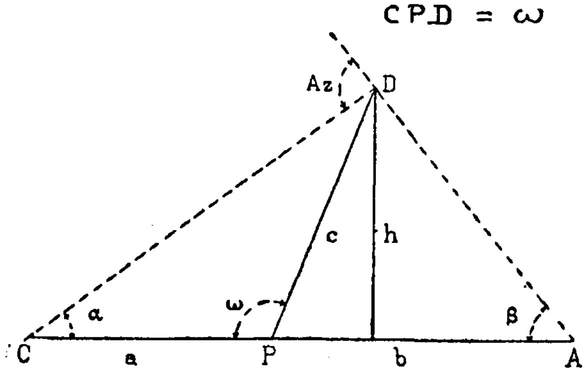


*Fig. 3*

In the triangle $D\ C\ A$ (Fig. 3) we have :-

$$Az = \alpha + \beta$$

$$\tan \alpha = \frac{h}{a - c \cos \omega}$$

$$\tan \beta = \frac{h}{b + c \cos \omega}$$

$$\text{cotang}(\alpha+\beta)=\left[1-\frac{h^2}{(a-c\cos\omega)(b+c\cos\omega)}\right]\frac{(a-c\cos\omega)(b+c\cos\omega)}{h(b+c\cos\omega)+h(a-c\cos\omega)}$$

and if $c' = c\cos\omega$

$$\text{cotang}(\alpha+\beta)=\frac{(a-c')(b+c')-h^2}{(a+b)\,h}$$

By developing the numerator and noting that

$$c'^2+h^2=c^2$$

we have :-

$$\text{cotang}(\alpha+\beta)=\frac{ab+(a-b)c\cos\omega-c^2}{(a+b)\,h}$$

now :-

$$ab = 1$$
$$a-b = -2\,\text{tang}\,\varphi$$
$$a+b = 2\sec\varphi$$

from which we infer :-

$$\text{cotang}(\alpha+\beta)=\frac{1-2\,\text{tang}\,\varphi\cdot c\cos\omega-c^2}{2\sec\varphi\cdot c\sin\omega}$$

This equation may be written :-

$$\text{cotang}(\alpha+\beta)\sec\varphi=\frac{1}{2c\sin\omega}\left(1-2\,\text{tang}\,\varphi\cdot c\cos\omega-c^2\right)$$

Now :-

$$1-c^2=1-\text{tang}^2\frac{90-d}{2}=2\,\frac{\text{tang}\frac{90-d}{2}}{\text{tang}(90-d)}=2\,\text{tang}\frac{90-d}{2}\,\text{tang}\,d$$

and the second part may be written :-

$$=\frac{\text{tang}\frac{90-d}{2}\,\text{tang}\,d-\text{tang}\,\varphi\cdot c\cos\omega}{c\sin\omega}=\frac{\text{tang}\,\cancel{\frac{90-d}{2}}\,\text{tang}\,d}{\cancel{c}\,\sin\omega}-\text{tang}\,\varphi\,\text{cotang}\,\omega$$

$$=\text{tang}\,d\,\text{cosec}\,\omega-\text{tang}\,\varphi\,\text{cotang}\,\omega$$

whence we obtain an alternate form of the classical formula :-

$$\text{cotang}\,Az=\frac{\text{tang}\,d\,\text{cosec}\,\omega-\text{tang}\,\varphi\,\text{cotang}\,\omega}{\sec\varphi}$$

giving the azimuth of a celestial body when the hour is known.

Consequently the angle $Az = \alpha+\beta$ of the Figure 3 is identical with the azimuth required.

*ACCURACY.*

The instrument is accurate to within about 1° if the altitude of the celestial body above the horizon does not exceed 30° to 40°. A more elaborately constructed instrument would give greater accuracy.

*EXAMPLES.*

1. To find the sun's azimuth on the 20th of April, 1929, at 5.00 p. m. local mean time in a given place

$$\begin{cases} \varphi = 38^\circ \text{ N.} \\ \lambda = 25^\circ \text{ E.} \end{cases}$$

The clockwork mechanish shows Greenwich sidereal time = 18 h. 52 m.

The instrument gives us : Azimuth = 87°.
Calculation » » : » = 86°51'.

2. To find the azimuth of *Algol* in a given place :–

$\begin{cases} \varphi = 0^\circ \\ \lambda = 30^\circ \text{ W.} \end{cases}$ at the instant when the watch indicates 10 h. 40 m. local sidereal time.

The instrument gives us : Azimuth = 311° 1/2.
Calculation » » : » = 311°.

NOTE —

A much more simple and easily-worked instrument, without any clockwork mechanism but on the same principle, may be constructed to calculate the course on the Great Circle, especially suited to aerial navigation.