THE USE OF AIRCRAFT FOR SURVEYING.

by

INGÉNIEUR HYDROGRAPHE GÉNÉRAL P. DE VANSSAY DE BLAVOUS,

PRESIDENT OF THE DIRECTING COMMITTEE.

The Air Survey Committee of the British War Office is continuing its studies on the use of aerial photography for surveys; they appear particularly interesting to hydrographic surveyors because they provide a method which does not necessitate the use of costly and complicated restitution apparatus.

In the Review of May 1928, we have already discussed publications N^os^ 2, 3 and 4. Since that date the Air Survey Committee has published the Professional Papers N^os^ 5 and 6.

a) In N° 5, entitled *Calibration of Surveying Cameras*, Captain M. HOTINE, R.E. gives a very detailed and precise description of the various methods employed for measuring the focal length of the objectives of the cameras for taking the views which are to be used for preparing charts. The focal length is one of the indispensable data and is constantly employed in the various problems encountered when using photographs and it must therefore be known with an accuracy comparable to that of the measurement of the position of a point on the photograph (*See Hydrographic Review*, N° 8, p. 74).

We cannot enter into the details of the methods which are described and discussed; some of them may be used in the field by the operator, while others require a laboratory and special instruments. Chapter III gives the methods, such as that of FOURCADE, which make use of angles only, while Chapter IV describes the method of Instructor Captain T. Y. BAKER, B. A., R. N., which utilizes distances only. Other methods, one of which is astronomical, are described in Chapter V for bi-lens or multiple-lens cameras. The operator will derive great benefit from reading the descriptions and by observing all the requirements.

b) Professional Paper N° 6, by the same author, is entitled *Extensions of the "Arundel" method:* it is a continuation of Publication N° 3 which gives a description of a graphic method easily employed without special apparatus, for photographs taken on nearly horizontal plates under good flying conditions — which method had been used with success in the survey of the moderately hilly region of Arundel. A new experiment was made in Scotland in the upper and more mountainous valley of the South Esk River, in the vicinity of Glen Clova. Here the altitudes varied from 210 m. (700 feet) to 960 m. (3200 feet) with differences amounting to 600 m. (2000 feet) on two successive photographs. The "Arundel" method proved inadequate and it was necessary to have recourse to calculation.

In the first two chapters, the author again touches on the "Arundel" method, on its rapidity, on certain details of its use and on the plotting of the contours.

In the third chapter he describes the BARR and STROUD Epidiascope — an instrument designed to give the photographs a transformed image, *i. e.* such as it would have appeared had the photographic plate been horizontal at the instant when the photograph was taken.

A photograph of this apparatus, which has many points in common with the ROUSSILHE apparatus (*See Hydrographic Review* Nº 12, page 100), is reproduced (Fig. 1), but, in principle, instead of making use of a photographic negative a sketch of the transformed image has to be made. The apparatus is primarily designed to permit the revision of old charts already in existence by the insertion of fresh detail which the photographs show to be necessary. It is intended to replace the "camera lucida" by avoiding its principal disadvantage. *i. e.* the lack of fixity of the images.

Chapter IV shows how, while retaining the "Arundel" method, graphic construction may be replaced by calculation, which is always more accurate.

Chapter V is the most important from the theoretical point of view. In this an effort is made to determine the angle of tilt of the plate at the instant the photograph is taken. With a knowledge of these tilts an improvment in the " Arundel " method of radial directions will result, because then not only can the principal point be determined but also the plumb point and, consequently, the isocentre at which the angles undergo no distortion.

As H. G. FOURCADE has shown, two photographs, which partially overlap, may be replaced in the relative positions which they occupied at the instant of exposure without having to use for this purpose control points previously established on the ground. Instead of regarding each plate as a separate unit to be put into place we may, thanks to this property, regard each pair of partially overlapping photographs as the unit. This pair forms a known figure, nearly to scale, such that it may be tranferred *en bloc* and fitted in by means of the control points. Considerable progress is made by always using this complex figure formed by all the luminous rays issuing from the two optical centres of a pair of photographs and mutually supporting each other in pairs instead of considering the single pyramid formed by one photograph.

An elegant demonstration, given as an appendix, shows how an equation may be established between the positions of the same point on the two photographs and the three angles which define the tilts of their optical axes with respect to the line of flight. By taking three common points on the two photographs in addition to the two principal points, we obtain three equations of this type which permit these angles to be determined and consequently the exact definition (approximately to scale) of the figure formed by the pair of photographs. Since the solution of the three equations is very complicated use is made of successive approximations. If the flight is well conducted the required angles will not exceed a very few degrees and by neglecting at first the infinitely small magnitudes of an order greater than the first, this leads to the solution of three equations of the first degree with three unknown

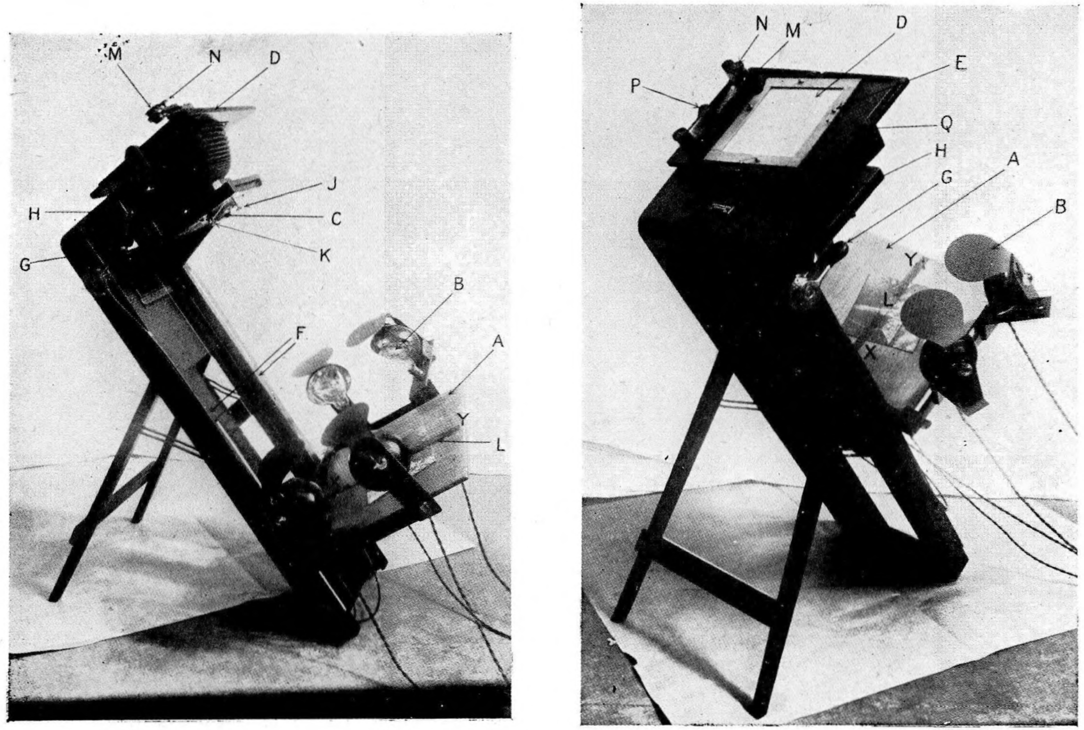

Fig. 1

The BARR & STROUD Epidiascope.

Epidiascope de BARR & STROUD

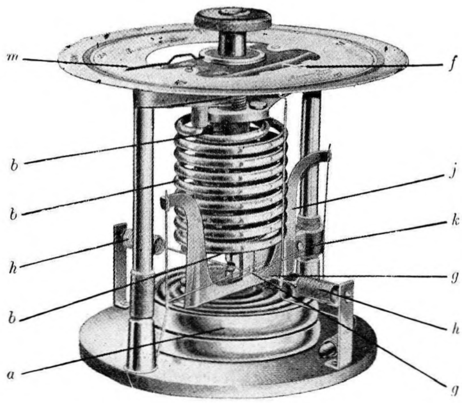

Fig. 2

Paulin Aneroid.

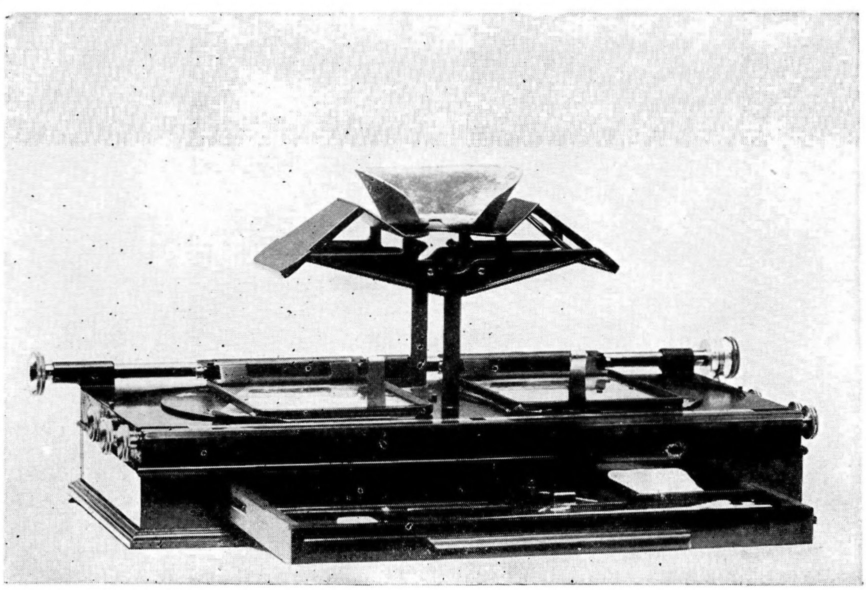

Fig. 3

Barr & Stroud Topographical Stereoscope.

Stéréoscope Topographique de Barr & Stroud quantities. By means of the values thus determined we calculate the terms of the second order and then recommence the calculation by incorporating the value of these terms in the constant term of the equations of the first degree. In practice, in addition to the principal points of the two photographs use is made of four points chosen near the angles of the quadrilaterals of overlap. Thus we use one more point than is necessary (we know that five points are needed), and have to solve four linear equations with three unknown quantities and a check should be obtained. The equations are so simple that their direct calculation is easy and rapid. The author proposes a graphic solution (page 49) which does not appear to us to give adequate verification. It would appear that the graphic solution given in Vol. XII of the *Hydrographic Review* of November 1929, page 110, would not be longer, would show whether there is agreement between the supplementary data and would permit the use of as many points as desired. This solution consists in considering the equations of the first degree with three unknown quantities as representing the faces of a pyramid. If an arbitrary value be given to one of these unknown quantities, each equation would represent a straight line which is easy to plot and with these a polygon is formed. With another arbitrary value of the same unknown quantity, we obtain a second polygon, homothetic to the first: the lines joining the corresponding apices of the two polygons should then intersect in the same point if all the supplementary data are in agreement. One may thus determine the accuracy of the result obtained, which may apparently be within a few minutes if the measurements of the co-ordinates are made to within a hundredth of a millimetre. When disagreement is found it may be due to the use of a shutter slit on the focal plane shutter with which the various parts of the photograph are not taken at the same instant; interlens shutters have been developed and are preferable.

We then proceed to the determination of the absolute tilts with reference to the vertical. This necessitates the use of some control points on the ground. If the altitudes of four points located in the part common to the two overlapping photographs are known, even without their positions on the ground being known, we may deduce therefrom the absolute tilt of the optical axes; the scale may even be deduced therefrom, though usually with but little accuracy.

It is preferable to know the position on the ground and the altitudes of two points on the common part, from which the scale and the orientation can be obtained with accuracy (if the two points are sufficiently far apart); but it is also necessary to know either the altitude, the zenith distance or the azimuth, taken from one of the extremities of the previous base, of a third point. If the third point is only visible on one of the photographs it would be necessary to know either its position (its altitude will not be necessary), or its altitude and azimuth, or its azimuth and zenith distance.

Several methods of calculation are given; it appears, however, that the process might be further improved. If this determination is made for each pair of photographs, the tilts of each optical axis (except for the first and the last of a strip), will be determined twice and thus furnish the desired check. But the principal advantage of the method consists in the fact that it is sufficient to determine the tilt of the optical axes, and the scale and orientation for the first pair of photographs, to be able to deduce from this the same data for all the photographs in the strip. By making the same determination for the last photograph of the strip we obtain a check which permits adjustment to be made. Treated in this manner the photographic method is very practical because it requires the use of only a few points of control on the ground.

One can go even further and omit the points of control on the ground if the altitudes of the aeroplane are given by a good altimeter and if a double system of strips perpendicular to each other be taken.

In this chapter we find a discussion of a large number of particular cases and numerous practical examples are given: chapter VII treats of the method employed at Glen Clova and the results obtained.

In chapter VI a study is made of the case of a reconnaissance survey with little or no help in the way of ground control points. A few simplified methods are given in which it is more than ever essential that the operator should succeed in flying in a straight line, horizontally, at a constant speed and making exposures at very regular intervals. He should be provided with an altimeter. In the appendix a description of an altimeter is given with which experiments were made at Glen Clova and which allows the altitude to be estimated within one foot (0.30 metre).

In this *Paulin Aneroid* (Fig. 2) the principle is applied of returning an indicator to the same place by establishing an equilibrium between the variations in the air pressure on the diaphragm of the vacuum box by means of a spiral spring, which is done by turning a pointer which, by its displacement, measures the variation in the pressure. Thus the deformation of the vacuum box is avoided (elastic errors) and friction errors are diminished.

c) The *Geographical Journal* for February 1930, gives the text of a very interesting lecture delivered by Captain HOTINE at the meeting of 11th November 1929 under the title:- *The application of stereoscopic photography to mapping.*

In this the author recounts the efforts which have been made, particularly in Great Britain, to solve the problem of the employment of photography for surveying for charts, the employment of stereoscopy, the researches made by FOURCADE and his Stereocomparator and the BARR and STROUD, Topographic Stereoscope, (Fig. 3) as well as the DAVILLE Stereoscopic Plotter.

He then describes the FOURCADE Stereogoniometer, which is based on the principles of stereoscopy established by this author (*see* above), and which appears to be a big step beyond all the automatic instruments for plotting charts developed up to the present (Fig. 4).

Finally, Captain HOTINE gives the description of a restitution apparatus in use by the Ordnance Survey, the BARR & STROUD Photogrammetric Plotter, which had never been described previously and which is remarkable for the simplicity of its conception (Fig. 5).

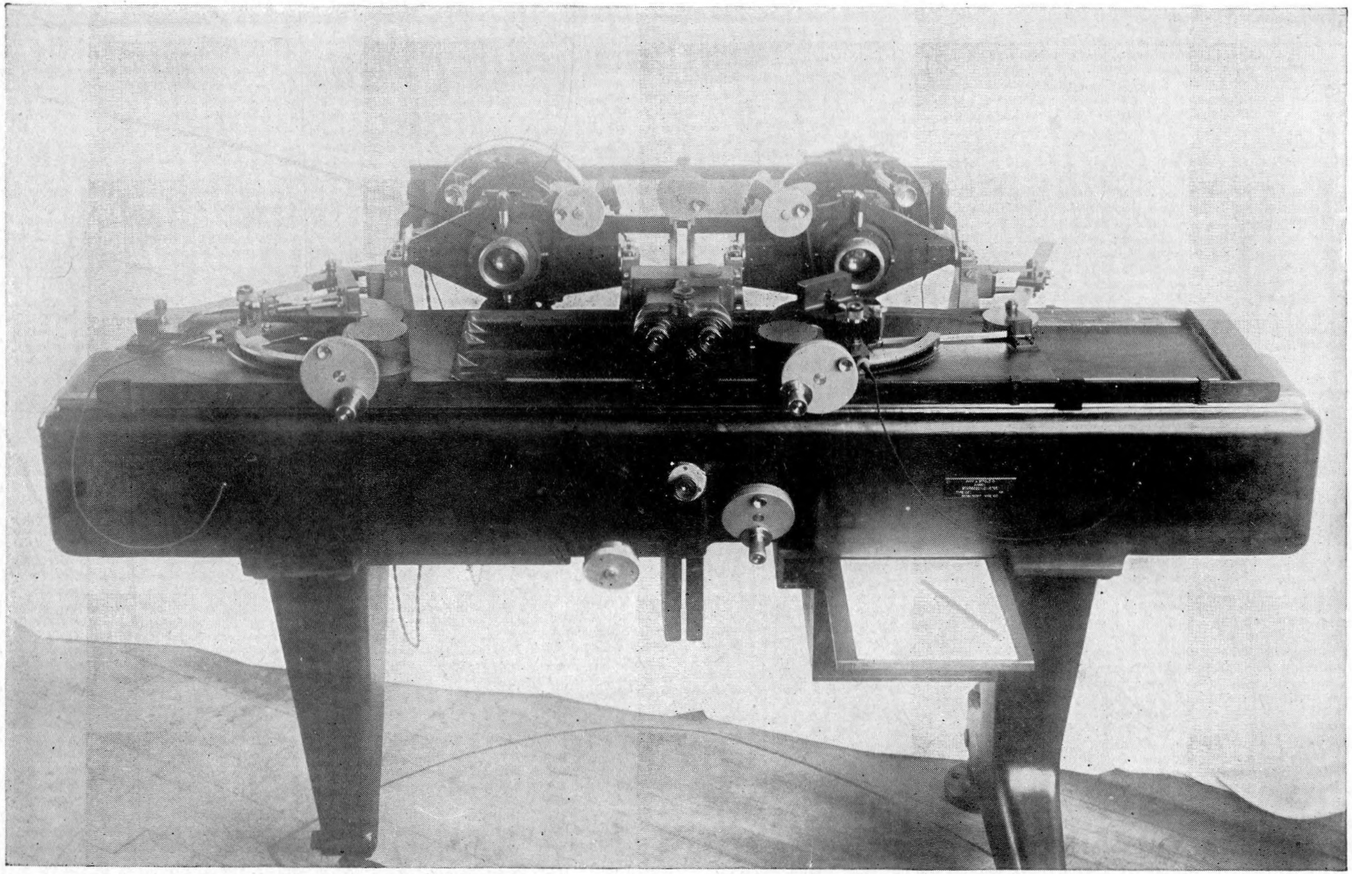

Fig. 4

The Fourcade Stereogoniometer.
Le Stéréogoniomètre de Fourcade.

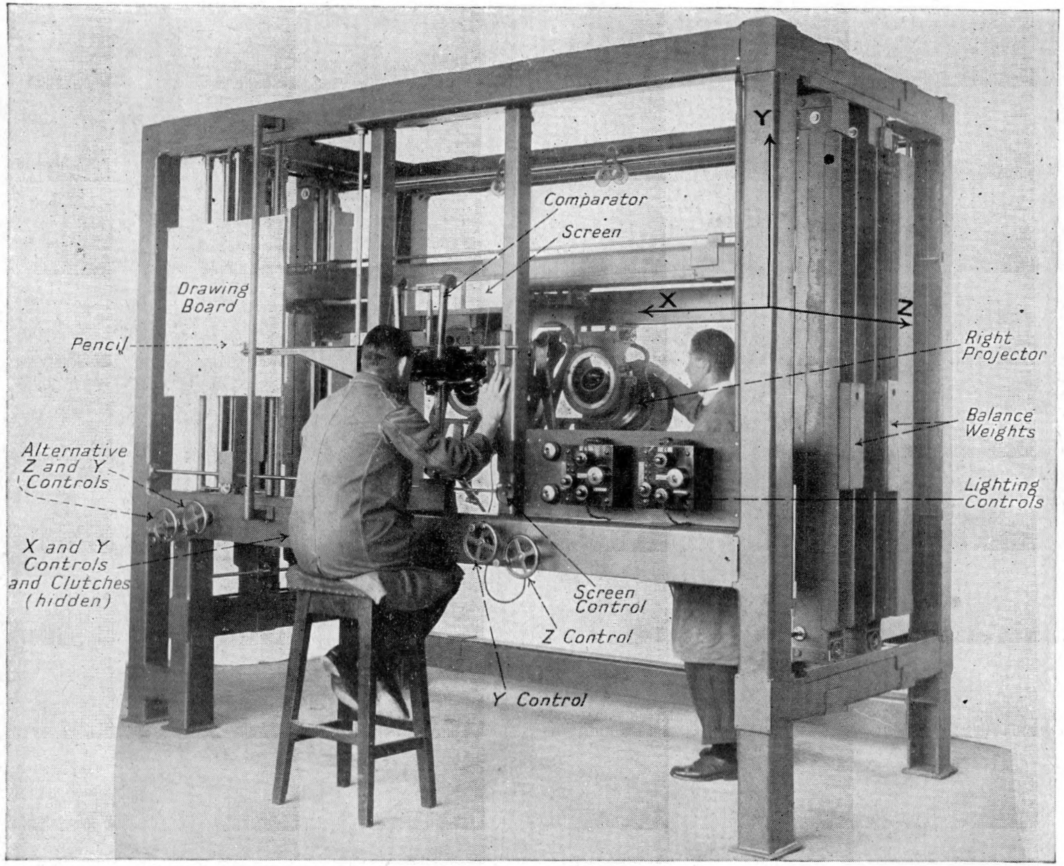

Fig. 5

The Barr & Stroud Photogrammetic Plotter.

d) The Italian Review, *Universo*, in its number for February 1930, published an article by Enzo PISANI on aerophotogrammetric surveys in the vicinity of Naples. The airplane carries two photographic cameras having their optical axes diverging at a certain fixed angle with a view to enlarging the strip of ground photographed.

The cameras are fitted with a special device consisting of two revolving prisms above the objective, to photograph the solar disc on one of the plates while simultaneously the dial of a small chronometer and the graduations of the drums which revolve with the prisms are photographed on the other. (*See Hydrographic Review* N° 8 - November 1927 - page 101).

In order that the photographs may be taken at intervals as nearly equal as possible and under full control, the apparatus is automatic, being controlled by a screw of variable pitch SANTONI patent, which is driven by the air current.

The axes of the successive strips which are to be photographed are carefully marked on the chart and on the ground, and strict attention is devoted to following these marks accurately with the aid of a sun dial oriented towards a clearly visible point which is brought into collimation with the shadow of a control mark, so that the pilot is able to follow a rectilinear course by keeping the shadow on the index.

The drift is estimated by means of a small drift-meter and the photographic apparatus is turned in such a manner that the axis of the plate always coincides with the axis of the resultant movement of the airplane. Generally all the strips are flown in the same direction.

e) We have described (*Hydrographic Review* N° 8, page 85) a method of calculation for placing a triangle ABC on a pyramid of apex S which is defined by the angle $\beta - \alpha$ of its face ASB, the angle α which the projection of the third edge SC on this face makes with the edge SA, and the angle θ of the edge SC with its projection.

We have solved, either by direct calculation or by a rapid method of successive approximations, the equation of the 4th degree which gives four values for the angle ω, made by the plane of the base ABC with the face ASB (pages 88 and 94).

We have shown also what algebraic conditions must obtain in order that the equation may have a double root and proved by calculation that the double root corresponds to the case where the apex S is located at a point, moreover well determined, on the right cylinder circumscribing the triangle ABC (page 93). So far as we know only one demonstration of this property has been given before, and therefore it appears that it might be of interest to give a very elementary geometrical proof which will make the conditions of the problem a little more readily comprehensible.

Let us draw on the straight line AB the segment of the arc subtending the angle $\beta - \alpha$; the apex S of the pyramid will be, on our plane of projection, somewhere on this segment. If the triangle ABC turns about AB, the point C will be projected somewhere on the straight line perpendicular to AB passing through the fixed point O (*See* Fig. 6). Besides the projection $C_1 S$ of CS will pass through a fixed point D of the circumference such that : *arc* $A\ D = 2\ \alpha$.

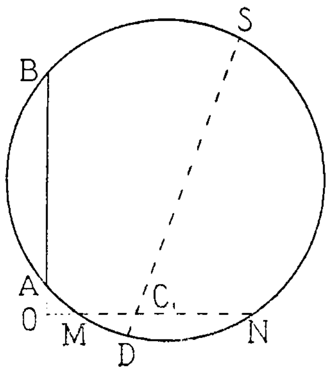

Fig. 6

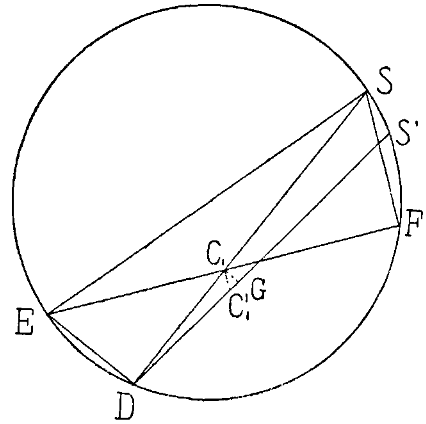

Fig. 7

If we describe the arc subtending the angle about point S, the straight line SC will make a constant angle θ with the plane of the projection and will constantly rest on the perpendicular dropped from D on to this plane. Thus it will generate a surface (called a conoid) which is intersected at four points (on the same side as the plane of the projection) by the circumference described by the point C in its rotation about AB. These are the four positions of the point C sought. Two of them will coincide if, at one of these points of intersection, the straight line OC is normal to the section of the conoid made by the plane $C_1 OC$ of the circumference.

The tangent to the conoid at the point C containing the generatrix DS, its projection on the plane of the projection will pass through the point S and is a straight line which we shall determine.

Let DS' be a generatrix very close to DS (*See* Fig.7) and let ε be the angle of the projections of these two straight lines on the face ASB. The difference in length $\overline{DS}$-$\overline{DS'}$ is equal to $\varepsilon\ \overline{DE}$ if E is symmetrical to S with respect to the centre of the circle. This is also the difference in length $\overline{DC_1}$-$\overline{DC'_1}$ if C'_1 is the projection of a point on DS' located at the same distance from the plane of the projection as the point C.

From C_1 drop a perpendicular on DS' which meets DS' at the point G. The sides of the right triangle $C'_1 C_1 G$ are :-

$$C'_1 G = \varepsilon \overline{DE}$$
$$C_1 G = \varepsilon \overline{DC_1}$$

It is therefore similar to the triangle $C_1 ED$ and the angle $\widehat{C_1 C'_1 G}$ is equal to the angle $\widehat{DEC_1}$. The straight line $C_1 C'_1$ is therefore parallel to the straight line SF obtained by joining S to the point of intersection of the straight line EC_1 and the circumference. The straight line SF is therefore the projection on the plane ASB of the plane tangent to the conoid at the point C. The tangent at the point C at the section of the conoid by the plane C_1OC will be the straight line CT contained in the tangent plane and projected on C_1T. (*See* Fig. 8).

1) If there is a double root, OC is perpendicular to CT and we have the relation :-

$$\overline{OC}^2 = \overline{OC_1} \,.\, \overline{OT}$$

The perpendicular SQ dropped from S on to the plane ABC will make, with the perpendicular SL at AB, a triangle SQL, similar to the triangle TCO, since angle $\widehat{QLS}$ is equal to $\widehat{COT}$.

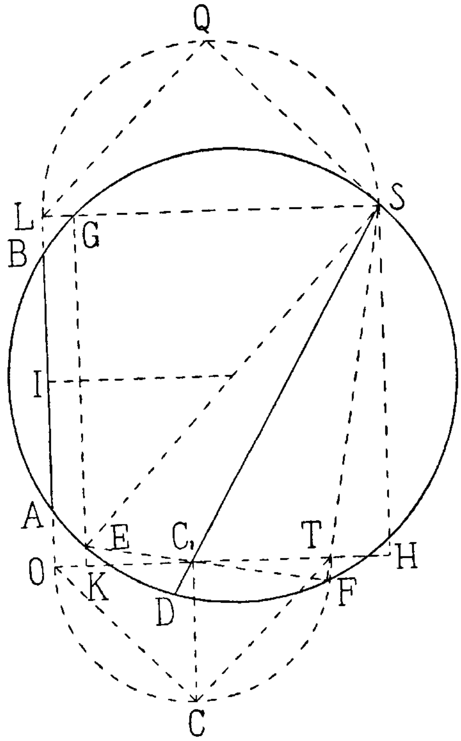

Fig. 8

We have therefore :-

$$\overline{LQ} = \overline{OC}\,\frac{\overline{LS}}{\overline{OT}}$$

and from the preceding formula

$$\overline{LQ} = \overline{LS}.\,\frac{\overline{OC_1}}{\overline{OC}}$$

The circle passing through the three points A,B,C will have its centre on a perpendicular to the straight line AB at its middle point I, at a distance d from AB, given by the equation :-

$$\overline{OA}\,.\,\overline{OB} = \overline{OC}\,(2d - \overline{OC})$$

The point Q will be on the same circle if at the same time we have :-

$$\overline{LA}\,.\,\overline{LB} = \overline{LQ}\,(2d - \overline{LQ}),$$

that is, by eliminating d between these two equations, if the relation holds

$$(1) \qquad \frac{\overline{OA}\,.\,\overline{OB}}{\overline{OC}} = \frac{\overline{LA}\,.\,\overline{LB}}{\overline{LQ}} + \overline{LQ} - \overline{OC}$$

Let us show that this is proven. For this purpose, drop the perpendiculars SH and KEG to the straight line OT and in equation (1) replace $\overline{OC}$ and $\overline{LQ}$ by their values obtained from the preceding equations. This equation then becomes

$$(1a) \qquad \overline{OA}\,.\,\overline{OB} = \overline{LA}\,.\,\overline{LB}\,(1 - \frac{\overline{TH}}{\overline{LS}}) + \overline{OC_1}\,.\,\overline{TH}$$

In this we may replace $\overline{LA}\,.\,\overline{LB}$ by $\overline{LG}\,.\,\overline{LS}$ or by $\overline{OK}\,.\,\overline{OH}$ and we have :-

$$(1b) \qquad \overline{OA}\,.\,\overline{OB} = \overline{LA}\,.\,\overline{LB} + \overline{KC_1}\,.\,\overline{TH}$$

From the similarity of the two right triangles STH and KEC_1, which have their sides perpendicular to each other, we obtain the relation :-

$$\overline{KC_1}\,.\,\overline{TH} = \overline{KE}\,.\,\overline{SH} = \overline{KE}\,.\,\overline{OL}$$

But we also have :-

$$\overline{OA}\,.\,\overline{OB} = \overline{OI}^2 - \overline{AI}^2$$

$$\overline{LA}\,.\,\overline{LB} = \overline{LI}^2 - \overline{AI}^2$$

$$\overline{OA}\,.\,\overline{OB} - \overline{LA}\,.\,\overline{LB} = \overline{OI}^2 - \overline{LI}^2 = (\overline{OL} - \overline{LI})\;(\overline{OL} + \overline{LI}) = \overline{KE}\,.\,\overline{OL}$$

Therefore the same circle passes through all the four points A,B,C,Q.

(2) Reciprocally, if the four points lie on the same circle, the straight line OC is perpendicular to the tangent at this point to the section of the conoid and the problem has a double root.

In this case equation (1) is proven. Further, the right triangles LSQ and OCC_1 are similar, the angles at O and L being equal. We have then :-

$$\overline{LQ} = \frac{\overline{OC_1}\,.\,\overline{LS}}{\overline{OC}}$$

from which :-

$$\overline{LQ} - \overline{OC} = \frac{\overline{OC_1}\,.\,\overline{LS} - \overline{OC}^2}{\overline{OC}}$$

The equation (1) becomes :-

$$(2)\; \overline{OA}.\overline{OB} = LA.LB\frac{\overline{OC}^2}{\overline{OC_1}\,\overline{LS}} + \overline{OC_1}\,\overline{LS} - \overline{OC}^2 = \overline{LA}.\overline{LB} + (\overline{OC_1}.\,\overline{LS} - \overline{OC}^2)\left(1 - \frac{\overline{LA}.\overline{LB}}{\overline{OC_1}\overline{LS}}\right)$$

But we have seen that

$$\overline{OA}\,.\,\overline{OB} - \overline{LA}\,.\,\overline{LB} = \overline{KE}\,.\,\overline{OL}$$

It is now possible to construct the tangent at C to the section of the conoid by the plane COC_1, and the similar triangles KEC_1 and THS then give us the following equation :-

$$\frac{\overline{KE}}{\overline{TH}} = \frac{\overline{KC_1}}{\overline{SH}} = \frac{\overline{KC_1}}{\overline{OL}}$$

from which :-

$$\overline{KC_1} = \frac{\overline{KE}\,.\,\overline{OL}}{\overline{TH}} \text{ and } \overline{OC_1} = \overline{OK} + \frac{\overline{KE}\,.\,\overline{OL}}{\overline{TH}}$$

$$\overline{OC_1}\,.\,\overline{LS} = \overline{OK}\,.\,\overline{LS} + \overline{LS}\,\frac{\overline{KE}\,.\,\overline{OL}}{\overline{TH}}$$

Since $\overline{OK} = \overline{LG}$, we may replace $\overline{OK} \cdot \overline{LS}$ by $\overline{LA} \cdot \overline{LB}$ and consequently equation (2) becomes :-

$$\overline{KE} \cdot \overline{OL} = (\overline{OC_1}\,\overline{LS} - \overline{OC}^2) = \frac{\overline{KE} \cdot \overline{OL}}{\overline{OC_1} \cdot \overline{TH}}$$

or :-

$$\overline{OC}^2 = \overline{OC_1}\ (\overline{LS} - \overline{TH}) = \overline{OC_1} \cdot \overline{OT}$$

which equation shows that OC is perpendicular to CT.