SLOPE CORRECTIONS FOR ECHO SOUNDING.

by

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The Director of the U. S. Coast and Geodetic Survey has forwarded to the International Hydrographic Bureau the following article which is the result of studies which have been made in the U. S. Coast and Geodetic Survey of data obtained by echo sounding.

PART I.

Until a few years ago, the measurement of depths of the sea was usually accomplished by means of a hand lead in comparatively shoal water and by the steam or electrically operated sounding machine in deep water areas. With the exception of pressure tubes, the apparatus used involved the measurement of the depths directly. In depths over 20 fathoms such methods were slow, laborious and costly, since they required slowing down or stopping the vessel completely for every depth obtained. As the depths became greater, the cost per sounding greatly increased until in depths over 100 fathoms, the cost became almost prohibitive. As a result fewer soundings were taken in deep water areas, important submarine configuration were inadequately developed or entirely missed, and information of signal value to both the mariner and the oceanographer was frequently lacking on nautical charts. With such definite limitations in the collection of navigational and scientific data, it was but natural that some means should be searched for that would not only reduce the excessive cost of deep sea surveying, but one that would also give a more detailed representation of the bottom relief even in shoal water.

Even as early as 1855 unsuccessful attempts were made to fathom the depths of the ocean by means of the transmission of sound waves. Maury in his picturesque work' "The Physical Geography of the Sea", published in 1855, speaks of the varied contrivances that were employed, but as he aptly phrases it, "every trial was only a failure repeated". After this there followed a period of inactivity with efforts chiefly concentrated on the development of direct methods of sounding. But with the *Titanic* disaster of 1912 and the war of 1914, the first real beginning was made in the practical application of acoustical methods to submarine surveying.

It was the experiments conducted by Professor Reginald FESSENDEN for detecting the presence and nearness of icebergs that led to the discovery of the basic principle of the Fathometer, the instrument that is successfully used today in the hydrographic work of the Coast and Geodetic Survey. FESSENDEN, while operating off the Grand Banks soon discovered that by measuring the elapsed time between the transmission of a sound impulse from his oscillator and the return of the echo from the bottom of the ocean, he was able to determine the depth of water under his vessel. This is the underlying principle of practically all echo sounding today. The various forms of the apparatus in use both in this country and abroad simply represent different devices for transmitting the sound, receiving the echo, and measuring the time interval. It is quite obvious that if the velocity of sound in sea is known, the depth of water below the point at which an oscillation takes place can be readily ascertained by multiplying the velocity of sound (as corrected for the local condition of temperature, density and salinity) by one-half the time it takes for the sound wave to reach the bottom and return to the ship.

The physical laws governing the reflection of light rays apply equally well to the reflection of sound waves. When a sound impulse emanates from a point near the surface of the water, the sound is diffused in all directions, and it is the present theory that one of these waves will reach the bottom and be reflected back by the shortest possible route and this will be the first echo received on the vessel by the hydrophone. From the familiar principle of physics, that the angle of incidence equals the angle of reflection, the depth D under the vessel is computed trigonometrically from the relation

$$D = \sqrt{(S)^2 - (L/2)^2}$$

where S equals the distance from the oscillator to the point of reflection on the bottom (velocity of sound × 1/2 elapsed time) and L equals the horizontal distance from oscillator to hydrophone, called the base line. (See Fig. 1.)

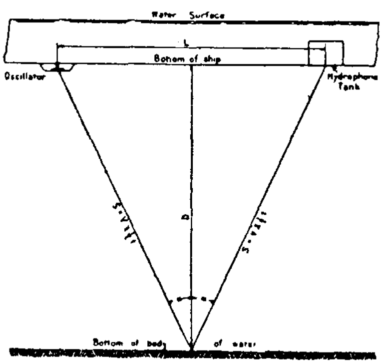


Fig. 1

Method of correcting for base line.

This computation is known as the correction for base line. In practice this correction is taken care of when the fathometer scale is designed, and if the ship installation of oscillator and hydrophone corresponds to the base line for which the fathometer scale was constructed the depths can be read directly on the fathometer dial. But where the base line on the vessel differs from that for which the scale has been designed, as occasionally happens, then the correction can be applied either by constructing a new scale based on the above relation and using that in place of the portion of the fathometer scale affected, or by changing the non-uniform portion of the fathometer scale to a uniform scale (a scale where the distance between oscillator and hydrophone is zero) and constructing a table of corrections for the particular base line as computed from the above expression. The depths read on the uniform scale are then corrected from the table of corrections. Obviously, as the depth increases, the difference between D and S becomes smaller, until in depths considerably in excess of the base line the difference disappears entirely, and when the bottom is horizontal or nearly so the depth under the vessel will for all practical purposes be the same as the echo distance from the oscillator to the point of reflection on the bottom.

But when the bottom is not horizontal, the echo distance will not represent the true depth under the vessel, but will be the perpendicular from the vessel to the sea bottom. (See Fig. 2.)

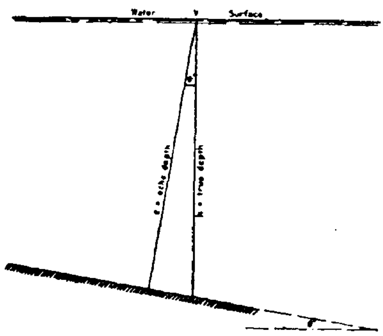


Fig. 2

Correction to echo depth for slope of bottom.

If the slope of the bottom is known, the true depth can be determined by applying a correction to the echo distance, which correction is arrived at in one of various ways, all of which, however, employ the basic relation $h = \frac{e}{\cos \theta}$, where h is the true depth under the vessel, e the echo distance as obtained by the echo sounding machine, and θ the slope of the bottom. Until some mechanical means is perfected whereby the vertical depth under the vessel is obtained instead of the normal to the bottom, these slope corrections will continue to be a problem.

It is the purpose of this paper, however, to demonstrate mathematically and graphically the practical unimportance and futility of applying slope corrections within certain limits, thereby reducing to a minimum the laborious and costly process of methodically correcting echo soundings for bottom slopes. While the conclusions reached are intended primarily for the guidance of the field engineers of the Survey, it is hoped they will also serve to allay the fears and doubts of those who believe that the charting of uncorrected echo soundings (within specified limits) will give the navigator an erroneous representation of the bottom. The paper is divided into two parts: Part I dealing with the problem of defining limits within which slope corrections may be disregarded, and Part II dealing with the problem of considering those special cases where the scale of the survey and the steepness of the slope require a closer approximation of the true depths under the vessel.

The whole complex problem of slope corrections is approached from the standpoint of the cartographer and the practical navigator rather than from the purely theoretical viewpoint. The problem is not to determine what degree of slope will make a considerable difference in the depth as obtained by the echo sounding machine (as has heretofore been considered), but at what distance from the position of the vessel will one obtain a vertical depth equal to the echo depth. This should be the basis of all slope corrections from the standpoint of practical navigation and chart construction. If *this distance*, considering the scale of the survey, is insignificant as compared with the height and width of sounding figures on the survey sheets of corresponding scales, it will profit no one to correct for slope, since the uncorrected echo depth at the plotted position will be equally as accurate as the vertical sounding. And if this condition holds true for the original survey sheet, it will with greater force apply to the printed chart, since the latter is usually on a smaller scale. For convenience this distance will be referred to as "displacement" in the discussion that follows.

It is not to be supposed, however, that the method is without its limitations. Two suggest themselves at once :

1. — In a configuration, such as a steep submarine valley, where the echo would never come from the deepest portion of the valley as long as the perpendicular distance from the vessel to the sloping sides is less than the vertical depths under the vessel, the conclusions reached below would indicate that for many scales no correction need be applied to the echo soundings. This would be undesirable since the incorporation or omission of an important submarine feature would be made to depend upon the scale of a survey sheet. On the other hand. if in accordance with the present practice of correcting for slopes, the echo soundings are increased, there is always danger of showing either a greater or less depth than actually exists and seldom will the true depth be indicated. This is illustrated in Fig. 3 where curves *A* and

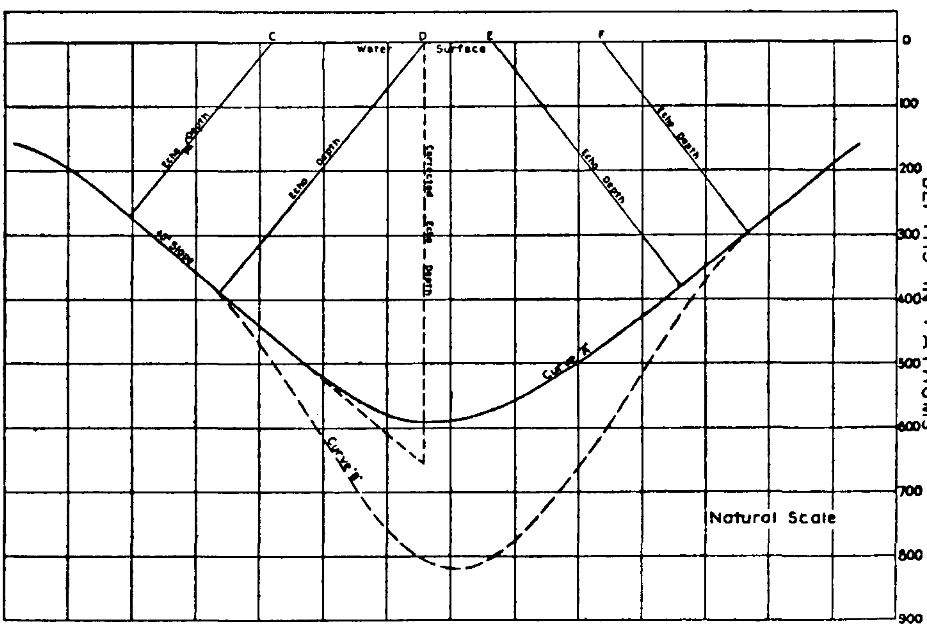


Fig. 3. — Echo sounding in a submarine valley.

B represent two submarine formations and *C, D, E,* and *F,* points at which successive echo soundings are obtained. It is clear that under the conditions shown the echo sounding machine would never give a depth equal to the greatest depth in the valley. Consequently it would be erroneous to use the uncorrected echo depths. But if a correction for slope is applied to the echo sounding at *D*, the depth obtained will be 70 fathoms greater than the correct depth for curve *A* and 150 fathoms less than the correct depth for curve *B*. Therefore, in such cases neither methods is satisfactory and the true solution would seem to be to take enough vertical soundings to accurately determine the thalweg lines or lines of greatest depth.

2. — When the bottom is very irregular, the whole theory of slope corrections breaks down, since these corrections are based on the assumption that the echo comes from a direction at right angles to the slope as determined by two consecutive or adjacent echo depths. In extremely irregular bottom the echo would be obtained from the nearest projection, and the actual slope would be difficult if not impossible to ascertain. In Fig. 4 the echo returns from projection *A* will no be perpendicular to a line joining projections *A* and *B* but will be perpendicular to the tangents to projection *A* at the various points of echo and hence any determinations of slope based on

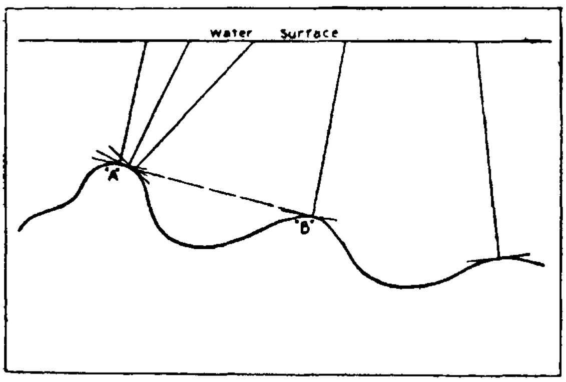


Fig. 4

Echo returns from irregular bottom.

the echo soundings from *A* and *B* will be erroneous, and an inspection of the figure will show the fictitious depths that would be introduced if an attempt was made to reduce the echo depths to vertical soundings.

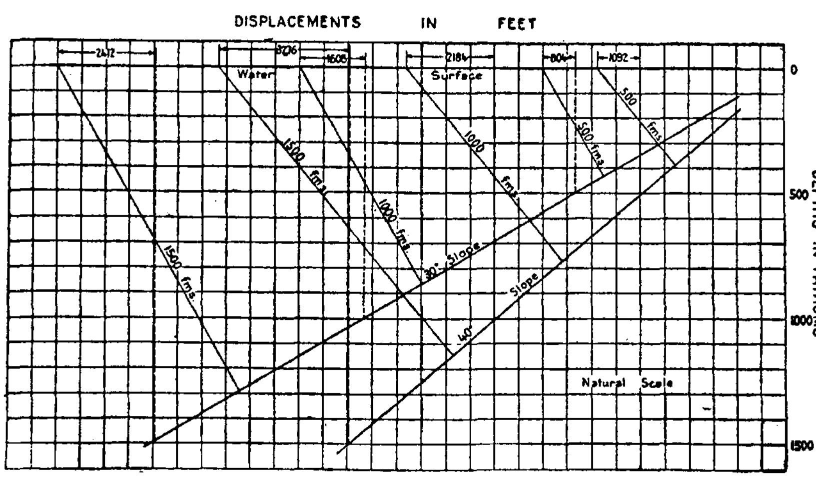


Fig. 5. — **Graphical method of determining displacements.**

But notwithstanding these limitations, the method proves extremely valuable when the slopes are long and uniform and the depth curves fairly regular. For example, from the analyses given in Tables I and II the interesting fact is disclosed that for a scale of 1:40,000 and depths less than 200 fathoms no corrections for slope need be applied even when the slope of the bottom is as much as 40°. The real significance of this becomes apparent when it is remembered that the usual echo sounding sheet is seldom plotted on a scale larger than 1:40,000 and the depths involved on such sheet are generally less than 200 fathoms. But even in the Hawaiian group where the 1000 fathoms curve is but a few miles offshore and well within the limits of a 1:40,000 scale survey sheet it will be found unnecessary to correct echo depths for slope on the greater portion of the sheet.

Table I is a table of displacements for depths ranging from 50 to 1500 fathoms and for slopes of 10 to 40 degrees. It is unnecessary to consider slopes of less than 10° since the difference between the uncorrected echo depth and the true depth would be less than 1.5 % of the true depth, (see Slope Correction Graph on page 95) a discrepancy well within the allowable limits for direct sounding methods. The *actual displacements* are given in Column 2 and are indicated in feet. These displacements can either be graphically determined (as in Fig. 5) or can be mathematically computed as was done in this paper from the expression,

$$\text{Displacement} = \frac{\text{Echo sdg}}{\textit{sin}\ \text{slope}} - \frac{\text{Echo sdg}}{\textit{tan}\ \text{slope}}$$

as developed in Problem 1 (page 92 of this paper). Columns 3 to 8 give the displacements in inches for the scales of 1:10,000, 1:20,000, 1:40,000, 1:60,000, 1:80,000, and 1:120,000 respectively. The scales of 1:10,000 and 1:20,000 have been incorporated in the table to take care of those surveys that might be made on these scales in the deep water areas of Washington Sound, Admiralty Inlet, Puget Sound and along some of the shores of the Hawaiian Islands. Column 9 gives the width and column 10 the height of a sounding figure in inches as it appears on the average hydrographic sheet. These latter values will be constant for all scales. The English system has been chosen for indicating displacements instead of the metric system in order to avoid the use of small decimal fractions that are not readily comprehended.

From a study of the Table of Displacements it is evident that many of the displacements, on various scales, are insignificant, when compared with the height and width of a sounding figure; which means that for practical purposes it is immaterial whether the uncorrected or corrected echo depth is plotted at the ship's position, since in many cases a vertical depth equal to the uncorrected echo depth would be obtained within the space of a sounding figure. This distance, it is submitted, is too small to be of any practical consequence and therefore, as a basis for determining when an echo depth should be corrected for slope, the following arbitrary rule was adopted:

"Begin correcting for slope when the displacement on any scale in inches equals the height or twice the width of a sounding figure in inches."

Table II, designated "Table of Limits", was arrived at by applying this rule to the data given in Table I, and shows at what depth to begin correc-

TABLE I

Table of Displacements
Table des Déplacements

Slope Pente	Actual (Feet) Réel	Displacements - Déplacements 1:10,000 Scale (Inches)	1:20,000 Scale (Inches)	1:40,000 Scale (Inches)	1:60,000 Scale (Inches)	1:80,000 Scale (Inches)	1:120,000 Scale (Inches)	Height of sounding figure Hauteur du chiffre de sonde (inches)	Width of figure Largeur de sonde (inches)
		50 Fathom Depths - Profondeurs de 50 brasses.							
40°	109	.13	---	---	---	---	---	.16	.08
		75 Fathom Depths - Profondeurs de 75 brasses.							
30°	120	.14	---	---	---	---	---	.16	.08
35°	142	.17	---	---	---	---	---	.16	.08
		100 Fathom Depths - Profondeurs de 100 brasses.							
10°	83	.06	.03	.02	.01	.01	---	.16	.08
15°	79	.09	.05	.02	.02	.01	.01	.16	.08
20°	106	.13	.07	.04	.02	.02	.01	.16	.08
25°	133	.16	.08	.04	.03	.02	.01	.16	.08
30°	161	.19	.10	.05	.03	.03	.02	.16	.08
35°	189	.23	.11	.06	.04	.03	.02	.16	.08
40°	218	.26	.13	.07	.04	.04	.02	.16	.08
		125 Fathom Depths - Profondeurs de 125 brasses.							
20°	132	.16	---	---	---	---	---	.16	.08
		150 Fathom Depths - Profondeurs de 150 brasses.							
30°	241	.29	.14	---	---	---	---	.16	.08
35°	284	.34	.17	---	---	---	---	.16	.08
		175 Fathom Depths - Profondeurs de 175 brasses.							
15°	138	.16	---	---	---	---	---	.16	.08
		200 Fathom Depths - Profondeurs de 200 brasses.							
10°	105	.13	.07	.03	.02	.02	.01	.16	.08
15°	108	.19	.10	.05	.03	.02	.02	.16	.08
20°	212	.25	.13	.06	.04	.03	.02	.16	.08
25°	266	.32	.16	.08	.05	.04	.03	.16	.08
30°	321	.39	.20	.10	.07	.05	.03	.16	.08
35°	379	.45	.23	.11	.08	.06	.04	.16	.08
40°	437	.52	.26	.13	.09	.07	.04	.16	.08
		250 Fathom Depths - Profondeurs de 250 brasses.							
10°	131	.16	---	---	---	---	---	.16	.08
20°	264	.32	.16	---	---	---	---	.16	.08

Table of Displacements (Cont.)
Table des Déplacements (suite)

Slope Pente	Actual (Feet) Réel	Displacements - Déplacements 1:10,000 Scale (Inches)	1:20,000 Scale (Inches)	1:40,000 Scale (Inches)	1:60,000 Scale (Inches)	1:80,000 Scale (Inches)	1:120,000 Scale (Inches)	Height of sounding figure Hauteur du chiffre de sonde (inches)	Width of figure Largeur de sonde (inches)
		300 Fathom Depths - Profondeurs de 300 brasses.							
10°	157	.19	.10	.05	.03	.02	.02	.16	.08
15°	236	.28	.14	.07	.05	.04	.02	.16	.08
20°	318	.38	.19	.10	.06	.05	.03	.16	.08
25°	399	.48	.24	.12	.08	.06	.04	.16	.08
30°	482	.58	.29	.15	.10	.07	.05	.16	.08
35°	567	.68	.34	.17	.11	.09	.06	.16	.08
40°	655	.79	.40	.20	.13	.10	.07	.16	.08
		400 Fathom Depths - Profondeurs de 400 brasses.							
10°	210	.25	.13	.06	.04	.03	.02	.16	.08
15°	312	.37	.19	.09	.06	.04	.03	.16	.08
20°	420	.50	.25	.13	.08	.06	.04	.16	.08
25°	532	.64	.32	.16	.11	.08	.05	.16	.08
30°	643	.77	.39	.19	.13	.10	.06	.16	.08
35°	756	.91	.46	.23	.15	.11	.08	.16	.08
40°	874	1.05	.52	.26	.17	.13	.09	.16	.08
		500 Fathom Depths - Profondeurs de 500 brasses.							
10°	263	.31	.16	.08	.05	.04	.03	.16	.08
15°	396	.48	.24	.12	.08	.06	.04	.16	.08
20°	528	.63	.32	.16	.11	.08	.05	.16	.08
25°	665	.80	.40	.20	.13	.10	.07	.16	.08
30	803	.96	.48	.24	.16	.12	.08	.16	.08
35°	946	1.13	.57	.28	.19	.14	.09	.16	.08
40°	1092	1.31	.66	.33	.22	.16	.11	.16	.08
		600 Fathom Depths - Profondeurs de 600 brasses.							
10°	313	.37	.19	.09	.06	.05	.03	.16	.08
15°	472	.57	.28	.14	.10	.07	.05	.16	.08
20°	637	.75	.38	.19	.13	.09	.06	.16	.08
25°	798	.96	.48	.24	.16	.12	.08	.16	.08
30°	966	1.16	.58	.29	.19	.15	.10	.16	.08
35°	1134	1.36	.68	.34	.23	.17	.11	.16	.08
40°	1310	1.57	.78	.39	.26	.19	.13	.16	.08
		700 Fathom Depths - Profondeurs de 700 brasses.							
10°	366	.44	.22	.11	.07	.06	.04	.16	.08
15°	551	.66	.33	.16	.11	.08	.06	.16	.08
20°	744	.89	.44	.22	.15	.11	.07	.16	.08
25°	931	1.12	.56	.28	.19	.14	.09	.16	.08
30°	1128	1.35	.68	.34	.22	.17	.11	.16	.08
35°	1320	1.58	.79	.40	.26	.20	.13	.16	.08
40°	1529	1.83	.92	.46	.31	.23	.15	.16	.08

Table of Displacements (Cont.)
Table des Déplacements (suite)

Slope Pente	Actual (Feet) Réel	1:10,000 Scale (Inches)	1:20,000 Scale (Inches)	1:40,000 Scale (Inches)	1:60,000 Scale (Inches)	1:80,000 Scale (Inches)	1:120,000 Scale (Inches)	Height of sounding figure Hauteur du chiffre de sonde (Inches)	Width of sounding figure Largeur du chiffre de sonde (Inches)
		800 Fathom Depths - Profondeurs de 800 brasses.							
10°	420	.50	.25	.13	.08	.06	.04	.16	.08
15°	631	.76	.38	.19	.13	.10	.06	.16	.08
20°	847	1.02	.51	.26	.17	.13	.09	.16	.08
25°	1064	1.28	.64	.32	.21	.16	.11	.16	.08
30°	1286	1.56	.78	.39	.26	.19	.13	.16	.08
35°	1514	1.82	.91	.46	.30	.23	.15	.16	.08
40°	1747	2.10	1.05	.52	.35	.26	.18	.16	.08
		900 Fathom Depths - Profondeurs de 900 brasses.							
10°	472	.57	.28	.14	.10	.07	.05	.16	.08
15°	710	.85	.42	.21	.14	.11	.07	.16	.08
20°	953	1.14	.57	.28	.19	.14	.10	.16	.08
25°	1197	1.44	.72	.36	.24	.18	.12	.16	.08
30°	1447	1.74	.87	.44	.29	.22	.15	.16	.08
35°	1703	2.04	1.02	.51	.34	.26	.17	.16	.08
40°	1965	2.36	1.18	.59	.39	.30	.20	.16	.08
		1000 Fathom Depths - Profondeurs de 1000 brasses.							
10°	525	.63	.32	.16	.11	.08	.05	.16	.08
15°	790	.95	.48	.24	.16	.12	.08	.16	.08
20°	1058	1.27	.64	.32	.21	.16	.11	.16	.08
25°	1330	1.60	.80	.40	.27	.20	.13	.16	.08
30°	1608	1.93	.96	.48	.32	.24	.16	.16	.08
35°	1892	2.27	1.14	.57	.38	.28	.19	.16	.08
40°	2184	2.62	1.31	.66	.44	.33	.22	.16	.08
		1100 Fathom Depths - Profondeurs de 1100 brasses.							
10°	578	.69	.35	.17	.12	.09	.06	.16	.08
15°	869	1.04	.52	.26	.17	.13	.09	.16	.08
20°	1164	1.40	.70	.35	.23	.18	.12	.16	.08
25°	1463	1.76	.88	.44	.29	.22	.15	.16	.08
30°	1770	2.12	1.06	.53	.35	.26	.18	.16	.08
35°	2081	2.50	1.25	.62	.42	.31	.21	.16	.08
40°	2402	2.88	1.44	.72	.48	.36	.24	.16	.08

Table of Displacements (Cont.)
Table des Déplacements (suite)

Slope Pente	Actual (Feet) Réel	1:10,000 Scale (Inches)	1:20,000 Scale (Inches)	1:40,000 Scale (Inches)	1:60,000 Scale (Inches)	1:80,000 Scale (Inches)	1:120,000 Scale (Inches)	Height of sounding figure Hauteur du chiffre de sonde (Inches)	Width of sounding figure Largeur du chiffre de sonde (Inches)
		1200 Fathom Depths - Profondeurs de 1200 brasses.							
10°	630	.76	.38	.19	.13	.10	.06	.16	.08
15°	948	1.14	.57	.28	.19	.14	.09	.16	.08
20°	1270	1.52	.76	.38	.26	.19	.13	.16	.08
25°	1596	1.92	.96	.48	.32	.24	.16	.16	.08
30°	1926	2.31	1.16	.58	.38	.29	.19	.16	.08
35°	2270	2.72	1.36	.68	.45	.34	.23	.16	.09
40°	2621	3.15	1.58	.79	.52	.39	.26	.16	.08
		1300 Fathom Depths - Profondeurs de 1300 brasses.							
10°	683	.82	.41	.20	.14	.10	.07	.16	.08
15°	1026	1.23	.62	.31	.21	.15	.10	.16	.08
20°	1376	1.65	.82	.41	.28	.21	.14	.16	.08
25°	1728	2.07	1.04	.52	.34	.26	.17	.16	.08
30°	2090	2.51	1.25	.63	.42	.31	.21	.16	.08
35°	2460	2.95	1.48	.74	.49	.37	.25	.16	.08
40°	2838	3.41	1.70	.85	.57	.43	.28	.16	.08
		1400 Fathom Depths - Profondeurs de 1400 brasses.							
10°	736	.88	.44	.22	.14	.11	.07	.16	.08
15°	1105	1.32	.66	.33	.22	.16	.11	.16	.08
20°	1482	1.78	.89	.45	.30	.23	.15	.16	.08
25°	1862	2.23	1.12	.55	.37	.28	.19	.16	.08
30°	2251	2.70	1.35	.68	.45	.34	.22	.16	.08
35°	2648	3.18	1.59	.80	.53	.40	.26	.16	.08
40°	3057	3.67	1.84	.92	.61	.46	.31	.16	.08
		1500 Fathom Depths - Profondeurs de 1500 brasses.							
10°	787	.93	.47	.23	.16	.12	.08	.16	.08
15°	1184	1.42	.71	.36	.24	.18	.12	.16	.08
20°	1587	1.91	.96	.48	.32	.24	.16	.16	.08
25°	1995	2.39	1.20	.60	.40	.30	.20	.16	.08
30°	2412	2.89	1.44	.72	.48	.36	.24	.16	.08
35°	2838	3.41	1.70	.85	.57	.43	.28	.16	.08
40°	3276	3.93	1.96	.98	.66	.49	.33	.16	.08

TABLE II

Table of Limits - Table des Limites

Slope Pente	Depth at which to begin correcting for slope Profondeur à partir de laquelle on devra commencer à faire la correction de pente (Fathoms) - (Brasses)	Distance between 100 fathom curves Distance entre les lignes de niveau de 100 bras. (inches)-(pouces)
	For 1/10,000 scale sheet - Minute à l'échelle du 1/10.000	
10°	250	4.15
15°	175	2.78
20°	125	2.10
25°	100	1.70
30°	75	1.44
35°	75	1.26
40°	50	1.12
	For 1/20,000 scale sheet - Minute à l'échelle du 1/20,000	
10°	500	2.08
15°	300	1.39
20°	250	1.05
25°	200	.85
30°	150	.72
35°	150	.63
40°	100	.56
	For 1:40,000 scale sheet - Minute à l'échelle du 1:40.000	
10°	1000	1.04
15°	700	.70
20°	500	.52
25°	400	.42
30°	300	.36
35°	300	.32
40°	200	.28

TABLE II

Table of Limits - Table des Limites

Slope Pente	Depth at which to begin correcting for slope Profondeur à partir de laquelle on devra commencer à faire la correction de pente (fathoms) - (brasses)	Distance between 100 fathom curves Distance entre les lignes de niveau de 100 brasses (inches)-(pouces)
	For 1:60,000 scale sheet-Minute à l'échelle du 1:60,000	
10°	1500	.69
15°	1000	.46
20°	700	.35
25°	600	.28
30°	500	.24
35°	400	.21
40°	400	.19
	For 1:80,000 scale sheet - Minute à l'échelle du 1:80,000	
10°	No correction to 1 500 fathoms - Pas de correction jusqu'à 1 500 brasses	.52
15°	1400	.35
20°	1000	.26
25°	800	.21
30°	600	.18
35°	600	.16
40°	500	.14
	For 1:120,000 scale sheet - Minute à l'échelle du 1:120,000	
10°	No correction to 1 500 fathoms Pas de correction jusqu'à 1 500 brasses	.35
15°	No correction to 1500 fathoms	.23
20°	1500	.18
25°	1200	.14
30°	1000	.12
35°	600	.10
40°	700	.09

ting an echo sounding having given a certain scale and a certain slope. For example, if working on a scale of 1:40,000 it is only necessary to determine the degree of slope of the bottom, by the method given below, or by any other method applicable, and entering the table with the degree of slope under the proper scale, the minimum depth at which echo soundings are to be corrected can be immediately ascertained. But whatever method is used for determining slopes it should be based on slopes between *depth curves* and not between successive echo soundings on a line, unless the line is normal to the slope. The reason is obvious as will be evident from an examination of Fig. 6 which represents a portion of a hydrographic sheet on a 1:40,000 scale. The soundings are uncorrected echo depths and the curves were drawn from these depths.

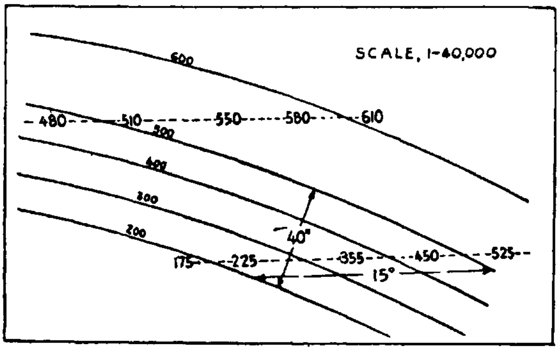


Fig. 6.

Determination of Slope from echo soundings.

It will be seen that while there is a 40° slope between the 200 and 500 fathoms curves and all soundings obtained between these curves should be corrected for such slope, yet if only successive soundings on a line be considered the apparent slope is only 15°. And if the sounding line should parallel a depth curve, as it frequently does, the successive depths obtained would indicate a level bottom when the depth curves would show a definite slope. There is one case, however, where adjacent soundings on a line can be utilized for determining slopes, and that is where the sounding line intersects the depth curves at right angles, thus giving the maximum slope possible.

Another advantage of the depth curve method is that small irregularities can be smoothed out in the drawing of the curves. And this is as it should be. The error that will be introduced from such generalization will be more than offset by the facility with which slopes can be ascertained. Moreover, the whole problem of the determination of bottom slopes from echo depths in irregular areas is far too complicated to justify attempts in refinement. Generalizations and approximations must be resorted to.

The last column of Table II gives the horizontal distance in inches between 100 fathom depth curves (based on echo soundings) at various slopes. These distances can be mathematically calculated from the expression

$$\text{Distance (in fathoms)} = \frac{100 \text{ fathoms}}{\textit{sin} \text{ slope}}$$

as developed in problem 2 (page 93 of this paper). The purpose of this information is to give the field and office engineers a means of determining mechanically the slopes on various portions of the field sheet. For convenience, small strips like that shown in Fig. 7 can be prepared for the various scales and the slopes in a given area determined by sliding the strip along the depth curves on either the boat sheet or the smooth sheet.

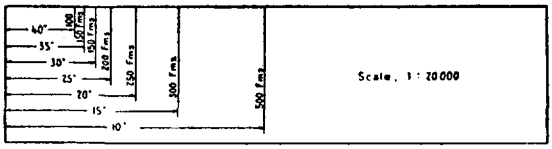


Fig. 7.

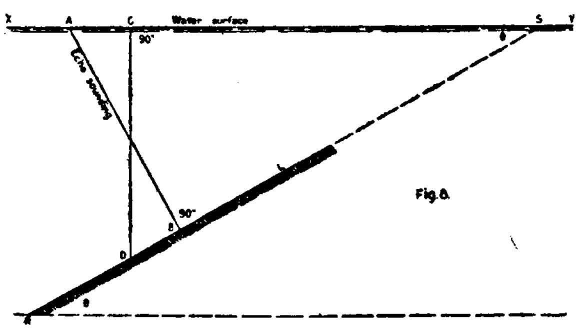
Slope scale.

When the slope of a particular area has been determined, a glance at the depth given on the strip or at Table II for that slope will indicate whether the soundings in that area need be given further consideration. Thus, if working on a 1:20,000 scale and the determined slope is 20°, a reference to the "slope scale" or to Table II will show a depth of 250 fathoms, indicating that all echo soundings of less depth than 250 fathoms can be used without applying any slope corrections whatever.

When working on scales of 1:80,000 or smaller, it will be found more convenient to construct these slope scales for 200 or 500 fathoms depth curves, and if working on a scale of 1:10,000 it will be advantageous to construct them for 50 fathoms depth curves in order to adapt them to the limiting depths for slope corrections shown in Table II. The distance between any two depth curves can readily be determined by using the equation developed in Problem 2, but substituting the desired depth curve for the 100. This necessarily follows from the fact that for a given scale and a given slope the distance between depth curves varies directly as the difference in depth between the curves.

PROBLEM 1.

To develop a formula for computing the displacement of an echo sounding for *any depth* and *any slope.*



In Fig. 8 let XY be the water level and RL the profile of the bottom where slope $= \theta^{o}$

Let A be the position of the vessel where an echo sounding AB is obtained and let C be the point where the *vertical depth* equals the echo sounding obtained at A.

To find the distance AC.

Proof :

Prolong the slope until it intersects the water level at S.
Then $ASD = \theta$
In the right triangle ABS :

$$\frac{AB}{AS} = \sin\theta \text{ or } AS = \frac{AB}{\sin\theta} \quad (1)$$

In the right triangle DCS :

$$\frac{CD}{CS} = \tan\theta \text{ or } CS = \frac{CD}{\tan\theta} \quad (2)$$

From Fig. 8,

$$AC = AS - CS$$

And substituting the values for AS and CS obtained in (1) and (2), we have,

$$AC = \frac{AB}{\sin\theta} - \frac{CD}{\tan\theta}$$

But the hypothesis,

$$CD = AB,$$

therefore $AC = \dfrac{AB}{\sin\theta} - \dfrac{AB}{\tan\theta}$ = *displacement of the sounding.*

PROBLEM 2.

To determine the horizontal distance between 100 *fathoms curves*, based on echo soundings, for *various* slopes.

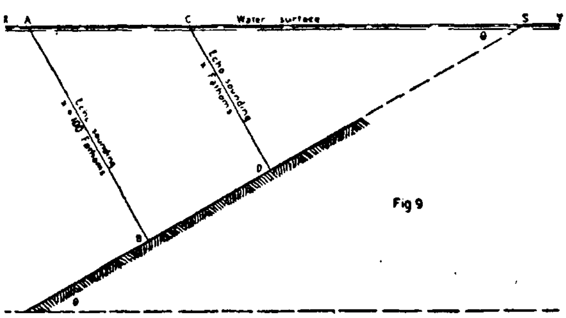


Fig 9

In fig. 9 :

Suppose XY to be the water level and x and $(x + 100)$ two echo soundings taken at C and A respectively and whose depths differ by 100 fathoms. And let θ be the slope of the bottom.
To find the distance AC.

Proof :

Prolong BD until it intersects the water level at S.
Then $ASB = \theta$
In the right triangle ABS,

$$\frac{x + 100}{AS} = \sin\theta$$

or

$$AS = \frac{x + 100}{\sin \theta} \qquad (1)$$

In the right triangle *CDS*,

$$\frac{x}{CS} = \sin \theta$$

or

$$CS = \frac{x}{\sin \theta} \qquad (2)$$

From Fig. 9,

$$AC = AS - CS$$

Substituting values (1) and (2) we have,

$$AC = \frac{x + 100}{\sin \theta} - \frac{x}{\sin \theta} = \frac{100}{\sin \theta}$$

PART II.

In Part I of this paper, the general principle of echo-sounding was discussed and some of the difficulties surrounding its execution noted. The problem there was to determine under what conditions of slope, corrections to echo soundings might be omitted. While it is believed that the limits defined in Table II will eliminate the necessity of applying slope corrections in the vast majority of cases, it is realized that certain conditions may exist where slope corrections will have to be considered if the survey sheet is to present a faithful picture of the bottom relief. It is, therefore, the purpose of this portion of the paper to submit a method for dealing with such slope corrections and incidentally to make observations upon certain precautions that should be taken in applying corrections to echo soundings.

Having determined the slope of the bottom and the echo depths to be corrected as explained in Part I, the curve in Fig. 10 gives a simple method for obtaining the correction. The coordinates for this curve were computed from the formula

$$\text{Percentage correction to echo sounding} = \frac{1 - \cos \text{slope}}{\cos \text{slope}}$$

as developed in Problem 3 (page 97 of this paper). For the convenience of those using this graph, the coordinates are given in Table III and can be used for constructing a graph on a workable scale. The degree of slope from column 1 is used as an abscissa and the percentage correction from column 4 as an ordinate.

To use the graph the diagram (Fig. 10) is entered with the degree of slope as an abscissa and from the curve the corresponding ordinate is read on the left marginal. This gives the percentage correction to be applied to any echo depth of corresponding slope. This correction will always be additive since at any point on the surface of the water the echo depth (the perpendicular to the bottom) is always less than the vertical depth.

When the bottom slopes are long and uniform, the problem is simple and the corrected echo soundings at various points will represent the true vertical depths at those points. In practice, however, such uniformity is seldom

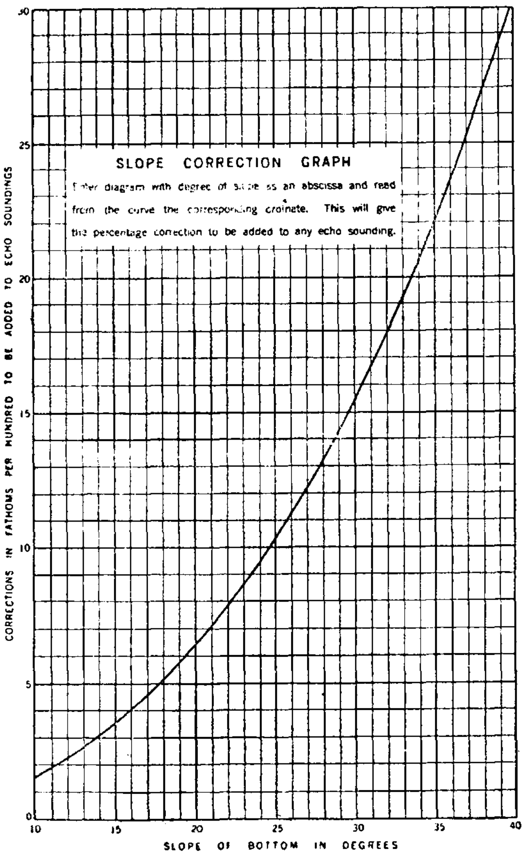


TABLE III.
Computations for Slope Correction Graph.
Calculs du Graphique de Correction de pente.

Slope Pente	Cos.	1 - Cos.	Correction %
10°	.9848	.0152	1.54 %
11	.9816	.0184	1.87
12	.9781	.0219	2.24
13	.9744	.0256	2.63
14	.9703	.0297	3.06
15	.9659	.0341	3.53
16	.9613	.0387	4.03
17	.9563	.0437	4.57
18	.9511	.0489	5.14
19	.9455	.0545	5.77
20	.9397	.0603	6.42
21	.9336	.0664	7.12
22	.9272	.0728	7.85
23	.9205	.0795	8.63
24	.9135	.0865	9.47
25	.9063	.0937	10.34
26	.8988	.1012	11.26
27	.8910	.1090	12.24
28	.8829	.1171	13.26
29	.8746	.1254	14.34
30	.8660	.1340	15.47
31	.8572	.1428	16.66
32	.8480	.1520	17.92
33	.8387	.1613	19.23
34	.8290	.1710	20.62
35	.8192	.1808	22.08
36	.8090	.1910	23.61
37	.7986	.2014	25.23
38	.7880	.2120	26.89
39	.7771	.2229	28.67
40	.7660	.2340	30.54

Fig. 10

encountered, and conditions may exist where an application of the ordinary methods of correcting for slope will result in a serious misrepresentation of the bottom relief, unless proper precautions are taken to meet such contingencies. It should be borne in mind, however, that slight errors due to undulations of the bottom must be overlooked if the echo sounding machine, in its present state of development, is to be utilized as an instrument in surveying operations. Therefore, it is with the major sources of error that this portion of the paper is primarily concerned. It is not intended, however, to discuss the various types of bottom formation that might to a greater or lesser degree affect the accuracy of echo soundings. It will be deemed sufficient for the purpose of this paper to consider a type of bottom which under certain conditions of slope will introduce discrepancies in the corrected echo soundings that cannot be ignored.

In Fig. 11 is shown the profile of a *shelving* or *terrace* bottom. It will be seen that at points *A*, *B*, and *E* no corrections for slope are necessary, since at these points the slope is too gentle to be of any consequence. But at *C* and *D* the echo soundings of 450 and 595 fathoms would indicate a slope of 40° ; and in accordance with Table II (assuming a work scale of 1:40,000) corrections would be required. If these depths are corrected for

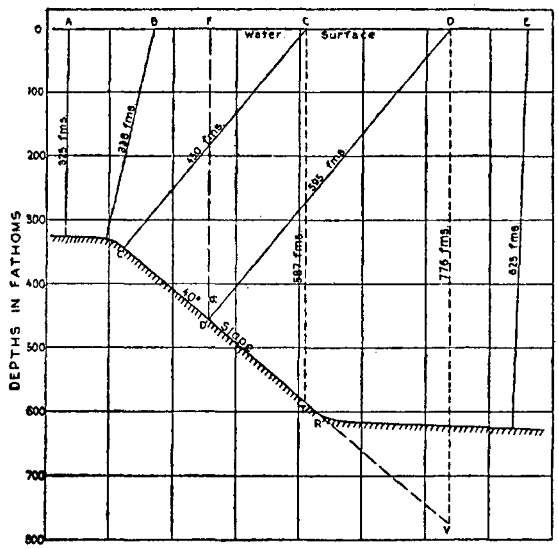


Fig. 11.

Profile of shelving bottom.

slope by the usual method, a vertical depth of 587 fathoms would be obtained at *C* and a depth of 776 fathoms at *D*. An inspection of the profile shows that the corrected depth at *C* represents the true vertical depth at this point, but the corrected depth at *D* is 155 fathoms greater than the actual vertical depth at this point. The reason is obvious, since from the point of echo for *C*, the slope of 40° continues to a point vertically below *C*, and hence when in the right triangle *CC'T* the hypotenuse *CT* is solved for (this is the basis of all slope corrections) the true depth at *C* is obtained. But from *D'*, the point of echo for *D*, the slope of 40°, for which the echo sounding is being corrected, extends only to a point *R* which is not vertically below *D*. Consequently when the hypotenuse *DV* is solved for in the triangle *DD'V* a fictitious condition is introduced by assuming that the slope continues to a point *V*, vertically below *D*. The result is a depth which has no real existence From this it may be concluded that theoretically, echo soundings should not be corrected for slope by the ordinary methods, unless the slope for which an echo sounding is being corrected continues, at least, for a horizontal distance from the point of echo equal to $e \sin \alpha$ where *e* is the echo depth and α the angle of slope (*see* distance *FD* in figure). A series of curves for various depths and slopes could be constructed and this theoretical limitation determined, but the difficulty in such cases would be in the application of this principle to practical problem. And the reason is that, *in cases of irregular bottom,* it is not possible to reconstruct from uncorrected echo soundings an accurate profile of the bottom. The actual point of echo for a particular echo sounding cannot be easily ascertained and the above relation cannot be utilized for determining, from the hydrographic sheet, whether the required condition has been fulfilled.

A practical solution of this problem would be to study the uncorrected echo soundings in a given area and if there is reason to believe that irregularities, such as steep slopes contiguous to gentle slopes, exist, or that submarine valleys are present, the work in the area affected should be supplemented by vertical soundings. Then too, since it can be mathematically demonstrated that if the slope of the bottom is uniform the corrected echo soundings will indicate the same degree of slope as the uncorrected ones, any circumstance such as the introduction of irregularities in the corrected echo soundings, not apparent in the uncorrected ones, should be viewed with suspicion and sufficient vertical soundings taken to completely develop the bottom configuration.

PROBLEM 3.

To determine the *percentage correction* to be applied to echo soundings for *any slope* to obtain the vertical depth under the vessel.

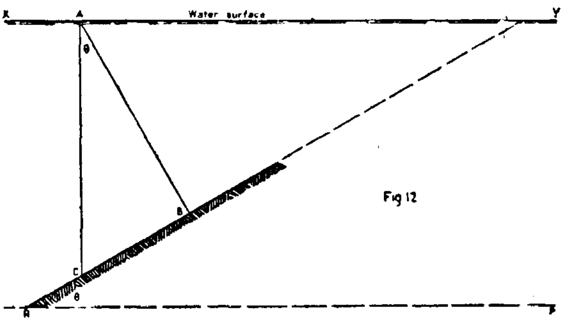


Fig 12

In Fig. 12.

Let XY be the water level, AB an echo sounding at A, and AC the vertical depth at A. And let $BRP = \theta$ the slope of the bottom. *To determine* the percentage correction to be added to AB.

Proof :

In the ABC,

$$CAB = \theta$$

Also $$\frac{AB}{AC} = \cos\theta \text{ or } AC = \frac{AB}{\cos\theta} \quad (1)$$

Now $AC - AB$ = correction to be applied to the echo sounding.

And $$\frac{AC - AB}{AB} = \text{percentage correction to echo sounding.}$$

Substituting the value of AC from (1), then

$$\frac{\frac{AB}{\cos\theta} - AB}{AB} = \text{percentage correction,}$$

or $$\frac{AB - AB\cos\theta}{AB\cos\theta} = \text{percentage correction,}$$

or $$\frac{1 - \cos\theta}{\cos\theta} = \text{percentage correction.}$$

SUMMARY.

Since the foregoing pages contain several digressions that were necessary for a proper presentation of the subject, a summary of the more important conclusions reached will be given, followed by an actual exemple showing the practical application of the method described:

1. In generally uniform bottom, slope corrections can be ignored when the scale of the survey and the depths come within the limits specified in Table II.

2. When the bottom is irregular, or where submarine valleys exist, the ordinary methods of correcting for slope are not applicable and the echo depths should be supplemented by a sufficient number of vertical soundings.

3. In determining degrees of slope, the depth curve method should be used and not slopes between consecutive echo soundings, unless the sounding line runs normal to the depth curves.

PROBLEM:

To determine the corrections to be applied to echo soundings taken on a 1:40,000 scale survey.

1. On the boat sheet carefully draw 100 fathom depth curves using the echo soundings as corrected for temperature and salinity.

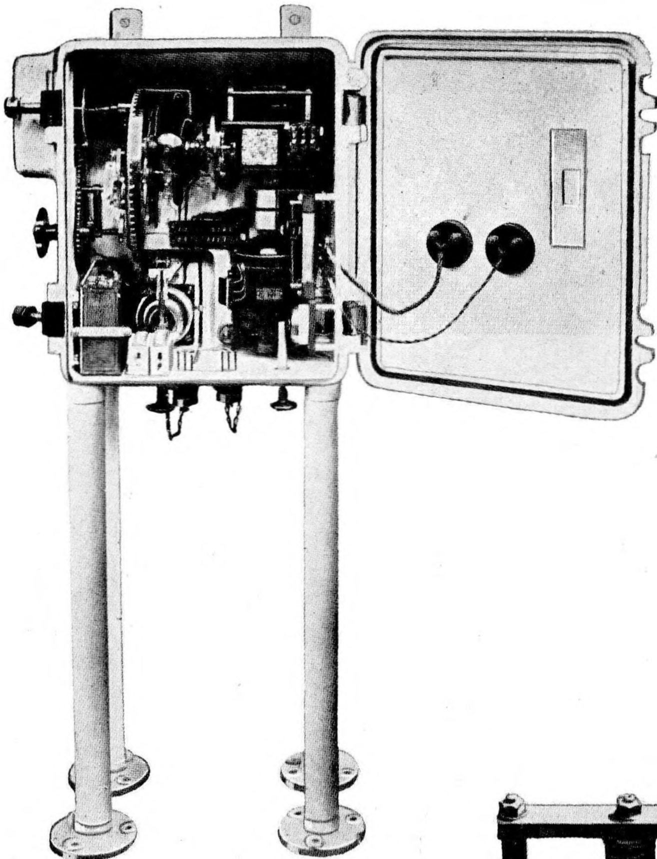
2. Construct a 1:40,000 slope scale in the form shown in Fig. 7 using for this purpose the information given in Table II.

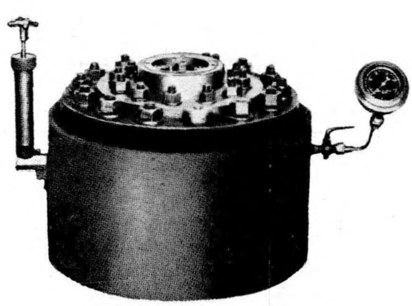
3. With the slope scale, determine the slopes for the various areas, considering only those depths that, in accordance with Table II, require slope corrections, and record these slopes in the "Remarks Column" of the sounding volume opposite the corresponding echo soundings.

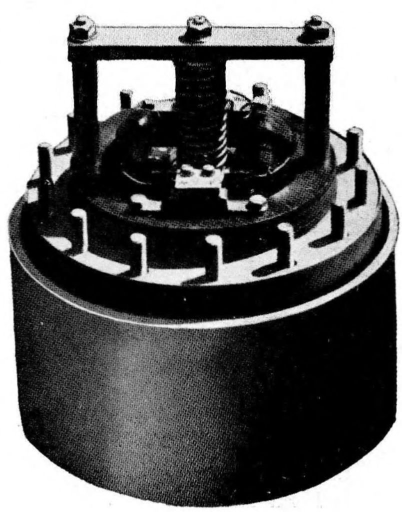
4. To compute the corrections, use the "Slope Correction Graph" (Fig. 10) and with the proper slope values determine the correction per hundred fathom to be added to the various echo soundings. Enter these percentage corrections to the nearest half per cent, in the "Remarks Column" immediately after the slope entry. (*See* paragraph 3 above).

5. Compute the total slope correction for the various soundings and enter these in the column headed "Corrections-Slope".

6. When plotting the smooth sheet, plot the corrected echo soundings and show the corresponding uncorrected ones immediately below or adjacent and enclosed in parentheses.





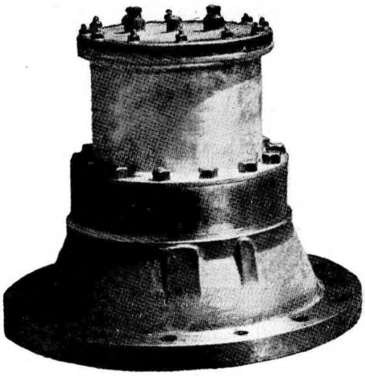


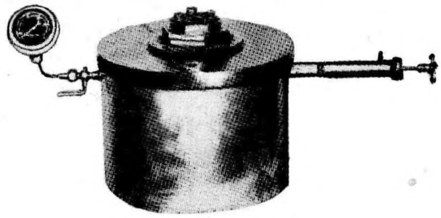
From left to right and from top to bottom of the page :

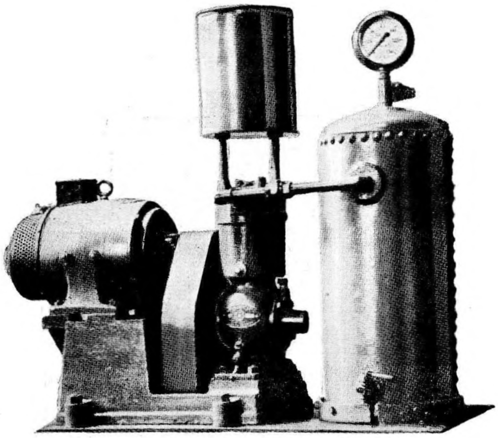
Receiving gear, opened.
Hydrophone Unit.
Transmitter Unit.

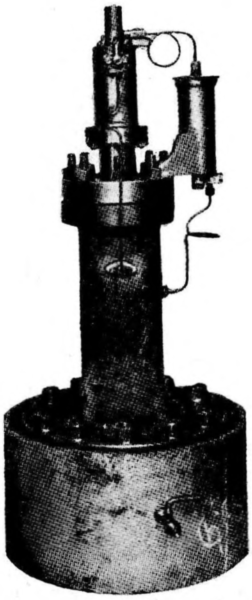
De gauche à droite et de haut en bas de la page :

Récepteur, ouvert.
Hydrophone.
Transmetteur.









From left to right and from top to bottom of the page :

Mark III Transmitter.
Top Surface Hydrophone.
Mark IV Compressor.
Mark IV Transmitter.

De gauche à droite et de haut en bas de la page :

Transmetteur Mark III
Hydrophone de Surface.
Compresseur Mark IV
Transmetteur Mark IV