MECHANICAL MEANS OF FINDING GEOGRAPHICAL POSITIONS IN NAVIGATION

Extract from an article by G. W. LITTLEHALES, Hydrographic Engineer, U. S. Hydrographic Office.

(Published in "*Journal of the American Society of Naval Engineers*"

February, 1929)

In the issue of February, 1929, the "*Journal of the American Society of Naval Engineers*" has published an interesting article by Mr. G. W. LITTLEHALES on a mechanical method of solving the astronomical triangle. The general equation for the spherical triangle in the form of:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

has been transformed into :

$$\text{versin } a = \text{versin } (b\text{-}c) + \tfrac{1}{2} \,[(\text{versin } (b + c) - \text{versin } (b - c)] \text{ versin } A.$$

Substituting for a, b, c, the following usual terms for the astronomical triangle:

z = zenith distance
L = latitude
d = declination
t = hour angle of celestial body (for time sight)

we obtain :

$$\text{versin } z = \text{versin } (L - d) + \frac{\text{versin } [180^\circ - (L + d)] - \text{versin } (L - d)}{\text{versin } 180^\circ} \text{ versin } t \qquad (10)$$

$$\text{versin } t = \frac{[\text{versin } z - \text{versin } (L - d)] \text{ versin } 180^\circ}{\text{versin } 180^\circ - (L + d) - \text{versin } (L - d)} \qquad (11)$$

to be used for solving the time sight.

When solving for azimuth, we have by analogy:

$$\text{versin } z = \frac{[\text{versin } (90^\circ\text{-}d) - \text{versin } (L - h)] \text{ versin } 180^\circ}{\text{versin } (180^\circ - (L + h) - \text{versin } (L - h)} \qquad (12)$$

in which Z represents the azimuth and h, the altitude.

Employing Cartesian rectangular coordinates, and letting

y = versin Z
a = versin $(L - d)$
x = versin t

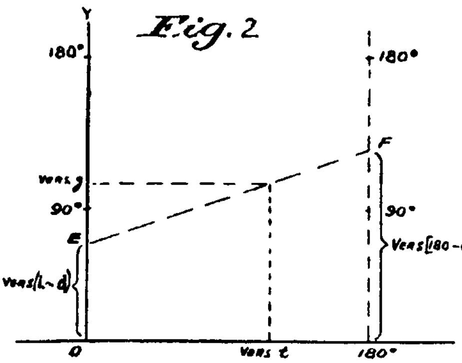
$$m = \frac{\text{versin } [(180^\circ - (L + d)] - \text{versin } (L - d)}{\text{versin } 180^\circ}$$

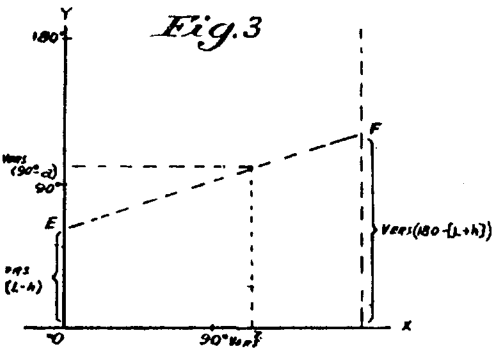
equation (10) will be transformed into

$$y = a + mx$$

which is the equation of a straight line intersecting the axis of Y at a distance a above the origin of coordinates and passing through the first quadrant at an angle of inclination to the axis of X whose tangent is m.

If the axes Y and X be graduated on the same scale to represent the versines of angles, commencing with 0 degrees at the origin designated by the letter 0 in Figure 2, and extending to 180 degrees in each case, the line EF will be the graph of the equation, $y = a + mx$, and hence the equation (10).





The length of the ordinate to the line EF corresponding to any given value of t on the scale, OX, of abscissae representing the values of versin t will mark the value of z on the scale, OY, of ordinates representing the value of versin z; and conversely, the abscissa corresponding to any given value of z (as when the altitude of a celestial body is measured in taking a time sight) on the scale of ordinates, OY, will mark the value of the hour angle t on the scale of abscissae, OX.

In like manner, equation (12) for finding the azimuth Z in which Z appears in place of t in equation (11) and the altitude, h, in place of the declination, d, may be represented by a straight line whose rectangular coordinates, as represented in Figure 3, are versin Z and versin $(90^\circ - d)$ respectively, and whose inclination to the axis of abscissae in versin Z is an angle whose tangent is equal to

$$\frac{\text{versin } [(180^\circ - (L + h)] - \text{versin } (L - h)}{\text{versin } 180^\circ}.$$

The necessity for drawing coordinates after the manner shown by the lines of fine dashes in Figures 2 and 3 may, to a large extent, be obviated by extending the ordinates from the division marks of the scales of versines constructed along OX and OY, and thus forming a square diagram as shown in Figure 4.

If the right-hand border of this diagram be numbered in the reverse order from the left-hand border, that is, from 180 degrees at the bottom to 0 degrees at the top, the right-hand ordinate will be $L + d$ in finding the hour angle and zenith distance and $L + h$ in finding the azimuth, instead of 180 degrees - $(L + d)$ and 180 degrees - $(L + h)$, as indicated in Figures 2 and 3 in these respective cases. Hence, the following rules provide for solving the equations (10), (11) and (12) to find, respectively, the zenith distance, the hour angle or time sight, and the azimuth, by means of the construction shown in Figure 4.

a) To find the zenith distance, z:

Mark the value of $(L - d)$ on the left-hand border scale of versines and the value of $(L + d)$ on the right-hand border scale of versines. The straight line drawn to connect these two markings is the graph of equation (10) representing the diurnal course of a celestial body whose declination is invariable between culmination on the upper branch of the meridian of the observer in latitude, L, and culmination on the lower branch of the same meridian. Mark the intersection on this graph of the vertical ordinate from the value of the hour angle, t, found on the top or bottom border scale of versines. The horizontal line from this intersection will mark, on the left-hand scale, the value of the zenith distance, z.

b) When the altitude, and hence the zenith distance, z, is known by measurement, by reversing the last two steps in (*a*), the hour-angle, t, may be found.

c) To find the azimuth, Z:

Mark the value of $(L - h)$ on the left-hand border scale of versines, and the value of $(L + h)$ on the right-hand border scale of versines. The straight line drawn to connect these two markings is the graph of equation (12).

Mark the intersection on this graph with the horizontal line from the value of the polar distance, (90 degrees - d), found on the left-hand border scale of versines. The vertical ordinate from this intersection will mark, on either the top or bottom border scale, the value of the azimuth, Z.

Fig. 4

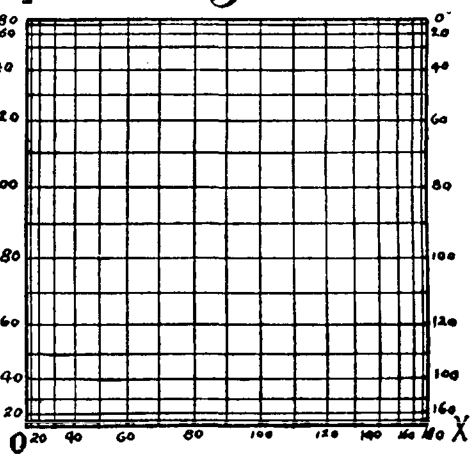
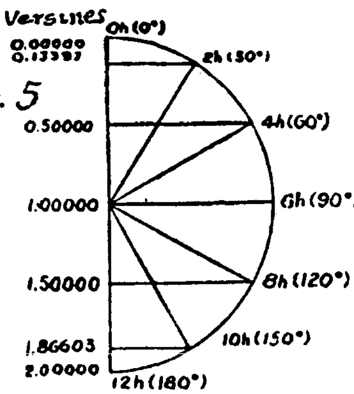


Fig. 5



In brief, these solutions consist in finding, in the form of a straight line connecting two points readily determined, the graph of the equation to be solved, whose coordinates, in terms of versines, are, in one case, hour angle and zenith distance, and, in the other, azimuth and polar distance; so that, one of a pair being given, the value of the other would be indicated by the graph in its established relation to the bordering scale of versines.

The object of the present mechanism is to perform these solutions by means designed to produce movements, to represent the versines of angles throughout the range of from 0 degrees to 180 degrees, for placing in position a ruler whose fiducial edge shall represent the required graph, and yet further for measuring the coordinates, in terms of versines, of any point of the graph represented by the fiducial edge.

In a semicircle, if a radius be conceived to revolve from a position of coincidence with the diameter in one direction to a position in coincidence with the diameter in the opposite direction, the versine of the angle of removal of the radius from the initial position will be represented by the distance of removal from the extremity of the diameter at the origin of movement of the foot of the perpendicular let fall from the outer end of the revolving radius upon the diameter, as shown in Figure 5.

The motion of the foot of the perpendicular will be derived from that of the outer end of the revolving radius by the arrangement shown in Figure 6, in which P represents a small pin and block set in the outer end of a radius arm CP pivoted at the center of the graduated semicircular plate RPQ. Let the pin work in the slot EF whose direction is at right angles to the sliding bar QB which forms a part of the yoke EF. Of the components which combine to produce the circular motion of P, that which occurs in the direction EF is rendered inoperative, and the whole of the other is imparted to the bar QB which is confined to a rectilinear movement by the sides of the slide ST.

Fig. 6

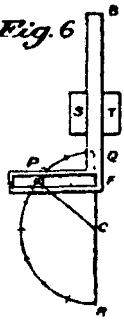
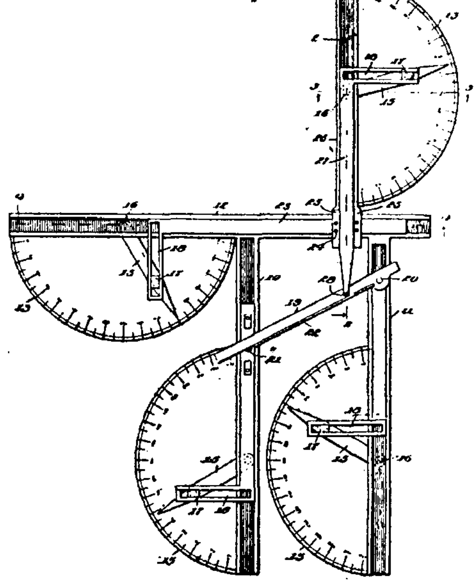


Fig 7



The mechanism whose principle is illustrated in Figure 6, being applied at the end-ordinates of the square forming the boundary of Figure 4, provides for determining the line of the graph of equation (10), or (11), or (12) by means of the fiducial edge of the bar or ruler pivoted to the head of one of the pair of members *QB* shown in Figure 6 and sliding over a support at the head of the other.

And, by means of a like mechanism placed in a position defined by the upper line of abscissae of the square forming the boundary of Figure 4, the sliding bar of the versine mechanism may be moved to define the known abscissa of a point of the edge of the graph bar, whose other coordinate is then to be identified by the operation through contact with the graph bar of the sliding mechanism carried by and fixed to the abscissa sliding bar in a position at right angles to it at the point selected to mark the reading of the abscissae.

Without going into details, the mechanism for carrying into effect the principles laid down is illustrated in Figure 7.

In the use of the instrument, assuming that the hour angle and latitude are known and it is desired to find the zenith distance of a celestial body of given declination, the values $(L - d)$ and $(L + d)$ being known, values are indicated by moving the arms *15* which are pivoted to the parallel guides, the hand on the left to indicate the value $(L - d)$ in terms of degrees, minutes and seconds, and the hand on the right to indicate the value $(L + d)$ in the same manner.

The upper surface of ledge *22* mechanically represents the graph of equation (10) representing the diurnal course of a celestial body whose declination is invariable between culmination on the upper branch of the meridian of the observer in latitude *L*, and culmination on the lower branch of the same meridian. The indicating arm which controls movement of the slide *23* is then moved to indicate the value of the hour angle. Movement of the slide *23* toward the right, as seen in Figure 7, results in a camming action of the roller or bearing on the end of slide *27* against the inclined face of ledge *22* thereby moving the slide *27* in its guide *26* and rotating the indicator arm *15* which is attached thereto to mechanically indicate the zenith distance in degrees, minutes and seconds as read from the dial underlying the indicator arm.

Similarly, to find the azimuth, the value $(L - h)$ is indicated by moving the indicator arm *15* over the lower left hand scale, and the value $(L + h)$ by moving the arm *15* over the lower right-hand scale. The plane of the upper surface of ledge *22* thereby indicates the graph of equation (12). The slide *23* having been moved to its extreme position on the side of the lesser end-ordinate of the graph bar, the indicator arm actuating slide *27* is then set to indicate the value of the polar distance (90 degrees - d), and, this being done, the slide *23* is then moved, by rotation of the indicator arm attached thereto, until the roller *28* of the slide *27* contacts with the ledge *22*. At this point, the indicator arm attached to the slide *23* will overlay the scale *13* at the point which directly indicates the value of the azimuth.

When the altitude, and hence the zenith distance, is known by measurement, the parallel slides being set for $(L - d)$ and $(L + d)$ as already described in finding the zenith distance, the hour angle may be found by moving slide *23* to its extreme position on the side of the lesser end-ordinate of the graph bar, by next setting, to the value of the zenith distance, the indicator arm which is attached to the slide *27*, and finally moving the slide *23* by rotation of the indicator arm attached thereto, until the roller *28* of slide *27* contacts with the ledge *22*. At this point, the indicator arm attached to slide *23* will overlay the scale *13* at a point which directly indicates the value of the hour angle.