NOTES ON NAUTICAL CARTOGRAPHY

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1. If a MERCATOR's chart of regions in high latitudes be examined (for instance Figure 1, which represents the seas of Norway and Greenland) and if the distribution of the lines of equal magnetic variation be considered, it would not be wrong to state that for many courses, the chief advantage of Rhumb line sailing, *i. e.* steering a constant course, becomes absolutely illusory; and besides, for long voyages, the *economical advantage* offered by Great Circle sailing over longer distances may become evident.

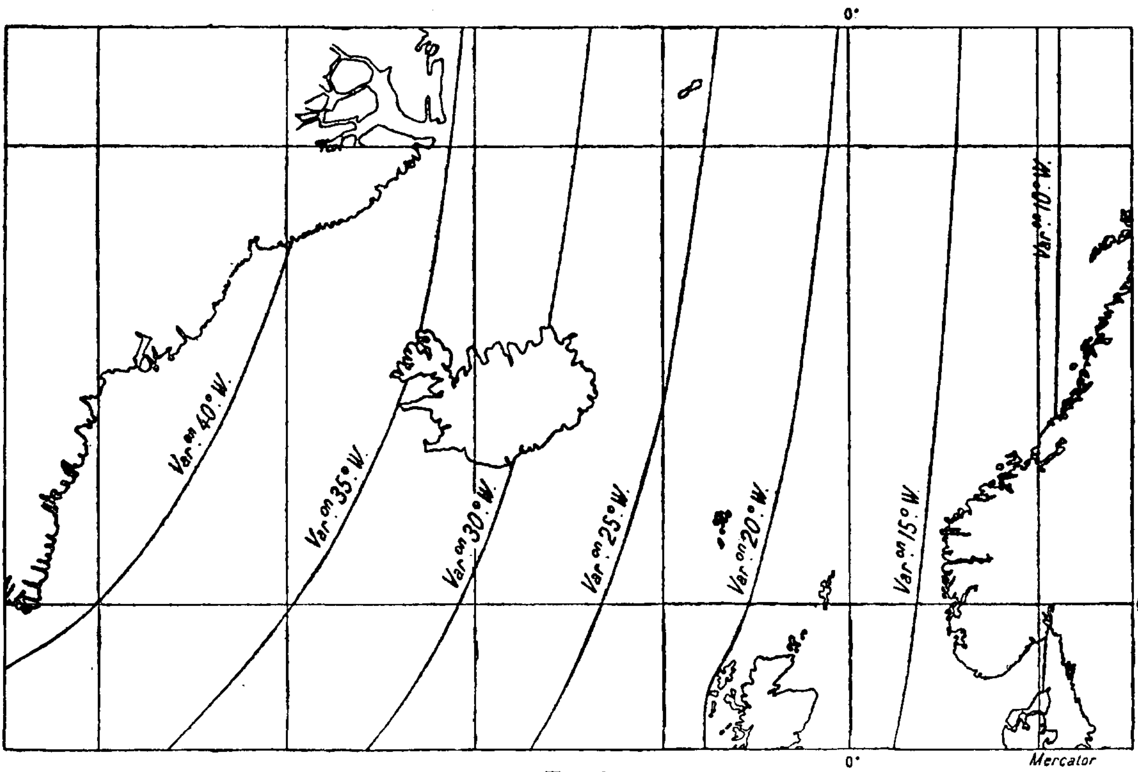

FIG. 1

It seems logical to assert that, when rapid alterations of variation make it necessary to alter the magnetic course very frequently, it might well be worth while to take into account the alteration of true course necessary for Great Circle sailing, or at any rate for sailing on a course very close to the latter.

2. If the representation of this region be considered (*which will be assumed to cover a small extent in latitude*) *on the* LAMBERT *conformal conical projection,* in which the parallel of least dilation (1) is not too far from the extreme parallels of the region represented (and where, under the most favourable conditions, this parallel is the *mean* parallel between these extreme parallels), it will be noticed that in such cases *the rectilinear courses drawn on the chart,* although they do not exactly coincide with the Great Circles drawn on the Earth's surface, are nevertheless not far removed therefrom, and thus *in practice* give the same economical advantages as these.

As may be seen, the determination of the *economical course* is quite simple in this type of chart. In practice it is reduced simply to drawing a straight line.

Besides, this same chart system has all the advantages of conformal charts and, consequently, lends itself to the graphic solution of navigational problems based on the measurement of angles, etc.

3. Figure 2 represents, on the LAMBERT projection, the same region as Figure 1. It is a reproduction, on a reduced scale, of a portion of the U. S. Coast & Geodetic Survey chart N° 3070 (North Atlantic Ocean - Scale: 1:10,000,000 for standard parallels 36° and 54°) for which the parallel of minimum dilation is φ_0 = about 45°11'. (2)

It is evident that, in this particular case, the most favourable condition, referred to at the beginning of the last paragraph, is not complied with; this example, however, may be used for demonstration. And besides, even under these conditions, which are not the most favourable, there is always a tangible economic advantage in favour of the rectilinear course on the chart as compared with a Rhumb line.

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(1) It is known that in LAMBERT'S projection :—

1st. The geographical Meridians are represented by a number of straight lines meeting at one and the same point (representing the Pole);

2nd. That there is a constant relation l between the angle γ formed by any two meridians on the chart and the corresponding angle ω at the Earth's surface (*i. e.* the difference of longitude of the two corresponding geographical Meridians);

3rd. That this relation is equal to *sin* φ_0, *i. e.* the sine of the latitude of the parallel of minimum dilation φ_0

$$l = \frac{\gamma}{\omega} = \sin \varphi_0.$$

This formula gives values for γ which will be called the *convergence of the meridians of the chart.*

$$\gamma = l\,\omega = \omega \sin \varphi_0$$

(2) The constant l of this chart is exactly $l = \sin \varphi_0 = 0.710105$ and consequently $\varphi_0 =$ 45°10'36"4 (See *Elements of Map Projection,* Special Publication N° 68 U. S. Coast & Geodetic Survey, pp. 84-85, where the method of construction of chart N° 3070 is described).

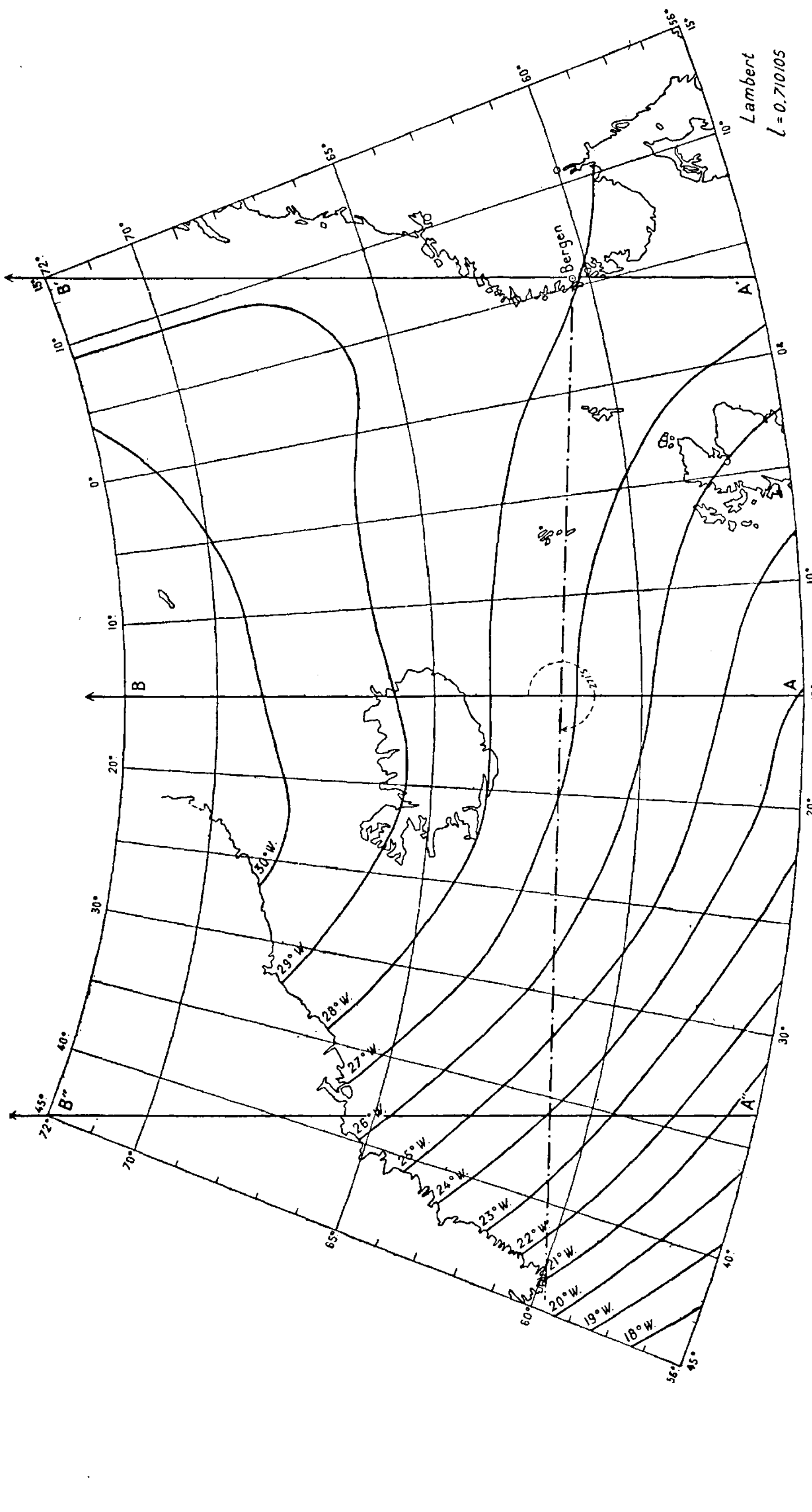
Lambert
l = 0,710105
Bergen
B'
B
B''
A'
A
A''
30° W.
29° W.
28° W.
27° W.
26° W.
25° W.
24° W.
23° W.
22° W.
21° W.
20° W.
19° W.
18° W.

Figure 30

4. The determination of the *magnetic course* corresponding to the various points of the rectilinear course on the LAMBERT chart can be made extremely simple by means of the following scheme (3):

a) The courses *on the chart* would be measured taking an arbitrarily chosen meridian as the sole direction of reference. For obvious symmetrical reasons the *central meridian* of the chart would be chosen.

In Figure 2 the meridian of 15° W has been adopted as the central meridian (Meridian *AB*). To facilitate the laying off of courses, the straight lines *A' B'* and *A'' B''* have been drawn, at the two lateral extremities, parallel to *AB*. The direction of the course measured as explained above will be described as the *Chart Course.*

b) The system of *lines of equal fictitious variation,* is then drawn on the chart. *Fictitious variation* at a given point on the chart is taken to mean the angle formed at this point by the vector representing the direction of the magnetic needle (unaffected by the iron of the ship) and the vector parallel to the direction *AB* of the central meridian.

It is evident that in *absolute value* the fictitious variation is equal to the sum or the difference of two known angles: the convergence γ taken *from the chart* between the meridian of the point under consideration and the central meridian and the magnetic variation, properly so called.

We know that this convergence (*See* Note 1 of paragraph 2) is proportional to the difference of geographical longitude ω between the two meridians under consideration. The coefficient of proportionality l (which is a *constant* of the projection and which should be indicated on every LAMBERT chart) is equal to the sine of the latitude φ_0 of the parallel of minimum dilation.

$$l = sin\ \varphi_0 \qquad\qquad \gamma = l\ \omega$$

The convergence γ will be called *Easterly* when, on the chart, the North direction of the meridian of the point under consideration is deviated to the Eastward of that of the North direction of the Central Meridian; it will be called *Westerly* in the contrary case (4). Thus, the value of the fictitious variation will be given, in quantity and in sign, by the algebric sum of the convergence and of the magnetic variation.

If an ordinary hydrographic chart is available (such as that which is represented in Figure 1) on which the lines of equal magnetic variation are drawn, the construction of the lines of equal fictitious variation on the LAMBERT chart becomes extremely simple.

In Figure 2 the lines of equal fictitious variation are shown for each degree of value.

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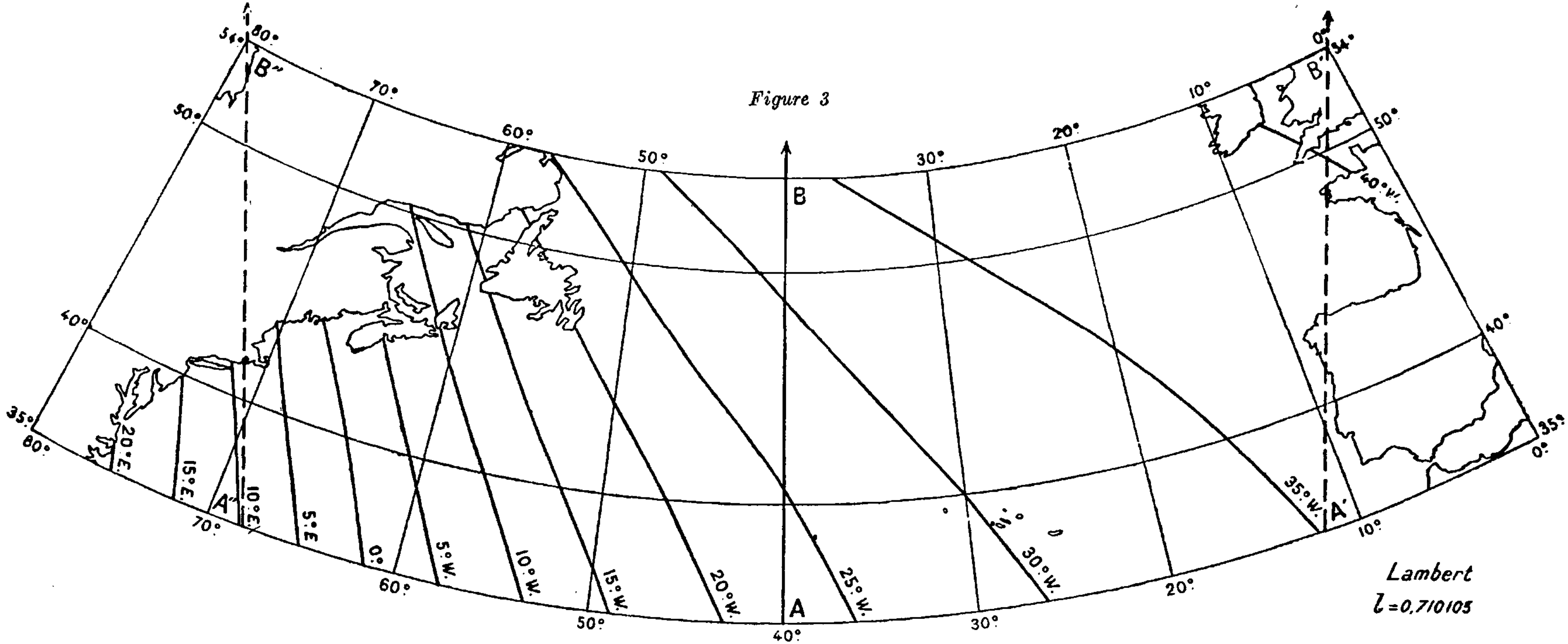
(3) This scheme is analogous to that suggested for adoption in connection with the type of polar chart described in the note entitled *A New Type of Polar Chart,* "*Hydrographic Review*" N° 10, November 1928.

(4) In charts for which φ_0 is North, the convergence will be Easterly for the region situated to the West of the central meridien. It will be Westerly for all the region situated to the East.

Figure 3



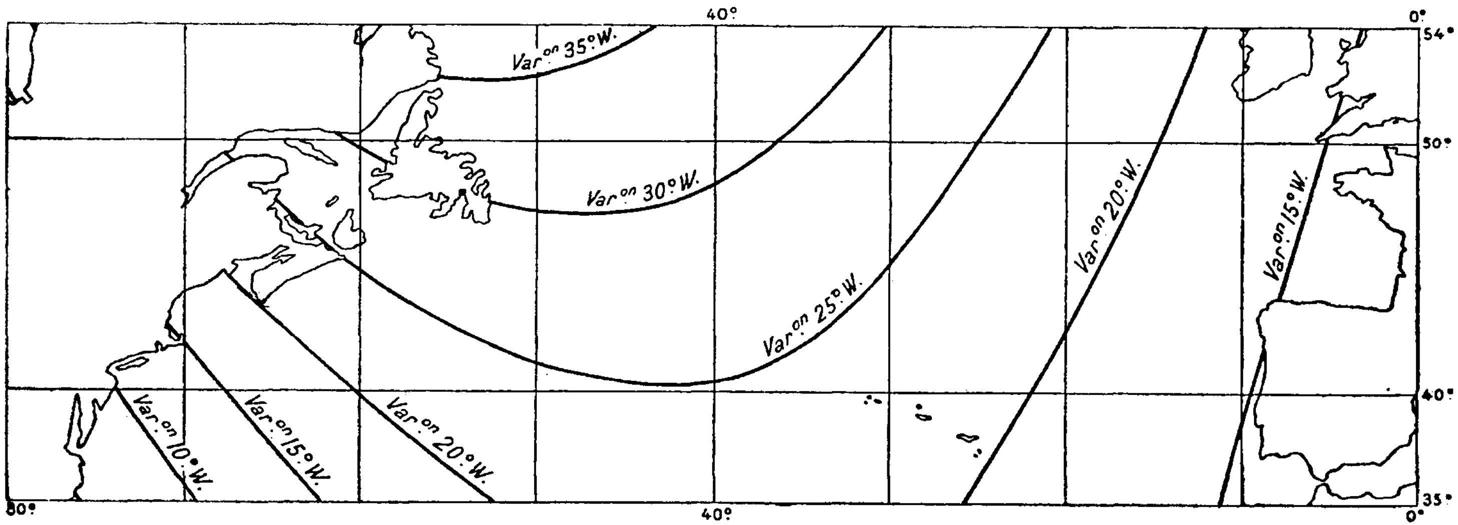


Figure 4

5. On the chart represented in Figure 2, the rectilinear course from Bergen to Cape Farewell has been drawn. The *chart course* is 271.5°. Between the point of departure and the point of arrival a change of fictitious variation of about 7° occurs (from 28° W to 21° W). Consequently the magnetic course varies between $271.5^\circ + 28^\circ = 299.5^\circ$ at the point of departure and $271.5^\circ + 21^\circ = 292.5^\circ$ at the point of arrival.

If the MERCATOR chart be referred to (Fig. 1), it is seen that the magnetic variation changes in the Rhumb line from Bergen to Cape Farewell about 25° (from 16° W to 41° W.). (5)

6. Based on the above considerations and still using U. S. C. & G. S. chart N° 3070, Figure 3 has been drawn which represents the Atlantic region comprised between the parallels 35° and 54° of North Latitude; it is in this region that the principal transoceanic crossings between the two continents take place.

In this chart, given the value of the constant l of the projection ($l = 0{,}710105$), the parallel of minimum dilation (45°11') is very near the mean parallel of the region represented and, therefore, the most favourable condition stated above in paragraph 2, is found to be nearly realised.

As central meridian, that of 40° W (meridian *AB*) has been chosen. To determine the fictitious variation and to draw the lines of equal fictitious variation, British Admiralty MERCATOR Chart N° 2598, 1922, partially reproduced in Figure 4, has been used.

There, on an equal course in an East to West direction, the alterations in the fictitious variation are greater than those of the magnetic variation, but it must be noted that the economic advantages presented by the rectilinear course drawn on the LAMBERT chart, as compared with the corresponding Great Circle course, are sufficiently marked that the long crossings between the two continents are found to be effectually shortened thereby.

7. It appears that the use of the LAMBERT projection offers truly remarkable advantages for navigation when the *Gyroscopic Compass* is used.

Figure 5 is a reproduction, on the usual LAMBERT projection, of the region represented in Figure 3. In this figure, instead of the lines of equal fictitious variation, the meridians whose convergence γ, with reference to the central meridian (40° W), is respectively 5° E., 10° E., etc... 5° W., 10° W., etc..., have been drawn (in dots and dashes).

If a rectilinear course is drawn and the *chart course* measured according to the rule explained in the preceding paragraph (*i. e.* by referring to the central meridian) the value of the *true course* for any point on the passage would be obtained by applying the value of the convergence γ, at the point under consideration, to the chart course. To determine γ rapidly, similar use would be made of the system of meridians which has just been referred to.

Besides, the calculation of the convergence is quite simple. (*See* Note 1).

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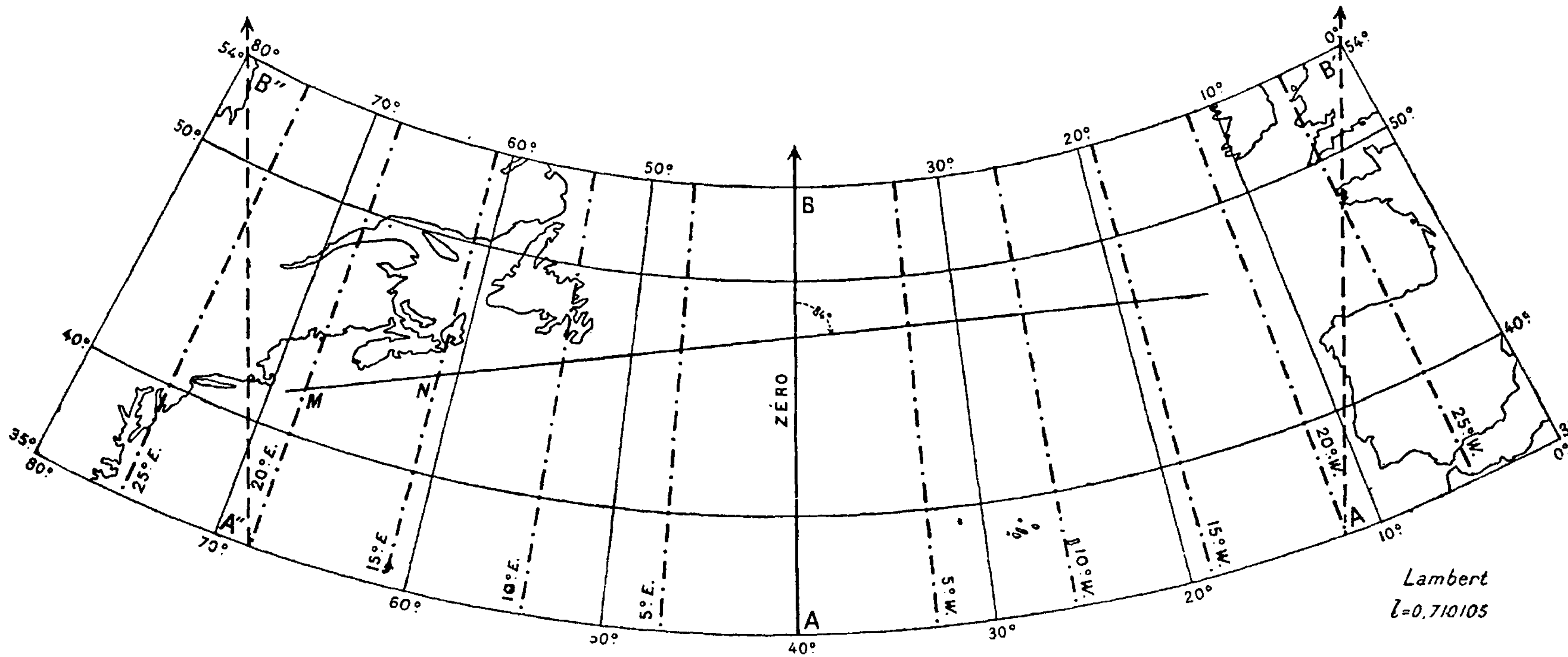
(5) The values of the magnetic variation, to which allusion is made, refer to the year 1917.



Lambert
l=0,710105
ZÉRO
A
B
A'
B'
A''
B''
M
N
25° W.
20° W.
15° W.
10° W.
5° W.
5° E.
10° E.
15° E.
20° E.
25° E.

Figure 5

8. The rectilinear course drawn on the chart could not in any case be followed accurately without *continual* change of course (magnetic or gyroscopic). In others words the magnetic courses, the determination of which has been shown on the chart in Figure 2, and the true courses on the chart in Figure 5, are *momentary* courses. In practice it is not possible to navigate on courses which continually vary. It is necessary therefore to establish a rational rule (as simple as possible) which enables the complete passage to be divided into a certain number of elements; from one end to the other of each such element it would be possible to sail on a *constant* course by compass and to follow a course of a length practically equal to that of the rectilinear course comprised between the same extremities. If the gyroscopic compass is used for navigation, the problem is immediately solved.

After having fixed the limits between which the Rhumb line can be substitued for the Great Circle without an appreciable difference (6) and after having determined, along the whole length of the rectilinear passage, a series of points the distances between which do not exceed the aforesaid limits, there is nothing to do but to follow the Rhumb line from one such point to the next.

It is but necessary to determine the course for each rhumb line element.

It should be noted that, in LAMBERT's conical projection, the rhumb line is represented by a *logarithmic spiral* (7).

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(6) It may be shown that the *maximum* value of the difference in length between the rhumb line m and the Great Circle M, for a *small* change of position on the sphere, is given, in minutes on the sphere, *i. e.* in miles, by the relation

$$m - M = \frac{m^3 \, arc^2 \, 1'}{24} \, tg^2 \, \varphi_m$$

in which φ_m is the middle latitude between the extremities of the course. For various values of m and of φ_m, this formula gives the following values :—

φ_m	m 100'	200'	300'	400'	500'
30°	$m - M \leqslant$ 0'.001	0'.01	0'.03	0'.08	0'.15
40	» $\leqslant$ 0 .002	0 .02	0 .07	0 .16	0 .31
50	» $\leqslant$ 0 .005	0 .04	0 .14	0 .32	0 .63
60	» $\leqslant$ 0 .011	0 .08	0 .29	0 .68	1 .32
70	» 0 .027	0 .21	0 .72	1 .70	3 .33
75	» 0 .049	0 .39	1 .33	3 .14	6 .14
80	» 0 .112	0 .91	3 .06	7 .26	11 .26

REMARK. — *In the demonstration it is implicitly assumed that the rectilinear passage on the Lambert chart has (very nearly) the same length as the corresponding Great Circle. This is exact to a very high degree of accuracy when the passage is effected in latitudes which are not far from the parallel of minimum dilation.*

(7) In fact, since the projection is conformal, the line corresponding to the rhumb line should cut the meridians of the chart at a constant angle; but these meridians are straight lines passing through the pole and they cannot be cut at a constant angle except by logarithmic spirals.

The rectilinear course MN (fig. 6) lying between the meridian PH and PK is the chord of the arc of the spiral MVN whose pole is at P, and this spiral is the representation of the rhumb line joining the extremities of the course.

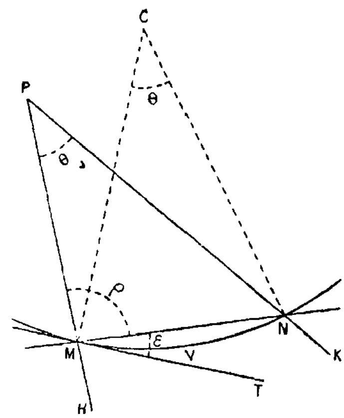

Figure 6

It is shown that an arc of the spiral MVN lying between the radius vector PH and the radius vector PK and inclined to the latter at an angle θ may, *within defined limits of* θ, be assumed to be the arc of the circle subtended by the arc θ. (8).

Thus the tangent MT to the spiral at the point M may be taken to coincide with the tangent to the arc of the circle at that point.

As $\varepsilon = NMT = \dfrac{\theta}{2}$ it may be deduced that the arc of the spiral MVN

cuts the meridians of the chart at the *constant* angle.

$$PMT = PMN + \frac{\theta}{2} = \rho + \frac{\theta}{2}$$

s representing the *true* course at M with relation to the rectilinear course MN. The angle PMT, thus determined, measures therefore the true course of the Rhumb line MVN. But $\theta = \gamma'' - \gamma'$, γ' and γ'' being the values of the convergence, in relation to the central meridian, of the meridians PH and PK of the chart; besides, according to the rule explained in paragraph 7.

$$\rho = \text{chart course} + \gamma'$$

and consequently

$$PMT = \text{chart course} + \frac{\gamma' + \gamma''}{2}$$

(8) It is also shown that (*independent of the amplitude of the angle* θ) the arc of the circle MN has, besides its extremities, another point in common with the spiral, — *viz.* : its point of intersection with the bisector of the angle MPN.

The following rule can therefore be enounced :—

The true rhumb line course which may be substitued for a given rectilinear element of the passage is equal to the chart course corrected by the mean of the convergences at the extremities of the element (9) :

The maximum limit of $\theta = \gamma'' - \gamma'$ within which it may be assumed, without appreciable error, that the arc of the circle coincides with the spiral, may be fixed at 10°. It must be remembered, however, that the application of this rule is subject to certain restrictions. Indeed, it is not sufficient that the element of passage under consideration be comprised between two meridians of the chart whose inclination to each other is not more than 10°, but it is necessary also that its length should not exceed the limit within which the difference in length between the Rhumb line and the corresponding rectilinear course (10) can be neglected.

It should be noted, further, that it is very simple to determine the maximum departure of the curvilinear course from its chord *MN*. It is but necessary to calculate the length *f* of the perpendicular at the centre of the circular arc subtended by the angle θ to the chord as a function of the length *C* of the chord (11).

EXAMPLE : In figure 5 a rectilinear passage has been drawn for which the *chart course* = 84°.

This passage cuts the meridians whose convergences (on the central meridian) are respectively

$$\gamma' = 20^\circ \text{ E}, \qquad \gamma'' = 15 \text{ E}$$

at the points *M* and *N*.

$$\text{Hence } \frac{\gamma' + \gamma''}{2} = 17^\circ.5 \text{ E}.$$

In order to go from the point *M* to the point *N* on a Rhumb line the following course (by gyroscopic compass) must be steered :

$$84^\circ - 17^\circ.5 = 66^\circ.5$$

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(9) It may be said also that it is equal to the chart course corrected by the convergence relating to the mean meridian between the two extreme meridians of the element ; or yet again, equal to the mean course between the courses at the extremities of the rectilinear passage.

(10) In the vicinity of the parallel of minimum dilation, as has already been pointed out, the rectilinear passage has practically the same length as the Great Circle. Consequently, the limit in question will be fixed on the basis of the data given in the table in Note 5.

(11) The practical formula which, for small values of θ, gives *f* in hundredths of *C*, may be used :

Length of perpendicular from centre of the arc to the chord in hundredths of the chord = $0.22\ \theta^\circ$ when θ° is the measure of θ in degrees.

Example : Chord *MN* = 95 millimetres, $\theta^\circ = 4.8^\circ$

$$f = (0.22 \times 4.8)\ \frac{95}{100} = \text{about 1 } \textit{millimetre.}$$

The curve is concave towards the pole and thus the departure is applicable on the side towards the equator.

9. An analogous rule may be given for the case where a magnetic compass is employed, and the LAMBERT chart represented by Figure 3 is used.

But then, in this case, the solution is not so accurate as in the last case because it is based on a hypothesis which can be applied approximately only in the general case and within defined limits of Δv = change of fictitious declination.

Assuming :—

a) That the vectors which represent, at each point of a given rectilinear passage *MN* (Figures 7*a*, 7*b*) on the chart (as also in its immediate neighbourhood), the directions of the local magnetic meridians, converge on the same point *O* (Figure 7*a*) or diverge, when produced, from the same point *O* (Figure 7*b*) :

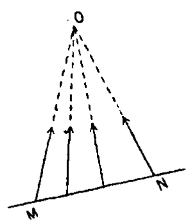

Figure 7 a

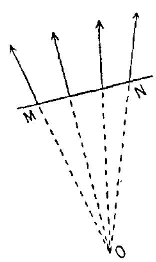

Figure 7 b

b) That the departures between the straight line *MN* and the arc of the spiral, of pole *O*, joining the extremities *M* and *N*, remain small.

On these hypotheses, one would proceed from *M* to *N* along the arc of the spiral in question, provided that one *steers along the whole length of the passage on the magnetic course obtained by applying the mean of the values of the fictitious variation at the extremities of the passage to chart course* (12).

A rapid examination of the distribution of the lines of equal fictitious variation will, in general, be sufficient to provide a certain criterion of the limits between which one may consider the hypothesis (*a*), which has been taken as the basis of the argument, as justified. An example from Figure 3 shows that, in the region represented, the rule is certainly applicable to passages along which there is a change of fictitious variation of less than, or, at most, equal to 5°.

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(12) It must be remarked that, in such cases, the spiral-shaped chart passage does not represent the geographical rhumb line. Only those spirals represent rhumb lines which have *P* for their pole, *viz.* the point of convergence of the meridians of the chart.

10. It is interesting to note that identical conclusions are reached when the rectilinear passages on MERCATOR's chart are considered (13). In such cases straight lines represent Rhumb lines.

The course by constant (*truly constant*) compass along which the vessel travels from one end of a given Rhumb line to the other is that which corresponds *to the magnetic course obtained by applying the mean magnetic variation, obtained by taking the mean of the variations for the two extremities of the passage, to the true Rhumb line course.*

Working thus a passage, represented (very nearly) on the chart by an arc of a spiral, is followed. The restrictions to the preceding cases apply here also. If the passage is short and if the change of magnetic variation in the length of the passage is slight, the curve does not deviate much from a straight line. (14)

11. If $\varphi_0 = 90^o$, *i. e.* $l = 1$, the case of the conformal polar projection is reached (15) which, on the hypothesis of a spherical earth, becomes identical with the polar stereographic projection.

To represent the Arctic cap, it would be best to choose, as the central meridian of the chart (it is to the direction AB (Fig. 8) of this meridian that the courses and the horizontal component of the earth's magnetic field, *i. e.* the fictitious variation, will be referred), the meridian which is at right angles to that of Greenwich, because this meridian passes within a small distance of the North magnetic pole.

To represent the Antarctic cap it is advisable, for a similar reason, to choose the meridian 30^o W — 150^o E as the central meridian.

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(13) This could be foreseen from the first because, as is known, MERCATOR's projection may be considered as a particular case of LAMBERT's conformed conical projection. It is the case in which the cone circumscribed round the sphere is reduced to a cylinder; the parallel of minimum dilation is the equator and the meridians of the chart are parallel to each other (the convergence is nil).

(14) Moreover, it is very simple to determine the maximum departure between the curvilinear passage and its chord (see Note 12). Here the departure is to the side opposite to the point O of convergence or of divergence of the magnetic meridians. (*See* hypothesis (a) of para. 9).

Example : British Chart 2182 *B*. True rhumb line course between Cape Lindesness and Firth of Forth (May I.) — 251^o

Cape Lindesness - Magnetic Variation	$9^o.9$ W	(1929).
May Island - » »	$14^o.5$ W	»
Difference	$4^o.6$	
Mean magnetic variation	12^o2 W	

The magnetic course $251^o + 12^o.2 =$ ***263°.2*** must be followed.

Length of the passage on the chart (chord) about 90 centimetres; maximum departure $4.6 \times 0.22 =$ about one-hundredth of this length; *i. e.* 0.9 centimetres (towards the North), which corresponds to about 3.3 miles.

(15) The properties of this projection have been considered and tables which enable it to be constructed have been given in "*Hydrographic Review*" N° 11, May 1929. (*See* also, for the advantages resulting from the consideration of the fictitious variation in Polar Navigation, the Note entitled: *A new type of Polar Chart*, "*Hydrographic Review*" N° 10, November 1928).

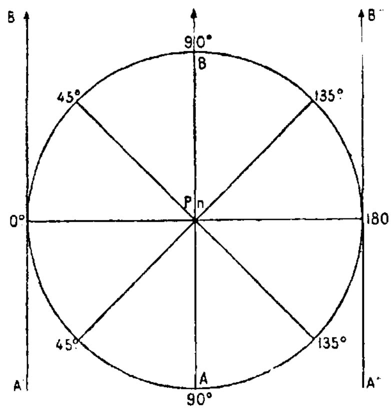

Figure 8

12. As a result of the above arguments, and taking into account other considerations also which, for brevity's sake, have not been developed (16), we are convinced that in very many circumstances, the LAMBERT projection can, with appreciable advantage, replace MERCATOR's projection as a basis for nautical charts.

Among these applications we include, not only the cases examined, *viz*: of Polar charts and track charts for long transoceanic passages by sea and by air, which extend mainly in a general East and West direction (for example between Europe and North America; North America and Japan), but also the case of hydrographic charts of subpolar regions, such as, for example, the coasts of the Arctic basin which undergo excessive (we will even say, inadmissible) distortion on MERCATOR's projection.

(16) For example the LAMBERT projection is ideal for drawing radiogoniometric bearings, position lines etc..., in a word for the graphic solution of the more important problems of modern navigation.