CORRECTION FOR INCLINATION OF SOUNDING WIRE.

by

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Although the increasing use of sonic depth finders makes inclined wire soundings of less importance than in the past, all vessels are not equipped with the more modern devices, and a correct formula for the inclined wire corrections is desirable.

A general formula may be developed as follows : —

Let h = distance, spool to water.

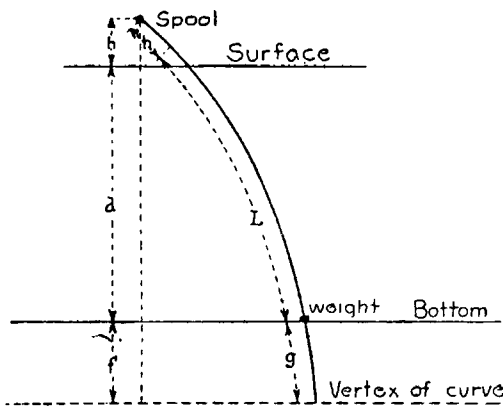L = length of wire out from the spool when weight touches bottom, less h.

g = length of wire which has the same horizontal resistance to the water as the weight, and f = its projection on the vertical.

φ = the angle of wire with the vertical at the spool.

α = the similar angle at the weight.

C = ratio of its projection on the vertical to length of arc of curve.

In the figure,

$$g = \frac{f}{C_\alpha}$$

$$d = \left(L + \frac{f}{C_\alpha} - h\ exsec\ \varphi \right) C_\varphi - f \ (*)$$

$$= LC_\varphi - h\ exsec\ \varphi\ C_\varphi - f \left(1 - \frac{C_\varphi}{C_\alpha} \right)$$

Add terms, $+ f(1 - C_\varphi)$ and $- f(1 - C_\varphi)$ and combine the 3rd and 4th terms.

Then, $d = LC_\varphi - h\ exsec\ \varphi\ C_\varphi$
$$- f(1 - C_\varphi) + fC_\varphi \left(\frac{1}{C_\alpha} - 1 \right)$$

In the second member of the equation, the first term is independent of the constants of the outfit and may be tabulated for all outfits, once the form of the curve is determined. The second and third terms are independent of the depth and may be tabulated for the particular outfit in use. The fourth term depends on all factors, but it will be shown that this term may be neglected in practice, as its value is small.

(*) $exsec\ \varphi = sec\ \varphi - 1$.

There have been several discussions of the nature of the curve taken by the wire in the water, notably by DE MARCHI, COURTIER, TONTA, and lately by GOUGENHEIM (*). It is improbable that the form of the actual curve can be determined except by assuming a theoretical curve and comparing experimental results with it.

The four curves which have been mentioned are the parabola, the catenary, a modified form of the catenary, and the circle. For small angles and depths, the differences between these are not very important. Experiments have shown that the circle certainly gives too large corrections for the greater depths. The corrections as found by the other three differ by about equal amounts from one curve to the other, these differences being, for example, about 1: 1,200 at 25°; 1: 500 at 30°; and 1: 100 at 45°.

The United States Lake Survey, in its sounding operations on the Great Lakes, has made experiments to determine the form of the curve. The latest was in 1928 on Lake Superior. The U. S. Lake Survey uses wire .08 inch in diameter and a 95 lb. iron fish-shaped weight. For these, it was found that the length of wire with horizontal resistance equal to the weight was about 15 feet.

The steamer made many runs at different speeds between two buoys one-half mile apart over an even bottom of about 650 feet depth. The corrections agreed very well with those determined by means of the parabolic curve, the curve which would seem to be theoretically the best.

Runs were also made at as nearly uniform speed as possible, letting the weight down by successive steps of 100 feet. The speed could not be regulated so closely but that there was a considerable range in the results for presumably similar conditions. The means of these were taken and compared with the theoretical formulas below : —

For the circle, $\dfrac{\sin \varphi}{\sin \alpha} = \dfrac{d + f}{f}$.

For the catenary, $\dfrac{\tan \varphi}{\tan \alpha} = \dfrac{L + h \, (1 - exsec \, \varphi)}{g}$.

For the parabola, $\dfrac{\tan \varphi}{\tan \alpha} = \dfrac{d + f}{f}$.

(*) Prof. DE MARCHI : *Teoria degli scandagli di alto mare* (Memoria XXI del R. Comitato Talassografico, Venezia, 1913) ;

M. A. COURTIER : *Note sur la Courbure de la ligne et la Correction d'inclinaison, etc.*, (Annales Hydrographiques, 1925-1926) ;

L. TONTA : *On the Curvature of the Lead Line and the Correction for its Inclination* (Hydrographic Review, Vol. IV, N° 2, Monaco, November 1927, p. 105) ;

M. A. GOUGENHEIM : *Nomographic determination of the inclination correction of soundings taken by means of a fish lead* (Hydrographic Review, Vol. IX, N° 2, Monaco, November 1932, p. 84).

MEAN OBSERVED ANGLE		CIRCLE				CATENARY				PARABOLA					
		L = 690		L = 400		L = 600		L = 400		L = 690		L = 500		L = 300	
	L	$\frac{\sin\varphi}{\sin\alpha}$	$\frac{d+f}{f}$	$\frac{\sin\varphi}{\sin\alpha}$	$\frac{d+f}{f}$	$\frac{\tan\varphi}{\tan\alpha}$	$\frac{L'}{g}$	$\frac{\tan\varphi}{\tan\alpha}$	$\frac{L'}{g}$	$\frac{\tan\varphi}{\tan\alpha}$	$\frac{d+f}{f}$	$\frac{\tan\varphi}{\tan\alpha}$	$\frac{d+f}{f}$	$\frac{\tan\varphi}{\tan\alpha}$	$\frac{d\times f}{f}$
2°	1														
13.5	100	.305	.185	.379	.307	.225	.189	.308	.280	.202	.196	.261	.251	.400	.384
23	200	.510	.340	.634	.562	.398	.351	.544	.521	.357	.355	.456	.458	.707	.697
31	300	.672	.488	.836	.805	.561	.514	.770	.762	.505	.510	.645	.653		
38	400	.804	.607			.730	.673			.657	.645	.836	.826		
43	500	.888	.756			.871	.837			.784	.781				
47	·600	.955	.866							.901	.900				
50	690														

In the Table above, the reduced depths are computed according to the curve in question and 15 feet added for vertex of curve. The ratio of the sine (or tangent) at the depth given at top of column to the sine (or tangent) at the partial depth at the left is compared with the ratio of $d + f$ to f (or L' to g) where d is depth derived from the L at left and f is remainder of depth to vertex of curve, for the depth as given at top of column.

The comparisons show a rather remarkable agreement for the parabola. An extension of this experiment with speed of boat kept reliably uniform would undoubtedly be decisive as to the form of the curve.

It may be shown that any reasonable error in reading the angle would not affect these comparisons noticeably.

If we adopt the parabolic curve and the distance $g = 15$, the value of the fourth term in the correction formula for certain depths and angles is shown in Table below : —

Depth =	0	15	30	75	285
$\varphi = 20°$	0.32	0.08	0.04	0.01	0.00
30°	0.76	0.20	0.08	0.02	0.00
40°	1.45	0.39	0.15	0.04	0.00
50°	2.52	0.70	0.32	0.08	0.01
60°	4.13	1.24	0.58	0.15	0.01

The use of the other curves would give results only slightly different.

The fourth term of the formula may therefore be entirely neglected in practice and the formula becomes : —

$$d = LC_\varphi - h \; exsec \; \varphi \; C_\varphi - f \; (1 - C_\varphi).$$

Taking the parabola, $y^2 = 4\,ax$, the angle φ which the tangent to the curve at the point x, y, makes with the vertical is expressed by : $\tan \varphi = \dfrac{2\,x}{y}$.

From the mathematics of the parabola it follows that C, in the formula, is expressed *exactly* by :

$$C = \tfrac{1}{2}\left[\, sec\,\varphi \,+\, cot\,\varphi \,\, log_\varepsilon\,\, tan\left(\frac{\varphi}{2} + \frac{\pi}{4}\right)\right].$$

The value of C has been computed for the following Table, where results are carried further than necessary for sounding, but may be of use in other connections.

φ	C	φ	C	φ	C	φ	C	φ	C	φ	C
0°	1.00000	15°	0.98830	30°	0.94961	45°	0.87124	60°	0.72455	75°	0.45383
1	0.99995	16	.98664	31	.94583	46	.86403	61	.71116	76	.42951
2	.99980	17	.98486	32	.94187	47	.85652	62	.69721	77	.40429
3	.99954	18	.98297	33	.93773	48	.84869	63	.68267	78	.37817
4	.99919	19	.98095	34	.93340	49	.84053	64	.66752	79	.35114
5	.99873	20	.97880	35	.92888	50	.83202	65	.65174	80	.32319
6	.99817	21	.97652	36	.92416	51	.82316	66	.63531	81	.29433
7	.99750	22	.97411	37	.91923	52	.81392	67	.61820	82	.26458
8	.99673	23	.97156	38	.91408	53	.80429	68	.60038	83	.23396
9	.99585	24	.96888	39	.90870	54	.79426	69	.58183	84	.20250
10	.99487	25	.96605	40	.90309	55	.78381	70	.56252	85	.17025
11	.99378	26	.96308	41	.89724	56	.77291	71	.54243	86	.13727
12	.99258	27	.95995	42	.89114	57	.76156	72	.52154	87	.10364
13	.99127	28	.95667	43	.88478	58	.74973	73	.49982	88	.06946
14	.98984	29	.95322	44	.87815	59	.73740	74	.47726	89	.03485

In practice, fine discriminations in the corrections are useless unless the angle be read correctly. A very satisfactory method lately in use by the U. S. Lake Survey is to hang on the spool a light wooden arc marked to degrees, so held by hooks to the sounding wire that its zero line is always parallel to the wire. Then a rather heavy ended pointer indicates the inclination of the wire at all times.

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