

NOTE ON A METHOD FOR A ROUGH PREDICTION OF DIURNAL TIDES.

by

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1. In the *Hydrographic Review*, Vol. X, N^o 1, May 1933, Captain J. L. LUYMES, Director of the Netherlands Hydrographic Service, made a comparison between two methods of predicting the time of diurnal tides :

one based on the use of the time of High Water of the K_1 tide, i.e. the preponderant constituent of the diurnal group (*Dutch method*) ;

the other based on the use of the time of the Moon's transit (*French method*).

In his article Captain LUYMES explored the theoretical foundations of the second method and gave the practical limits of its application, basing his conclusions on the relative magnitude of the harmonic constituents K_1 and O_1 .

We propose to indicate the origin of the French method, and the reasons for which this method is mentioned in the preface of the *Tables de Marées des Colonies françaises des Mers de Chine*.

2. It is a well-known fact on the coasts of Indo-China that to obtain the time of High Water the native fishermen merely watch the moon and reckon the time that this body will take to reach a position in the sky which regularly corresponds to the moment of High Water at the place in question.

This practical procedure for predicting the tides, used long ago in European countries under the title of the method of *situations lunaires*, is easily explained in the case where the semi-diurnal tide is preponderant, and has given birth to what have been called "establishments" and "intervals". But at first sight it would appear somewhat strange that it could give practical results in countries such as Indo-China where the tide is complicated by a strong diurnal wave.

Consideration of the formulae however shows that such is not the case. Also it was by using the time of the moon's transit that Ingénieur Hydrographe HÉRAUD in 1873 worked out his first Annual of the Tides of Lower Cochin China and Tonkin, and this method of prediction was retained until 1898, the year in which it was replaced by harmonic prediction.

For Doson (Tonkin) in particular, for which predictions were given for the first time in the year 1878, HÉRAUD used the following rule (1) : "When the declination is Northerly the time of High Water at Doson is obtained by adding 5^h 30^m to the time of the moon's transit of the meridian of Brest, and when the declination is Southerly the time of Low Water at Doson is obtained by adding 6^h 00^m.

"The predictions thus obtained must be applied to a date two days later than that considered."

For the Doson calculations HÉRAUD used the calculations already worked out for the tide at Brest ; he added the quantities 5^h 30^m and 6^h 00^m

(1) In this connection see an article by HÉRAUD on the prediction of the tides at Tonkin in the 1877 *Annual*.

mentioned above, not to the time of the moon's transit of the meridian of Brest, but to what is called the time of "maximum action" (1) which makes a slight difference to the rule enunciated above.

The predictions thus obtained and shown for 20 years from 1878 to 1898 in the Annuals of the Tides of Cochin China and Tonkin were very near the truth; this showed that it was possible for countries with diurnal tides, as for countries with semi-diurnal tides, to use an "establishment" which would be valid enough for the rough prediction of the time of the tides.

3. The *diurnal tide* is, as we know, composed broadly of two terms, one *lunar* due to the action of the moon, and the other *solar* due to the action of the sun; the lunar term is usually preponderant and its theoretical value is 2.17 times that of the solar term.

If then we assume, with a view to a rough prediction, that the solar term is negligible, the result is that the time of the diurnal High Water is lost in the time of the High Water of the lunar term; consequently it follows the time of the moon's transit by a practically constant quantity. At the same time we must distinguish the case where the declination of the moon is positive (Northerly) from the case where this declination is negative (Southerly).

Let us consider the development of the potential as a function of the hour angle of the heavenly bodies.

For the diurnal part we have

$$V = \tau_1 i^3 \frac{\sin 2 D}{2} \cos Ha + \tau_1' i'^3 \frac{\sin 2 D'}{2} \cos Ha'$$

where Ha represents the hour angle of the moon,

D represents the declination of the moon,

i represents the parallax of the moon,

Ha' , D' , i' represent the corresponding quantities for the sun, and $\frac{\tau_1}{\tau_1'} = 2.17$.

The resulting height of tide by the elementary equilibrium theory is

$$y = \frac{V}{g}$$

In dynamic theory, LAPLACE, after carrying out a synthesis of the periodic terms representing the tide and assuming that certain coefficients of the formula thus obtained were zero, as he verified in the case of Brest, adopted in his "Mécanique Céleste" (Book IV) the following expression of the diurnal tide :

$$(1) \quad y_{t+T_1} = B_1 i^3 \sin 2 D \cos (Ha - \mu) + B_1' i'^3 \sin 2 D' \cos (Ha' - \mu)$$

in which formula the quantities T_1 , μ are two constants ;

y_{t+T_1} represents the height of the sea above the level of equilibrium at an instant $t + T_1$, later by the quantity T_1 than the instant t for which the second member is calculated.

In this form, it will be seen that when the second term of the second member is neglected, we get, for the time of High Water, the time of High Water of the lunar term. In the case where D is positive, High Water takes

(1) See ROLLET DE L'ISLE : *Observation, Etude et Prédiction des Marées*, p. 104.

place at a time which is later by T_1 than the moment at which the moon reaches the constant hour angle $Ha = \mu$.

In other words, *when the declination is Northerly*, High Water follows the moon's upper transit by a quantity $\frac{2\mu}{q_{M_2}} + T_1$,

$\frac{1}{2} q_{M_2}$ representing the angular velocity of the mean motion of the moon and applying this result to the moon's transit immediately preceding the high water in question, it follows the transit by

$$(2) \quad \frac{2\mu}{q_{M_2}} + T_1 - n \times 24^h 50.5^m$$

n representing a whole number 0, 1, 2 and $24^h 50.5^m$ the mean interval between the moon's transit one day and the next.

When the declination is Southerly, the coefficient $\sin 2D$ changes sign, and there is a low water at the moment considered above instead of a high water.

This reasoning is only of use when the solar term of expression (1) is negligible in proportion to the lunar term; consequently this method fails when the declination D of the moon is near zero. All the same, as at this epoch the total tide, reduced to the solar term, is small, and as the water level varies but little, any uncertainty in the time of the tide is of less importance in practice. Besides, in those ports where there is a residual semi-diurnal tide, of which the time of high water is equal to that of the diurnal high water — and this applies at Doson (1) — the error is reduced in value.

4. There remain for investigation the conditions under which formula (1) given by LAPLACE can be considered as sufficiently valid to represent the diurnal tide of a place.

For this purpose let us compare formula (1) with the harmonic formula of the place and assume that the latter exactly represents the tide.

Let us develop formula (1) in periodic terms. From the well-known development of the potential, and the resulting table of constituents, we get the two following identities:—

$$\begin{aligned} B_1 i^2 \sin 2D \cos Ha &= (2 B_1) \times 0.189 f_{O_1} \cos \left(T + h - 2s - v + 2\xi + \frac{\pi}{2} \right) \\ &+ (2 B_1) \times 0.181 f_{K_1} \cos \left(T + h - v - \frac{\pi}{2} \right) \\ &+ (2 B_1) \times 0.037 f_{Q_1} \cos \left(T + h - 3s + p - v + 2\xi + \frac{\pi}{2} \right) \\ &+ \dots \end{aligned}$$

$$\begin{aligned} B_1' i'^2 \sin 2D' \cos Ha' &= (2 B_1' \times 2.172) \times 0.088 f_{P_1} \cos \left(T - h + \frac{\pi}{2} \right) \\ &+ (2 B_1' \times 2.172) \times 0.084 f_{K_1} \cos \left(T + h - \frac{\pi}{2} \right) \\ &+ \dots \end{aligned}$$

(1) At Doson, in fact, $\frac{\kappa_{M_2}}{2} = 56^{\circ}5$, $\frac{\kappa_{K_1} + \kappa_{O_1}}{2} = 63^{\circ}$. These quantities do not differ by much.

In these formulae T represents the hour angle of the mean sun, and $s, h, \phi, \nu, \xi \dots$ represent the astronomical quantities defined in the tidal text-books.

Let us put down what the first equation becomes at the time $t - \frac{2\mu}{q_{M_2}}$ and the second equation at the time $t - \frac{2\mu}{q_{S_2}}$; $\frac{1}{2} q_{M_2}$ and $\frac{1}{2} q_{S_2}$ representing the mean angular velocities of the moon and the sun.

We thus obtain the two new equations

$$B_1 i^3 \sin 2D \cos(Ha - \mu) = \left(2 B_2\right) \times 0.189 f_{O_1} \cos\left(T + h - 2s - \nu + 2\xi + \frac{\pi}{2} - q_{O_1} \frac{2\mu}{q_{M_2}}\right) + \left(2 B_2\right) \times 0.181 f_{K_1} \cos\left(T + h - \nu - \frac{\pi}{2} - q_{K_1} \frac{2\mu}{q_{M_2}}\right) + \left(2 B_2\right) \times 0.037 f_{Q_1} \cos\left(T + h - 3s + \phi - \nu + 2\xi - \frac{\pi}{2} - q_{Q_1} \frac{2\mu}{q_{M_2}}\right)$$

$$B_1' i'^3 \sin 2D' \cos(Ha' - \mu) = \left(2 B_1' \times 2.1722\right) \times 0.088 f_{P_1} \cos\left(T - h + \frac{\pi}{2} - q_{P_1} \frac{2\mu}{q_{S_2}}\right) + \left(2 B_1' \times 2.1722\right) \times 0.084 f_{K_1} \cos\left(T + h - \frac{\pi}{2} - q_{K_1} \frac{2\mu}{q_{S_2}}\right)$$

If in the terms $i^3 \sin 2D, i'^3 \sin 2D'$ we neglect the small intervals $\frac{2\mu}{q_{M_2}}, \frac{2\mu}{q_{S_2}}$, we can take it that the two expressions $B_1 i^3 \sin 2D \cos(Ha - \mu)$ and $B_1' i'^3 \sin 2D' \cos(Ha' - \mu)$ which appear in formula (1) are furnished by the second members of the foregoing equations.

Taking these second members at the time $t - T_1$, we thus have, for the expression of the height of sea-level at the time t ,

$$(1 a) \quad y_t = 2 B_1 \times 0.189 f_{O_1} \cos\left[T + h - 2s - \nu + 2\xi + \frac{\pi}{2} - q_{O_1} \left(\frac{2\mu}{q_{M_2}} + T_1\right)\right] + 2 B_1 \times 0.181 f_{K_1} \cos\left[T + h - \nu - \frac{\pi}{2} - q_{K_1} \left(\frac{2\mu}{q_{M_2}} + T_1\right)\right] + 2 B_1 \times 0.037 f_{Q_1} \cos\left[T + h - 3s + \phi - \nu + 2\xi + \frac{\pi}{2} - q_{Q_1} \left(\frac{2\mu}{q_{M_2}} + T_1\right)\right] + \left(2 B_1' \times 2.1722\right) \times 0.088 f_{P_1} \cos\left[T - h + \frac{\pi}{2} - q_{P_1} \left(\frac{2\mu}{q_{S_2}} + T_1\right)\right] + \left(2 B_1' \times 2.1722\right) \times 0.084 f_{K_1} \cos\left[T + h - \frac{\pi}{2} - q_{K_1} \left(\frac{2\mu}{q_{S_2}} + T_1\right)\right] + \dots$$

Such is the transformation of formula (1) of LAPLACE.

By equating this expression term for term with the harmonic formula of the place, the following conditions can be deduced:—

$$\left\{ \begin{array}{ll} H_{O_1} = 2 B_1 \times 0.189 & q_{O_1} \left(T_1 + \frac{2 \mu}{q_{M_2}} \right) = \kappa_{O_1} \\ H_{K_1 \text{ lun.}} = 2 B_1 \times 0.185 & q_{K_1} \left(T_1 + \frac{2 \mu}{q_{M_2}} \right) = \kappa_{K_1 \text{ lun.}} \\ H_{Q_1} = 2 B_1 \times 0.036 & q_{Q_1} \left(T_1 + \frac{2 \mu}{q_{M_2}} \right) = \kappa_{Q_1} \\ H_{P_1} = (2 B_1' \times 2.172) \times 0.088 & q_{P_1} \left(T_1 + \frac{2 \mu}{q_{S_2}} \right) = \kappa_{P_1} \\ H_{K_1 \text{ sol.}} = (2 B_1' \times 2.172) \times 0.084 & q_{K_1} \left(T_1 + \frac{2 \mu}{q_{S_2}} \right) = \kappa_{K_1 \text{ sol.}} \end{array} \right.$$

Assuming that $\kappa_{K_1 \text{ lun.}} = \kappa_{K_1 \text{ sol.}}$, $\frac{H_{K_1 \text{ lun.}}}{2.172} = \frac{H_{K_1 \text{ sol.}}}{1.00} = \frac{H_{K_1 \text{ lunisol}}}{3.1722}$ as well as the equality

of the terms $T_1 + \frac{2 \mu}{q_{M_2}}$ and $T_1 + \frac{2 \mu}{q_{S_2}}$, we find that for LAPLACE'S

formula (1) to be applicable to the tide of a port the latter must fulfil the two following well-known conditions, viz :

(1) that the 1/2 amplitudes H_{O_1} , H_{K_1} , H_{P_1} , ... of the constituents should be proportional to the theoretical amplitudes

$$\frac{H_{O_1}}{0.189} = \frac{H_{K_1}}{0.265} = \frac{H_{P_1}}{0.088} = \frac{H_{Q_1}}{0.0365} = \dots$$

(2) that the difference of phase lags of the constituents K_1 , O_1 , P_1 be proportional to the difference of the angular velocities

$$\frac{\kappa_{K_1} - \kappa_{O_1}}{q_{K_1} - q_{O_1}} = \frac{\kappa_{P_1} - \kappa_{O_1}}{q_{P_1} - q_{O_1}} = \dots$$

Under these conditions the values B_1 , B_1' , T_1 , μ are given by the formulae

$$(2) \left\{ \begin{array}{l} B_1 = \frac{H_{O_1}}{0.377} = \frac{H_{K_1}}{0.528} = \frac{H_{Q_1}}{0.073} = \dots \\ B_1' = \frac{H_{P_1}}{0.381} = \frac{H_{K_1}}{1.161} = \dots \\ \mu = \kappa_{K_1} - 15.0^\circ T_1 = \kappa_{O_1} - 13.9^\circ T_1 = \kappa_{Q_1} - 13.4^\circ T_1 = \dots \\ T_1 = \frac{\kappa_{K_1} - \kappa_{O_1}}{q_{K_1} - q_{O_1}} = \frac{\kappa_{P_1} - \kappa_{O_1}}{q_{P_1} - q_{O_1}} = \dots \end{array} \right.$$

These are the conditions to be realised (1).

5. Let us apply the foregoing results to the case of Doson (Tonkin).

The harmonic constants of Doson (Hondau) obtained by the analysis of a year's observations are :—

	<i>H</i>	κ		<i>H</i>	κ
	<i>cm</i>	$^{\circ}$		<i>cm</i>	$^{\circ}$
K_1	65.2	92.3	M_1	4.0	26.0
O_1	74.0	37.2	J_1	2.3	50.4
P_1	20.5	82.7	S_1	2.6	243.4
Q_1	14.0	11.0			

For LAPLACE'S formula (1) to represent the tide of Doson correctly we must have

$$B_1 = \frac{74 \text{ } \frac{\text{cm}}{\text{m}}}{0.377} = \frac{65 \text{ } \frac{\text{cm}}{\text{m}}}{0.528} = \frac{14 \text{ } \frac{\text{cm}}{\text{m}}}{0.073}$$

(1^m95) (1^m24) (1^m92)

$$B_1' = \frac{20.5}{0.381} = \frac{65.2}{1.161}$$

(0^m54) (0^m56)

$$T_1 = \frac{92.3^{\circ} - 37.2^{\circ}}{1.10^{\circ}} = 50.1^{\text{h}}$$

$$\mu = 92.3^{\circ} - 15.04^{\circ} T_1 = 37.2^{\circ} - 13.94^{\circ} T_1 = 58.8^{\circ}$$

Taking $B_1 = \frac{1.95 + 1.24}{2} = 1^{\text{m}}60$

$$B_1' = 0^{\text{m}}54$$

the formula of the tide at Doson is

$$(3) \ y_{t+T_1} = 1^{\text{m}}60 \ i^3 \sin 2D \cos (Ha - 58.8^{\circ}) + 0^{\text{m}}54 \ i^3 \sin 2D' \cos (Ha' - 58.8^{\circ})$$

with $T_1 = 50.1^{\text{h}}$,

a formula correct within a limit of $\pm 0^{\text{m}}35$ in the term $1^{\text{m}}60$.

These results show that formula (3) is not strictly applicable to the tide at Doson, but is only an approximation.

(1) Applying the same reasoning to the semi-diurnal tide, we find that the expression

$$Y_{t+T_2} = B_2 \ i^3 \cos^2 D \cos 2(Ha - \lambda) + B_2' \ i^3 \cos^2 D' \cos 2(Ha' - \lambda)$$

exactly represents the semi-diurnal tide if we have

$$\frac{H_{M_2}}{0.909} = \frac{H_{N_2}}{0.176} = \frac{H_{K_2}}{0.115} = \frac{H_{L_2}}{0.026} = \dots = B_2$$

$$\frac{H_{S_2}}{0.930} = \frac{H_{K_2}}{0.248} = \frac{H_{T_2}}{0.054} = \dots = B_2'$$

$$\lambda = \kappa_{M_2} - q_{M_2} T_2 = \kappa_{S_2} - q_{S_2} T_2 = \kappa_{N_2} - q_{N_2} T_2 = \dots$$

and consequently $T_2 = \frac{\kappa_{S_2} - \kappa_{M_2}}{q_{S_2} - q_{M_2}}$ which gives the expression of the quantities B_2, B_2', λ, T_2 as a function of the harmonic constituents.

With regard to the "Etablissement lunaire" or the quantity by which High Water follows the moon's upper transit when the declination is North-erly, this is given by formula (2)

$$\frac{2\mu}{q_{M_2}} + T_1 - n \times 24^h 50.5^m,$$

i.e. $4.06^h + 50.1^h - 2 \times 24^h 50.5^m = 4^h 29^m$

This value is practically equal to the value 4 h, 27 m given by M. LUYMES and to the values

$$\begin{cases} 5^h 30^m + 15^m - 2 \times 50.5^m = 4^h 05^m \\ 6^h 00^m + 15^m - 2 \times 50.5^m = 4^h 35^m \end{cases}$$

obtained by the rule adopted by HÉRAUD.

6. It follows from the above that the *French method*, based on the use of the moon's transit, has only an approximate character; it is in default at the epoch where the declination of the moon is zero.

The *Dutch method* is based on the fact that the solar diurnal tide and the lunar diurnal tide each contain a constituent K_1 , and the combination of these two constituents constitutes in a great number of places the main part of the diurnal tide. This method does not suffer from the French method's disadvantage of failing at the epoch when the moon's declination is zero; at the same time, as at this epoch the diurnal tide is weak the residual semi-diurnal tide assumes some importance and the various summary methods of tidal prediction give results which at this period are not very accurate.

To conclude, let us show by harmonic formula that in the special case of those ports where the K_1 and O_1 constituents have practically the same amplitude the French method can be applied to advantage.

The formula for the diurnal tide reduced to the two principal constituents K_1 and O_1 is

$$(4) \quad y = f_{O_1} H_{O_1} \cos \left(t + h - 2s - v + 2\xi - \frac{\pi}{2} - \kappa_{O_1} \right) + f_{K_1} H_{K_1} \cos \left(t + h - v - \frac{\pi}{2} - \kappa_{K_1} \right).$$

If we have $f_{O_1} H_{O_1} = f_{K_1} H_{K_1}$ we can combine the two terms, and the formula becomes

$$y = \left(f_{O_1} H_{O_1} + f_{K_1} H_{K_1} \right) \sin \left(s - \xi - \frac{\kappa_{K_1} - \kappa_{O_1}}{2} \right) \cos \left(t + h - s - v + \xi - \frac{\kappa_{O_1} + \kappa_{K_1}}{2} \right)$$

Now the quantity $t + h - s - v + \xi$ represents the hour angle Ha of the moon so well that, neglecting the small term ξ , the total of the two constituents K_1 and O_1 is given, for practical purposes, by the formula

$$(5) \quad y = \left(f_{O_1} H_{O_1} + f_{K_1} H_{K_1} \right) \sin \left(s - \frac{\kappa_{K_1} - \kappa_{O_1}}{2} \right) \cos \left(Ha - \frac{\kappa_{O_1} + \kappa_{K_1}}{2} \right)$$

It is easy to see that at the time of diurnal springs, the phases of the two constituents K_1 and O_1 being then the same, the expression (5) represents the initial formula (4), even without the hypothesis that $f_{K_1} H_{K_1} = f_{O_1} H_{O_1}$.

Formula (5) readily gives the explanation of the various peculiarities of the lunar tide : age, establishment, etc. ; in particular the factor $\sin\left(s - \frac{\kappa_{K_1} - \kappa_{O_1}}{2}\right)$ shows how the amplitude of the tide varies as a function of the declination of the moon since for

$$D = 0 \text{ we have practically } s = 0, \pi$$

and for

$$\left\{ \begin{array}{l} D = \text{maximum we have practically } s = \frac{\pi}{2}, \\ D = \text{minimum we have practically } s = \frac{3\pi}{2}. \end{array} \right.$$

7. To summarise : for countries with diurnal tides it is possible, by using the time of the moon's transit, to have an approximate value of the time of High Water, and this value is so much the better as the epoch in question is nearer springs.

All the same, as Captain LUYMES has shown, in a certain number of ports (those where the K_1 tide is strongly predominant) this method will only give very mediocre information ; and it is preferable for these ports to use the time of High Water of the K_1 constituent.

In most ports with diurnal tides, these methods will only give approximate indications in any case. If greater precision is necessary in the time of the tide and to know the exact height, it will be convenient to use other processes — either working out the principal harmonic constants with the aid of rapid-use tables given in text-books and tidal annuals, or else taking detailed predictions from the annuals for a port whose tidal regime is near to that of the port in question and using the method of "differences" or "tables of tidal differences".

