NOTES AND TABLES ON POLAR CARTOGRAPHY.

by

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I.

THE LAMBERT CONFORMAL POLAR PROJECTIONS. (*)

1. — These belong to that class of conformal conical projections which were first studied by LAMBERT, (but frequently attributed to GAUSS) and therefore represent a particular case of the latter. Another particular case of these projections is the well-known MERCATOR projection used for hydrographic charts.

The geographic meridians are represented by straight lines radiating from the point of origin P, the image of the pole, making angles to each other, equal to their respective differences in longitude. (Fig. 1).

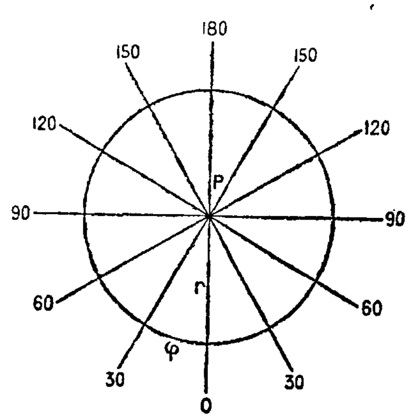

Fig. 1

(*) It would appear advisable, in a publication of international character, and for reasons of clearness, to give an exact definition of the terms employed in the text. The term *conformal* (French: conforme) is the term which is, without doubt, most generally employed to designate those projections which have *the property of not changing the angles.* D'AVEZAC: *Coup d'œil historique sur la projection des Cartes géographiques,* 1863 (General historical review of the projections of geographic charts) and those who followed him, called such a type of projection *orthomorphic* (French: orthomorphe); TISSOT (*Mémoire sur la Représentation des Surfaces,* etc., 1878) calls them *autogonal.* The term *isogonic* has also been employed (isogonique or isogone) (FIORINI: *Le projezion delle carte geografiche,* 1881).

The geographic parallels of latitude are represented by circles having a common centre at the origin, *P*. The circle which represents the parallel of latitude φ has a radius r given by the equation:

$$(1) \qquad r = k \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right)^{\frac{e}{2}} \cot\left(45^\circ + \frac{\varphi}{2}\right)$$

in which

k is a constant.
e is the excentricity of the terrestrial spheroid.

The *linear modulus* (or the ratio of linear alteration) (*) m is given by the equation

$$(2) \qquad m = \frac{k\sqrt{1 - e^2 \sin^2 \varphi}}{2\, a \sin^2\left(45^\circ + \frac{\varphi}{2}\right)} \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right)^{\frac{e}{2}}$$

in which a is the equatorial radius.

Consequently the modulus has the same value for all points on the same parallel and has a *minimum value* when

$$\sin \varphi = 1,$$

that is to say, at the pole. This minimum value, which we shall designate as m_{90} is given by the formula:

$$(2\,a) \qquad m_{90} = \frac{k}{2\,a}\sqrt{1 - e^2}\left(\frac{1 + e}{1 - e}\right)^{\frac{e}{2}}$$

In order to determine the value of the constant k, take the condition that *at the pole, the modulus equal unity, i. e.*

$$\frac{k}{2\,a}\sqrt{1 - e^2}\left(\frac{1 + e}{1 - e}\right)^{\frac{e}{2}} = 1.$$

From this equation we deduce

$$(3) \qquad k = \frac{2\,a}{\sqrt{1 - e^2}\left(\frac{1 + e}{1 - e}\right)^{\frac{e}{2}}}$$

By definition, the values of r obtained from equation (1) in which this value of k is substituted, are those suited to a method of representation in which the linear elements radiating from the pole are maintained in their actual magnitude, or in other words, in which the *scale of distances* at the pole is equal to unity.

(*) That is, the ratio between the linear element ds_1 as distorted by this method of projection and the corresponding element ds on the earth's surface.

If, as happens in practice, we decide that the scale of distances at latitude 90 deg. (pole) should be equal to $\frac{1}{p}$, the values r_c of the radius to be used in tracing the chart are given by the *practical formula :* —

$$(4) \qquad r_c = \frac{k}{p} \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right)^{\frac{e}{2}} \cot \left(45^\circ + \frac{\varphi}{2}\right)$$

in which k is a constant given by formula (3). For the other latitudes we will have different scales given by the equation :

$$(5) \qquad \textit{Special scale for the latitude } \varphi = \frac{m_\varphi}{p}$$

in which m_φ is the value of the modulus obtained by equation (2) for the parallel φ under consideration.

It should be noted here that the quantity $\frac{k}{p}$ represents the length of the radius employed for tracing the equator on the chart. In fact, by making φ equal to zero in equation (4) we have :

$$\textit{The radius of the equator on the chart} = \frac{k}{p}$$

2. — If in equation (1) we put e = zero, that is on the hypothesis that the earth is spherical, we obtain (*)

$$(6) \qquad r = k \cot \left(45^\circ + \frac{\varphi}{2}\right)$$

This is the formula relating to the *polar stereographic projections of the sphere, i. e.* the *perspective* projection of the sphere viewed from the

South } pole projected on the plane tangent to the North } pole
North } South }

(*) In the case of a spherical earth, the value of the constant k (which we shall suppose has been obtained in the same manner as for the spheroid, *i. e.* $m_{90} = 1$) depends on the value given to the radius of the terrestrial sphere, which in practice will be different from a. Putting e = zero and substituting for a the terrestrial radius R, we obtain from equation (3)

$$k = 2\,R$$

If, for instance, we substitute for R the value of the mean radius ($R = \frac{a + a + b}{3}$) of the international ellipsoid, we obtain k = 12,742,458.630 metres. Consequently, if we adopt, for instance, the scale $\frac{1}{p} = \frac{1}{1{,}000{,}000}$, the practical formula for calculating the radius for the parallels on the chart becomes

$$r_c = \frac{k}{p} \cot \left(45^\circ + \frac{\varphi}{2}\right) = 1274{,}246 \cot \left(45^\circ + \frac{\varphi}{2}\right) \textit{ centimetres.}$$

We shall find that for the spheroid, under the same conditions we obtain :

$$\frac{k}{p} = 1271.392 \textit{ centimetres.}$$

The conformal polar projection of the spheroid which we are considering is thus analogous to the *polar stereographic projection* but it is not at all, like the latter, a perspective projection of the surface represented.

It should be noted that, with this method of representing the spheroid, the radii of the parallels may be determined with sufficient accuracy without resort to formula (1) by making use of an expression similar to the equation (6) for the sphere, provided that in the latter expression the *geographic* latitude φ s replaced by the *geocentric* latitude ψ.

(7) (spheroid) $$r = k \cot \left(45^\circ + \frac{\psi}{2}\right)$$

Formulae (1) and (7) give results which differ only by an amount of the order of e^4 (provided of course that the same value of k is used in both cases).

However, it seems to us that it is more convenient to make use of the more accurate formula in calculating the radii of the parallels, the more so, since the latter calculation, which appears complicated, may be made very simple by means of an appropriate artifice. (*)

We do not know whether there are tables in existence giving the values of r_c sufficiently expanded to permit the construction of the chart using the

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(*) From formula (4) we have:

$$\log r_c = \log \frac{k}{p} + \log \cot \left(45^\circ + \frac{\varphi}{2}\right) + \frac{e}{2} \log \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right)$$

Developing the series of $\log \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right)$ and putting M as the modulus of the decimal system of logarithms ($M = 0{,}43429\ldots$), we have

$$\frac{e}{2} \log \left(\frac{1 + e \sin \varphi}{1 - e \sin \varphi}\right) = M e^2 \sin \varphi + \frac{M e^4}{3} \sin^3 \varphi + \frac{M e^6}{5} \sin^5 \varphi + \ldots$$

The calculation of Table I was made with the aid of this formula neglecting the terms of e^6 and the following terms:

$$\log r_c = \log \frac{k}{p} + \log \cot \left(45^\circ + \frac{\varphi}{2}\right) + M e^2 \sin \varphi + \frac{M e^4}{3} \sin^3 \varphi.$$

The terms $M e^2 \sin \varphi$ and $\frac{M e^4}{3} \sin^3 \varphi$ may be calculated rapidly by logarithms.

For the International Ellipsoid (Madrid 1924), if we determine k under the conditions set forth in § 1, formula (3) and use for $\frac{1}{p} = \frac{1}{1{,}000{,}000}$, we have:

$$\frac{k}{p} = 1271{,}392 \text{ centimetres} \quad ; \quad \log \frac{k}{p} = 3{,}1042795$$

$$M e^2 = 0{,}0029196 \quad \text{»} \quad ; \quad \log M e^2 = \bar{3}{,}4653261$$

$$\frac{M e^4}{3} = 0{,}0000065 \quad \text{»} \quad ; \quad \log \frac{M e^4}{3} = \bar{6}{,}8157466$$

We may calculate the value of the modulus (formula 2) by a similar process.

Mon. Hasse, Chef du Bureau des Calculs du Service Géographique in Paris, calculated Tables I and II in this manner under our direction.

LAMBERT conformal polar projections. Certainly, at the moment there are no tables of this nature calculated for the International Ellipsoid (Madrid 1924). For that reason it has appeared to us advantageous to compile Table I, which gives the values in centimetres for r_c at the pole calculated to the third decimal place of for the chart scale $\frac{1}{100.000}$ and calculated especially for the international ellipsoid $\left(\alpha = \frac{1}{297}; a = 6{,}378{,}388^m\right)$

Taking into consideration the approximation involved in drawing, this Table may be employed for the construction of charts on the scale of $\frac{1}{100.000}$ or to a smaller scale. It is evident that when the scale chosen is so great that the circles (parallels) on the chart may no longer be described with compasses, they may be plotted from points calculated from the rectangular system of coordinates (x, y) by the formulae:

$$\begin{cases} x = r_c \cos \omega \\ y = r_c \sin \omega \end{cases}$$

(The x axis is the central meridian of the chart and ω is given by the difference in longitude between the point in question and the central meridian).

Table II gives the value of the linear moduli for the different latitudes given in Table I. These values, as we stated in the preceding paragraph, may be employed for the determination of the special scales at the various latitudes.

3. — Let us now consider the properties of this type of projection (which is one of the principal objects of this paper). In our opinion these properties are particularly appropriate for the construction of hydrographic charts in the polar regions and in general for charts of the Arctic and Antarctic (both marine and aviation). (*)

a) It possesses the one great and indispensable quality for navigational charts of being *conformal*.

b) Within definite limits of latitude (and certainly within the polar cap within the parallel 30°) the distortion is not excessive. In this connection it is very interesting and instructive to make a comparison with the MERCATOR chart for the equatorial and subequatorial regions.

Before making this comparison and enumerating the other properties, it should be noted that in this discussion we shall make the comparison between

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(*) In the reports of the 13th session of the International Commission for Aerial Navigation (Rome, 24-28 Oct. 1927) we note the following resolution under the heading: *Publication of Charts for Aerial Navigation. Projection to be adopted for the general International Aeronautical Chart o the polar Regions.* "The Commission, after having listened to the report of the Secretary-General "on the state of the work in progress, has decided to hold over the study of this question until "the next session."

We are not aware of the present state of the current studies.

the polar stereographic projection of the sphere and the MERCATOR cylindrical projection of the same sphere. This approximation is made for the purpose of simplifying the analysis; while, for the rest, the conclusion obtained may be applied without appreciable error to the spheroid and its mode of representation. (*)

This being understood, let us imagine a *composite map of the world* showing the terrestrial sphere on a MERCATOR projection for the equatorial zones and on polar stereographic projections for the polar regions.

Assume that we have:

for the MERCATOR projection................................ modulus = 1 at equator.
for the stereographic projection........................... modulus = 1 at the pole.

Let us consider the values which this modulus will have in the other latitudes represented in both types of projections. (**)

LATITUDE.	MERCATOR $m = \frac{1}{\cos\varphi}$	STEREOGRAPHIC POLAR. $m = \frac{1}{\sin^2\left(45^\circ + \frac{\varphi}{2}\right)}$
0°	1.00000 ↓	....
10°	1.01543	....
20°	1.06418	....
30°	1.15470	....
36°52'12"	1.25000 ↓	1.25000 ↑
40°	....	1.21744
50°	....	1.13247
60°	....	1.07180
70°	....	1.03109
80°	....	1.00765
90°	....	1.00000 ↑

(*) It should be noted that in treating the subject in this manner, we are following the method usually employed to avoid unnecessary complications, in treatises on navigation and hydrography in the elementary discussions of the various properties of the MERCATOR chart, and also in particular for the study of the special lines traced on the earth's surface (loxodrome, orthodrome etc...).

(**) In formula (2) by putting e = Zero and replacing a by R, we have:

$$m = \frac{k}{2R} \frac{1}{\sin^2\left(45^\circ + \frac{\varphi}{2}\right)}$$

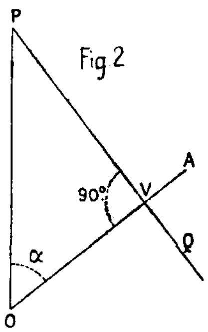
If we have the modulus at the pole equal to unity, $m_{90} = \frac{k}{2R} = 1$ and we obtain $k = 2R$.

Therefore, for the stereographic projection we have: $m = \frac{1}{\sin^2\left(45^\circ + \frac{\varphi}{2}\right)}$

A study of the above table shows that on the MERCATOR chart, a linear terrestrial element, represented by the length *ds* at the equator will actually be represented by the length $ds_1 = 1.25\ ds$ at latitude 36° 52' 12" ; or, in other words, the extension to which the element of length is subjected as a result of the change in latitude from the equator to lat. 36°52' 12" amounts to one quarter of its original length. In the polar stereographic chart an identical extension occurs in passing from the pole to latitude 36° 52' 12", or, in other words, for a change in latitude of 53° 07' 48". (*)

In general it may be stated that, starting *from the pole*, the expansion of the elements is smaller than that which occurs for the same element in moving away *from the equator* on the MERCATOR chart.

c) In the stereographic projections, it will be remembered that the great circles of a sphere are projected as circles or straight lines (the latter being a particular case of the circle) (**)



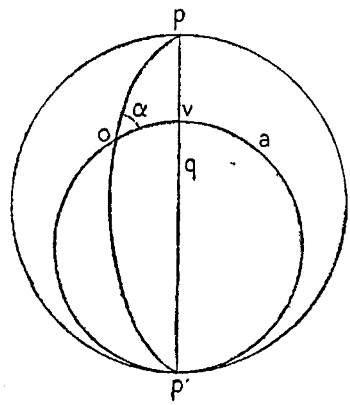


Fig. 2 bis

In the polar stereographic projection let us consider the point *O* (fig. 2) at latitude φ_0. A straight line *OA* drawn from the point *O* and making an angle α (azimuth) with the meridian *PO*, is the representation of a small circle of the sphere which passes through the pole of the earth opposite to the point *P*. This circle has as spherical radius

$$\theta = 45° + \frac{\varphi_0}{2},$$

(*) The parallel for which the same expansion results in the two types of projection is evidently given by the equation $\cos\varphi = \sin^2\left(45° + \frac{\varphi}{2}\right)$ which, by reduction, becomes $\sin\varphi = \frac{6}{10}$, or again $\frac{1}{\cos\varphi} = \frac{10}{8} = 1.25$; therefore φ = about 36°52'12"

(**) It is evident that the straight lines represent the circles of the sphere which pass through the centre of the perspective projection.

φ_v being the latitude of the point V at which OA intersects the meridian PQ which is perpendicular to it. This circle has its centre on the meridian of the sphere represented by PQ, it intersects the meridian represented by PO at an angle α. The above is evident from a glance at Fig. 2 *bis* (in which the points on the sphere corresponding to the points in the projection are indicated by the small letters). Applying the formula which gives the radius of the parallels in the polar stereographic projections, we obtain (Fig. 2)

$$PO = c \times \cot\left(45^\circ + \frac{\varphi_o}{2}\right)$$

$$PV = c \times \cot\left(45^\circ + \frac{\varphi_v}{2}\right)$$

$$= c \times \cot \theta$$

in which c is a constant.

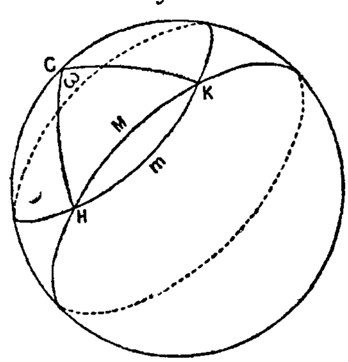
On the other hand, in the triangle OVP we have :

$$PV = PO \sin \alpha, \quad \text{and therefore}$$

$$\cot \theta = \sin \alpha \cot\left(45^\circ + \frac{\varphi_o}{2}\right) \tag{8}$$

This formula gives the spherical radius θ of the small circle of which OA is the representation, as a function of φ_o and of α

Fig. 3



This being granted, let us note that the difference $m - M$ between the length m of a given track HK (fig. 3) made along a small circle having a spherical radius θ, and a length M of the *orthodromic* track (great circle) comprised between the same extremities H and K, may be expressed with close approximation, between the limits determined for θ and m, by the formula :

$$m - M = \frac{m^3 \ arc^2 \ 1'}{24} \cot^2 \theta, \tag{9}$$

in which we assume that m and M are measured in miles (that is, in minutes of arc of the great circle). (*)

We repeat that this formula is valid between the limits determined for θ and m. In the actual case under consideration for points in the polar and sub-polar regions, θ is always comprised between the limits and the formula is always applicable for m less than 1000 miles and even outside these limits.

For $m = 1000'$ $$\frac{m^3 \, arc^2 \, 1'}{24} = 3'.53 \text{ (approx) and therefore}$$
$$m - M = 3'.53 \quad cot^2 \theta$$
This formula gives for $\theta \geqslant 75°$
$$m - M \leqslant 0'.25 \text{ (about), } i.\ e. \leqslant \frac{1}{4{,}000} \text{ of the distance}$$

Let us note that in accordance with formula (8), among all the circles represented by the group of straight lines issuing from the given point O of the projection, that having the minimum radius (θ *min.*) is the one corresponding to $\alpha = 90°$
$$\theta \, min = 45° + \frac{\varphi_0}{2}$$

(*) In the triangle HCM (fig. 3) in which $CH = CK = \theta$, $HCK = \omega$, we have
$$sin \frac{M}{2} = sin \, \theta \, sin \frac{\omega}{2}.$$
Developing as a series $sin \frac{M}{2}$ and $sin \frac{\omega}{2}$ and assuming that $\frac{M}{2}$ and $\frac{\omega}{2}$ are sufficiently small (for example $< 10°$) and that we may neglect the terms **above** the third order, we have:
$$\frac{M}{2} - \frac{M^3}{48} = sin \, \theta \left(\frac{\omega}{2} - \frac{\omega^3}{48}\right),$$
$$(a) \qquad M = \frac{M^3}{24} - sin \, \theta \left(\omega - \frac{\omega^3}{24}\right).$$
But the arc of the *small circle* $HK = m$ is given by $m = \omega \, sin \, \theta$ (in terms of the radius of the sphere). Substituting in the formula (a) the values of ω given as the results of this expression, we obtain, after reduction:
$$m - M = \frac{1}{24}\left(M^3 - \frac{m^3}{sin^2 \, \theta}\right).$$
In this expression it is interesting to note that M as well as m are expressed in radians and are small quantities in accordance with our original hypothesis. Consequently the difference $m - M$ is a very small quantity, of the third degree. In order to obtain an approximate value it is permissible to put $M = m$ in the second part of the expression, therefore we obtain:
$$m - M = \frac{m^3}{24} cot^2 \theta.$$
Designating by m_1 and M_1 the lengths of the tracks m and M and taking as unit of length a minute of arc of the great circle (one mile), we have:
$$m_1 = m \; arc \, 1' \quad ; \qquad M_1 = M \; arc \; 1'$$
$$m_1 - M_1 = \frac{m_1{}^3 \, arc^2 \, 1'}{24} \; cot^2 \, \theta$$
It is easy to prove that the necessary conditions for the development of the series, indicated below, are practically satisfied for $\theta > 60°$ and for $m_1 = 1000'$.

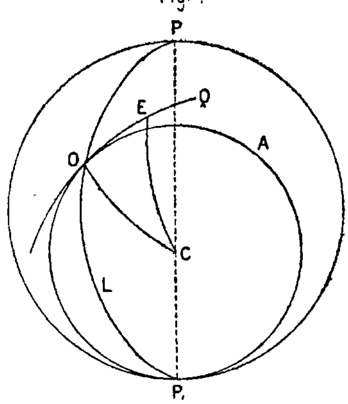
Further, the more φ_0 increases the more θ *min.* increases. Therefore if we consider the tracks in which the points of departure are located inside of the polar cap limited by the parallel 60°, we shall always have θ *min* $\geqslant$ 75°

From this we conclude that, within the latitudes and distances under consideration here, the length of the track represented on the polar stereographic projection by a segment of a straight line is practically equal to the orthodromic track comprised between the limits of this segment.

Extending this analysis, we might demonstrate that, for all the tracks which may be plotted within the limits of the cap bounded by parallel 60°, the difference $m - M$, between the length m of the track represented by the segment of a straight line and the length M of the corresponding orthodromic track is in *every case* about $\frac{4}{5000}$ ths smaller than the other track.

Therefore, we may state that on the polar chart the rectilinear segment traced between two points represents a track which has *practically* the same *economic advantages* as the orthodromic. It is evident that this orthodrome is represented by the arc of a circle which is concave towards the pole *P*. (*)

Fig. 4



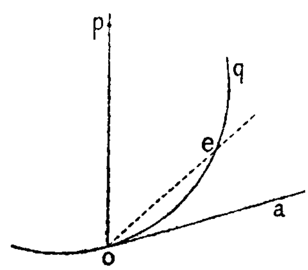


Fig. 4 bis

(*d*) Let us consider the orthodrome *OEQ* on the sphere (fig. 4) which join the point *O* (latitude φ_0) with another point *E*. Let us designate the spheri cal distance *OE* by δ, and the angle *POQ* by α (azimuth of *E* taken at point *O*). Let us consider also the small circle OAP_1, tangent at *O* to the orthodrome *OQ* and having a spherical radius $CO = \theta$, θ being determined as a function of φ_0 and α by the formula (8) of paragraph 3.

(*) It is easy to show by elementary spherical analysis that within the cap of the sphere limited by the usual parallel of 60°, the maximum departure between the two tracks cannot exceed about 51' in any case.

We may now demonstrate that for a track of considerable length from the origin at *O*, the departure between the points of the orthodrome *OQ* and the arc of the small circle *OA* is very small and therefore within certain definite limits, the two lines are approximately identical. That is to say, one may be replaced by the other.

The importance of this demonstration becomes evident, when we consider that on the chart (fig. 4 *bis*) the two corresponding lines are respectively the arc of the circle *o e q* and its tangent at *o*; that is, the straight line *oa* making an angle α with the meridian *po* on the chart. By substituting this straight line for the circle we greatly simplify the solution of nautical problems based on measurement of bearings, etc...

Let us consider the figure drawn on the sphere (fig. 4). Strike the arc of the great circle *CE*. This arc intersects the small circle OAP_1 orthogonically and therefore the arc *DE* equals ε, which is a measure of the departure of the point *E*, located on the orthodrome *OQ* at a distance δ from the point *O*, and the arc of the small circle *OA*. This departure is measured perpendicular to the arc *OA*.

It is demonstrable that between certain limits of φ_0 and of δ, the departure ε measured in minutes of arc of the sphere (in miles) at a distance δ also measured in miles, is given by the formula: (*)

$$\varepsilon = \frac{arc\ 1'}{2}\ \delta^2 \cot\left(45^\circ + \frac{\varphi_0}{2}\right) \sin\alpha \qquad (10)$$

(*) In the rectangular spherical triangle *COE* (fig. 4) we have:

a) $\cos CE = \cos\theta \cos\delta \quad (\theta = CO,\ \delta = OE)$;

and also

$$\varepsilon = DE = CE - \theta$$

The formula (*a*) may also be written

b) $\cos(\theta + \varepsilon) = \cos\theta\cos\delta$

For small values of ε

$$\cos(\theta + \varepsilon) = \cos\theta - \varepsilon\sin\theta$$

and for small values of δ

$$\cos\delta = 1 - \frac{\delta^2}{2}$$

Therefore the formula (*b*) becomes after reduction:

c) $\varepsilon = \dfrac{\delta^2}{2}\cot\theta,$

in which ε and δ are measured in radians, and therefore if ε_1 and δ_1 are measured in minutes of arc

$$\varepsilon = \varepsilon_1\ arc\ 1', \qquad \delta = \delta_1\ arc\ 1'$$

Substituting the formula (*c*) we have

$$\varepsilon_1 = \frac{arc\ 1'}{2}\ \delta_1^2 \cot\theta.$$

Finally from formula (8) we obtain:

$$\varepsilon_1 = \frac{arc\ 1'}{2}\ \delta_1^2 \cot\left(45^\circ + \frac{\varphi_0}{2}\right) \sin\alpha$$

For $\delta = 100'$

(10 *bis*) $$\varepsilon_{100} = 1'.45 \; cot \left(45^\circ + \frac{\varphi_o}{2}\right) sin \; \alpha$$

The maximum value of the difference ε, at the given distance δ and for the given latitude φ_o is obtained for $\alpha = 90^\circ$. Therefore the formula (10 *bis*) gives the following results for the various values of φ_o

$\varphi_o = 60^\circ$	$\varepsilon_{100}\, max = 0'.39$
» $= 70^\circ$	» $= 0'.26$
» $= 80^\circ$	» $= 0'.13$

It might be interesting to determine for which values δ_a and δ_b of δ the difference ε takes the values 0.'5 and 1' respectively.

From the formula (10) we obtain

$$\delta_a = \sqrt{\frac{2}{arc\ 1'}} \sqrt{\frac{tan \left(45^\circ + \frac{\varphi_o}{2}\right)}{sin\ \alpha}}$$

$$\delta_b = \sqrt{\frac{1}{arc\ 1'}} \sqrt{\frac{tan \left(45^\circ + \frac{\varphi_o}{2}\right)}{sin\ \alpha}}$$

$$\left(\sqrt{\frac{2}{arc\ 1'}} = 82', 92 \; ; \quad \sqrt{\frac{1}{arc\ 1'}} = 58', 63\right)$$

The minimums of δ_a and δ_b are obtained when $\alpha = 90^\circ$

$\varphi_o = 60^\circ$	$\delta_a\, min = 160.'2$	$\delta_b\, min = 113.'3$
» $= 70^\circ$	» $= 197.'7$	» $= 139.'6$
» $= 80^\circ$	» $= 280.'3$	» $= 198.'2$

We see therefore that the replacement of the circle representing the orthodrome by the straight line *o a* is fully justified to considerable distances from the point *O*.

For the rest we might note that it is precisely in the case where, given the azimuth of the point *E* from the given point *O*, we desire to plot on the chart the accurate rectilinear bearing of the point *E*, that it is necessary to calculate the small angle of the chart *aoe* $= \gamma$ formed by the chord *oe* and the straight line *oa*.

In making this calculation one can employ, at any rate for the first approximation, the approximate value of the distance δ which may be deduced from the estimated position of the point *E*.

Seeing the smallness of the magnitude of ε with respect to δ, we may, by a simple geometric analysis (fig. 4 *bis*) and taking formula (10) into account, write the following formula:

(11) $$\gamma = \frac{\delta}{2} \; cot \left(45^\circ + \frac{\varphi_o}{2}\right) sin \; \alpha$$

in which γ is given in minutes of arc if δ is measured in miles.

It is evident that the angle γ here plays the same role as the so-called *Givry correction* on the MERCATOR chart. In effect, this is the correction which it is necessary to apply to the observed or given azimuth of an object when plotting the bearing of that object *on the chart.* (*)

4. — In the preceding discussion we have omitted all mention of the loxodrome. This omission was made purposely, because it is essential to remember that in the polar regions the ordinary methods of loxodromic navigation based on the use of the magnetic compass must be modified. (**)

We have explained the reasons in a note published in the November 1928 issue of the *Hydrographic Review,* entitled *A new type of polar chart.* This being understood, it appears advisable in any event to conclude the study of the conformal polar projections with a brief study of the lines representing the loxodrome. In polar conformal projections (and in general with all conformal conic projection, of which this is a particular case) the geographic loxodromic curve is represented by a *logarithmic spiral.* In fact, since the projection is conformal the plane line corresponding to the loxodromic curve must intersect the meridians on the chart at the same angle, because the meridians are straight lines passing through the pole and they cannot be intersected at a constant angle except by logarithmic spirals.

Two considerations will suffice as an argument against the use of the loxodromic curve for polar navigation: 1st) In the vicinity of the pole the loxodromic track is quite *uneconomic ;* 2nd) If polar navigation is carried out with the magnetic compass, as ordinarily, the very rapid changes in magnetic variation along the loxodromic curve completely nullify the principal advantage which accrues from the employment of a track of this nature in other latitudes; that is to say, the advantage of the constancy (or rather the very slow and regular change in magnetic variation) of the *magnetic course* along this track.

(*) With regard to the sign to be applied to the *correction* γ, it is necessary to remember that the angle $aoe = \gamma$ (Fig. 4) must always be applied towards the pole p from the direction oa. Thus, in effect, the arc oeq is concave towards the pole.

The GIVRY correction also might be given an analogous form to the formula (11) that is, the

GIVRY correction $= \frac{\delta}{2} \tan \varphi_m \sin \alpha$, in which φ_m is the middle latitude between the two points under consideration.

(**) All navigators and arctic explorers are agreed in their contention that in the Arctic and the Antarctic the magnetic compass is the best and most practical instrument for determining direction.

5. — In our opinion, fig. 5 shows the limits which should be adopted in the construction of a general chart of the arctic and the antarctic regions, and it is not necessary to add that these limits are suggested by the particular configuration of the two basins.

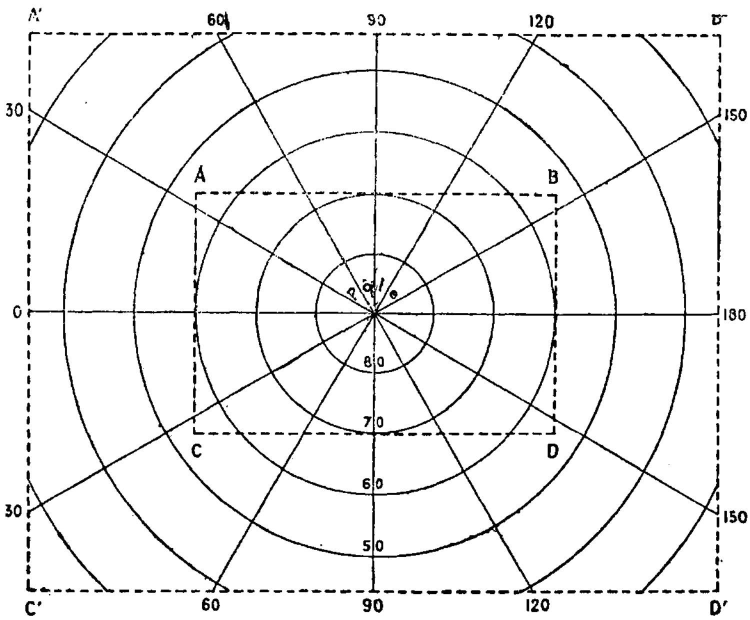


Fig. 5.

The small rectangle *A B C D* limits the Arctic chart, and the large rectangle *A' B' C' D'* shows the limits of the Antarctic chart. In both cases the centre of the chart coincides with the pole and the long sides are parallel to the prime meridian (Greenwich). In the Arctic chart the long sides and the short sides are respectively tangent to the parallels 70° and 60°. In the chart of the Antarctic the long sides and the short sides are respectively tangent to the parallels 45° and 35°.

The large expanse of the chart for the Antarctic is demanded by convenience (one might also say by necessity) in order to represent the Southern Ocean in its entirety and to include the southern extremities of all the adjacent continents (South America, Africa and Australia). It is only by this means that a clear representation of the Antarctic and its relations with respect to position and size to the neighbouring regions may be obtained. This relationship is not at all clearly shown on the ordinary maps of the world constructed on cylindrical projections.

II

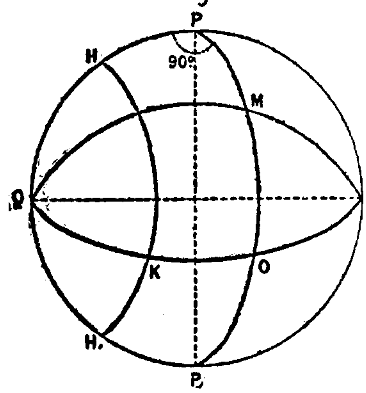
THE CONFORMAL INVERSE CYLINDRICAL PROJECTION (or THE INVERSE MERCATOR PROJECTION) (*)

1. — If we agree to the assumption of a *spherical* shape for the earth, we may then represent its surface as a MERCATOR projection by considering any desired great circle as the equator in place of the true equator of the earth.

If in place of the equator we take one of the *geographic meridians*, we then obtain the *inverse cylindrical projection*, which we shall discuss in this second part. (**)

In other words, this is the conformal projection of the earth, *assumed to be spherical*, obtained by the development of a cylinder placed in the *inverse position*, *i. e.* circumscribed about the sphere on a geographic meridian.

Fig.6



In fig. 6, $P\ P_1$ are the geographic poles, $Q\ O\ Q_1$ represents the geographic equator, POP_1 represents the geographic meridian chosen as the new equator of the sphere, which we shall designate as the *fictitious equator*. In the same manner the poles QQ_1, the meridians (such as QMQ_1) and the parallels (such

(*) See : *Hydrographic Review*, Vol. V, N° 2, page 51 : *A new type of Polar Chart.* This article was communicated to the International Society for the Exploration of the Arctic Regions by means of Airships (Aeroarctic) of Berlin, which replied that after having taken note with lively interest of the proposal of a new type of polar chart, its Directing Committee considered that the problem merited serious consideration in connection with the work of exploration undertaken by the Society and especially for the solution of navigational problems.

(**) On the other hand we call *transverse* cylindrical projections those projections in which the equator chosen for the projections coincides neither with the true equator nor with any of the geographic meridians.

LAMBERT was the first to suggest such inversion of the cylindrical projection.

as HKH_1) of the new system of orthogonal spherical coordinates, of which the fictitious equator is the fundamental reference circle, are designated as *fictitious*. Every point on the spherical surface will therefore be determined by its *fictitious* longitude and its *fictitious* latitude.

The geographic meridian $PQP_1\ Q_1$ at 90° from the geographic meridian $P\,O\,P'$ which we have chosen as the fictitious equator, is also a fictitious meridian; it is represented in the inverse projection (fig. 7) by the straight line YY perpendicular to the straight line XX the ıepresentation of the fictitious equator. We shall call the straight line YY the *principal meridian* of the chart.

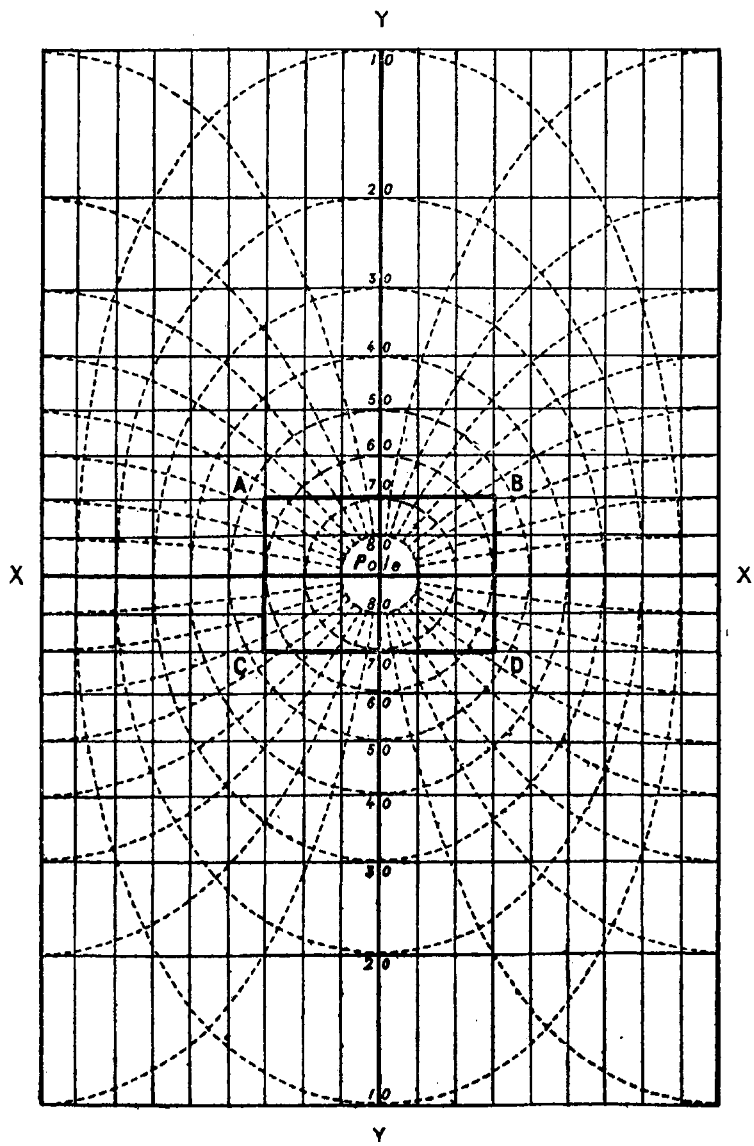


Fig. 7

In fig. 7, the system of rectangles drawn in full is the representation of the MERCATOR projection composed of the *fictitious* meridians and parallels.

On the other hand the system of *geographic* meridians and parallels is represented by dotted lines. The geographic parallels are oval curves having as their axes of symmetry the fictitious equator XX and the principal meridian YY. The geographic meridians have a sinusoidal form with a point of inflexion at the geographic poles.

Nothing is easier than to determine the relation between the geographic coordinates and the fictitious coordinates of a given point of the sphere, and with the aid of these relations, to determine the rectangular coordinates x and y (relative to the axes XX and YY) for the corresponding point on the chart as a function of the geographic coordinates (*). In this manner it is possible to plot the system of meridians and parallels point by point. Table III gives the coordinates x y of the points of the network located at the intersection of the geographic meridians for every 5° and the parallels for every 2°. This suffices for plotting the points on a small scale chart and also for solving the navigational problems described in the following paragraph.

As a unit of measure the minute of the fictitious equator on the chart has been adopted; that is, the same unit of measure as for the meridional parts (fictitious) of the MERCATOR system. The geographic longitude ω, which is employed as one of the arguments of the table, is computed from the geographic meridian $P\,Q\,P_1$ which corresponds to the principal meridian on the chart YY.

Since the curves are symmetrical with respect to the orthogonal axes XX and YY, the table simply gives the values of ω from 0° to 90°. In the construction of the north polar chart (**) and taking into consideration the special configuration of the arctic basin, we propose to assume as the fictitious equator XX the meridian of Greenwich and therefore as the principal meridian of the chart YY the geographic meridian 90° E - 90° W, which passes not far from the north magnetic pole.

2. — The "novelty" of this type of polar chart lies in the fact that we have considered the lines of the fictitious coordinates and also we have actually represented these lines as a MERCATOR projection.

(*) The abscissa x is equal in magnitude to the minutes of the angle σ (less than 90°) given by the formula :

$$\tan \sigma = \cot \varphi \sin \omega$$

The ordinate y is the meridional part for the angle r given by the equation :

$$\sin r = \cos \varphi \cos \omega$$

$$y = \frac{1}{arc\ 1'} \log_e \operatorname{tang} \left(45^\circ + \frac{r}{2} \right)$$

(φ = geographic latitude ; = ω the geographic longitude reckoned from the meridian corresponding to the principal meridian YY on the chart).

(**) It will be recalled that in our note entitled *A new type of polar Chart* we proposed the employment of the inverse projection, *particularly* for the charts of the *arctic regions*.

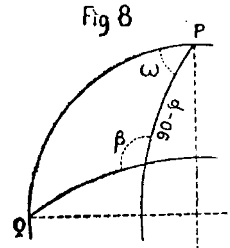
This method of presentation permits us to apply practically identical navigational methods within the polar region, based on the use of the magnetic compass, as those which are ordinarily used in the usual latitudes with hydrographic charts. This result is obtained by referring the courses and the direction of the horizontal component of the terrestrial magnetic field with the system of fictitious meridians (represented on the chart by a group of parallel straight lines) as is done in the system of geographic meridians, that is, by taking as the *pole of reference* the fictitious pole in the hemisphere containing the north magnetic pole.

This fictitious pole we shall designate by the *conventional* name of north pole. We shall also term the courses thus plotted the *fictitious courses*, and the *fictitious magnetic variation* will be understood to mean the angle between the horizontal direction of the earth's magnetic field (the direction in which the north end of the needle points when uninfluenced by local disturbances) and the fictitious meridian.

It will naturally be necessary to plot on the chart the lines of *fictitious equal magnetic variation* which constitute a group of curves having a common origin at the point representing the magnetic pole. (*)

The clear and simple distribution of the above-mentioned lines makes it easy and convenient to convert the courses, *i. e.*, the conversion of the fictitious course to the magnetic course (course reckoned as usual from the direction of north indicated by the needle unaffected by compass deviation). This conversion is accomplished in exactly the same manner as ordinarily in converting *true* courses into magnetic courses.

Fig 8



It is necessary to note that the tracks which are represented as straight lines on the inverse cylindrical projection are spherical spirals of which the pole coincides with the fictitious north pole; and that therefore these are the loxodromic curves of the fictitious system. These loxodromes which are developed in the equatorial region may be considered as having about the same lengths as those of the orthodromic arcs comprised between the same two

(*) At the present moment this pole is located at about $\varphi = 70°51'N$, $L = 96°00'\ W$, that is, at a point *outside* the arctic polar basin.

terminal points. Their length is measured in miles (*) at the scale of the *fictitious latitude* (which we assumed to be plotted on the margin of the chart). In other words the scale of the fictitious latitudes constitutes in the inverse projection the *scale of distances*, in the same manner that the scale of geographic latitudes gives the scale of distances on the ordinary MERCATOR chart.

3. — For converting the known value of the variation referred to the geographic meridian to the corresponding value of the fictitious vaiiation, it naturally suffices to determine the angle β (fig. 8) which the geographic meridian makes with the fictitious meridian at the point in question *B*. This determination is extremely simple.

We have :

$$\cot \beta = - \cot \omega \sin \varphi$$

Table IV gives the values of β as a function of the argument ω (geographic longitude of the point reckoned from the meridian $P\,Q\,P_1$) from 0° to 180° and for every 10 degrees, and of φ (geographic latitude of the point) from 60° to 90° for every two degrees.

(*) For calculating these distances in accordance with the usual practice of navigation and assuming that *the polar arctic basin proper* is comprised within the cap limited by the parallalel 70°, we might choose for instance, the length of the mile equal to one minute of meridian taken at the pole, *i. e.*, 1861.67 metres (International ellipsoïd), in round numbers 1862 m.

CONFORMAL (LAMBERT) POLAR PROJECTION — PROJECTION POLAIRE CONFORME (LAMBERT)

RADII OF THE PARALLELS (r_c) — RAYONS DES PARALLÈLES (r_c)

Scale at the Pole 1 : 1.000.000 Echelle au Pôle

International Ellipsoid $\alpha = 1 : 297$; $a = 6.378.388$ m. Ellipsoïde International

LAT.	r_c	DIFF.	LAT.	r_c	DIFF.	LAT.	r_c	DIFF.
	centimèt.			centimèt.			centimèt.	
30°0'	736,511	2,459	40°0'	595,430	2,251	50°0'	465,141	2,098
10	734,052	2,455	10	593,179	2,247	10	463,043	2,096
20	731,597	2,451	20	590,932	2,245	20	460,947	2,093
30	729,146	2,448	30	588,687	2,242	30	458,854	2,092
40	726,698	2,443	40	586,445	2,239	40	456,762	2,089
50	724,255	2,440	50	584,206	2,236	50	454,673	2,088
31°0'	721,815	2,435	41°0'	581,970	2,233	51°0'	452,585	2,085
10	719,380	2,431	10	579,737	2,230	10	450,500	2,083
20	716,949	2,428	20	577,507	2,227	20	448,417	2,082
30	714,521	2,424	30	575,280	2,225	30	446,335	2,079
40	712,097	2,420	40	573,055	2,221	40	444,256	2,077
50	709,677	2,416	50	570,834	2,219	50	442,179	2,075
32°0'	707,261	2,412	42°0'	568,615	2,216	52°0'	440,104	2,073
10	704,849	2,409	10	566,399	2,213	10	438,031	2,071
20	702,440	2,405	20	564,186	2,211	20	435,960	2,069
30	700,035	2,401	30	561,975	2,208	30	433,891	2,067
40	697,634	2,397	40	559,767	2,205	40	431,824	2,066
50	695,237	2,394	50	557,562	2,202	50	429,758	2,063
33°0'	692,843	2,390	43°0'	555,360	2,199	53°0'	427,695	2,061
10	690,453	2,386	10	553,161	2,197	10	425,634	2,059
20	688,067	2,382	20	550,964	2,195	20	423,575	2,058
30	685,685	2,379	30	548,769	2,191	30	421,517	2,055
40	683,306	2,375	40	546,578	2,189	40	419,462	2,054
50	680,931	2,372	50	544,389	2,186	50	417,408	2,051
34°0'	678,559	2,368	44°0'	542,203	2,184	54°0'	415,357	2,050
10	676,191	2,365	10	540,019	2,181	10	413,307	2,048
20	673,826	2,361	20	537,838	2,178	20	411,259	2,046
30	671,465	2,357	30	535,660	2,176	30	409,213	2,045
40	669,108	2,354	40	533,484	2,173	40	407,168	2,042
50	666,754	2,350	50	531,311	2,171	50	405,126	2,041
35°0'	664,404	2,347	45°0'	529,140	2,168	55°0'	403,085	2,038
10	662,057	2,344	10	526,972	2,166	10	401,047	2,037
20	659,713	2,340	20	524,806	2,163	20	399,010	2,036
30	657,373	2,337	30	522,643	2,161	30	396,974	2,033
40	655,036	2,333	40	520,482	2,158	40	394,941	2,032
50	652,703	2,330	50	518,324	2,155	50	392,909	2,030
36°0'	650,373	2,326	46°0'	516,169	2,153	56°0'	390,879	2,028
10	648,047	2,323	10	514,016	2,151	10	388,851	2,026
20	645,724	2,320	20	511,865	2,148	20	386,825	2,025
30	643,404	2,316	30	509,717	2,146	30	384,800	2,023
40	641,088	2,313	40	507,571	2,144	40	382,777	2,021
50	638,775	2,310	50	505,427	2,141	50	380,756	2,020
37°0'	636,465	2,307	47°0'	503,286	2,139	57°0'	378,736	2,018
10	634,158	2,303	10	501,147	2,136	10	376,718	2,016
20	631,855	2,300	20	499,011	2,134	20	374,702	2,015
30	629,555	2,297	30	496,877	2,132	30	372,687	2,013
40	627,258	2,294	40	494,745	2,129	40	370,674	2,011
50	624,964	2,291	50	492,616	2,127	50	368,663	2,009
38°0'	622,673	2,287	48°0'	490,489	2,125	58°0'	366,654	2,008
10	620,386	2,284	10	488,364	2,122	10	364,646	2,007
20	618,102	2,281	20	486,242	2,120	20	362,639	2,005
30	615,821	2,278	30	484,122	2,118	30	360,634	2,003
40	613,543	2,275	40	482,004	2,116	40	358,631	2,001
50	611,268	2,272	50	479,888	2,113	50	356,630	2,000
39°0'	608,996	2,268	49°0'	477,775	2,111	59°0'	354,630	1,999
10	606,728	2,266	10	475,664	2,109	10	352,631	1,997
20	604,462	2,263	20	473,555	2,107	20	350,634	1,996
30	602,199	2,259	30	471,448	2,104	30	348,638	1,994
40	599,940	2,257	40	469,344	2,102	40	346,644	1,992
50	597,683	2,253	50	467,242	2,101	50	344,652	1,991
40°0'	595,430		50°0'	465,141		60°0'	342,661	

CONFORMAL (LAMBERT) POLAR PROJECTION — PROJECTION POLAIRE CONFORME (LAMBERT)

RADII OF THE PARALLELS (r_c) — RAYONS DES PARALLÈLES (r_c)

Scale at the Pole 1 : 1.000.000, Échelle au Pôle

International Ellipsoid $\alpha = 1 : 297$; $a = 6.378.388$ m. Ellipsoïde International

LAT.	r_c	DIFF.	LAT.	r_c	DIFF.	LAT.	r_c	DIFF.
	centimèt.			centimèt.			centimèt.	
60°0'	342,661	1,989	70°0'	225,604	1,917	80°0'	111,973	1,875
10	340,672	1,988	10	223,687	1,915	10	110,098	1,875
20	338,684	1,987	20	221,772	1,915	20	108,223	1,874
30	336,697	1,985	30	219,857	1,914	30	106,349	1,874
40	334,712	1,983	40	217,943	1,913	40	104,475	1,873
50	332,729	1,982	50	216,030	1,913	50	102,602	1,873
61°0'	330,747	1,981	71°0'	214,117	1,911	81°0'	100,729	1,873
10	328,766	1,979	10	212,206	1,910	10	98,856	1,872
20	326,787	1,978	20	210,296	1,910	20	96,984	1,871
30	324,809	1,976	30	208,386	1,908	30	95,113	1,872
40	322,833	1,975	40	206,478	1,908	40	93,241	1,871
50	320,858	1,974	50	204,570	1,907	50	91,370	1,870
62°0'	318,884	1,972	72°0'	202,663	1,906	82°0'	89,500	1,871
10	316,912	1,971	10	200,757	1,905	10	87,629	1,870
20	314,941	1,969	20	198,852	1,905	20	85,759	1,869
30	312,972	1,968	30	196,947	1,903	30	83,890	1,869
40	311,004	1,967	40	195,044	1,903	40	82,021	1,869
50	309,037	1,966	50	193,141	1,902	50	80,152	1,869
63°0'	307,071	1,964	73°0'	191,239	1,901	83°0'	78,283	1,868
10	305,107	1,963	10	189,338	1,901	10	76,415	1,868
20	303,144	1,961	20	187,437	1,899	20	74,547	1,867
30	301,183	1,960	30	185,538	1,899	30	72,680	1,868
40	299,223	1,959	40	183,639	1,898	40	70,812	1,867
50	297,264	1,958	50	181,741	1,898	50	68,945	1,866
64°0'	295,306	1,956	74°0'	179,843	1,896	84°0'	67,079	1,867
10	293,350	1,955	10	177,947	1,896	10	65,212	1,866
20	291,395	1,954	20	176,051	1,895	20	63,346	1,866
30	289,441	1,953	30	174,156	1,895	30	61,480	1,865
40	287,488	1,951	40	172,261	1,893	40	59,615	1,866
50	285,537	1,950	50	170,368	1,893	50	57,749	1,865
65°0'	283,587	1,949	75°0'	168,475	1,893	85°0'	55,884	1,865
10	281,638	1,948	10	166,582	1,891	10	54,019	1,865
20	279,690	1,947	20	164,691	1,891	20	52,154	1,864
30	277,743	1,945	30	162,800	1,890	30	50,290	1,865
40	275,798	1,944	40	160,910	1,890	40	48,425	1,864
50	273,854	1,943	50	159,020	1,889	50	46,561	1,864
66°0'	271,911	1,942	76°0'	157,131	1,888	86°0'	44,697	1,863
10	269,969	1,941	10	155,243	1,888	10	42,834	1,864
20	268,028	1,940	20	153,355	1,887	20	40,970	1,863
30	266,088	1,938	30	151,468	1,886	30	39,107	1,864
40	264,150	1,937	40	149,582	1,886	40	37,243	1,863
50	262,213	1,937	50	147,696	1,885	50	35,380	1,863
67°0'	260,276	1,935	77°0'	145,811	1,885	87°0'	33,517	1,862
10	258,341	1,934	10	143,926	1,884	10	31,655	1,863
20	256,407	1,933	20	142,042	1,883	20	29,792	1,863
30	254,474	1,932	30	140,159	1,883	30	27,929	1,862
40	252,542	1,930	40	138,276	1,882	40	26,067	1,863
50	250,612	1,930	50	136,394	1,882	50	24,204	1,862
68°0'	248,682	1,929	78°0'	134,512	1,881	88°0'	22,342	1,862
10	246,753	1,928	10	132,631	1,881	10	20,480	1,862
20	244,825	1,926	20	130,750	1,880	20	18,618	1,862
30	242,899	1,926	30	128,870	1,879	30	16,756	1,862
40	240,973	1,924	40	126,991	1,879	40	14,894	1,862
50	239,049	1,924	50	125,112	1,879	50	13,032	1,862
69°0'	237,125	1,923	79°0'	123.233	1,878	89°0'	11,170	1,862
10	235,202	1,921	10	121,355	1,877	10	9,308	1,861
20	233,281	1,921	20	119,478	1,877	20	7,447	1,862
30	231,360	1,919	30	117,601	1,877	30	5.585	1,862
40	229,441	1,919	40	115.724	1,876	40	3,723	1,861
50	227,522	1,918	50	113,848	1,875	50	1,862	1,862
70°0'	225,604		80°0'	111,973			0,000	

NATURAL VALUES AND LOGARITHMS OF THE MODULUS OF THE CONFORMAL (LAMBERT) POLAR PROJECTION

VALEURS NATURELLES ET LOGARITHMES DU MODULE DE LA PROJECTION POLAIRE CONFORME (LAMBERT)

International Ellipsoid — $\alpha = 1 : 297$; a = 6 378 388 m. — Ellipsoïde International.

Lat.	Module	Log.	Lat.	Module	Log.
30°	1,33221	0,124 57269	60°	1,07173	0,030 08610
31°	1,31905	0,120 26206	61°	1,06683	0,028 09375
32°	1,30628	0,116 03787	62°	1,06212	0,026 17166
33°	1,29390	0,111 89931	63°	1,05760	0,024 31953
34°	1,28187	0,107 84554	64°	1,05326	0,022 53712
35°	1,27021	0,103 87580	65°	1,04912	0,020 82410
36°	1,25889	0,099 98931	66°	1,04515	0,019 18025
37°	1,24792	0,096 18535	67°	1,04137	0,017 60531
38°	1,23727	0,092 46321	68°	1,03777	0,016 09907
39°	1,22694	0,088 82217	69°	1,03434	0,014 66129
40°	1,21692	0,085 26159	70°	1,03108	0,013 29174
41°	1,20720	0,081 78082	71°	1,02799	0,011 99027
42°	1,19779	0,078 37924	72°	1,02508	0,010 75663
43°	1,18866	0,075 05621	73°	1,02233	0,009 59069
44°	1,17981	0,071 81117	74°	1,01975	0,008 49224
45°	1,17123	0,068 64355	75°	1,01733	0,007 46116
46°	1,16293	0,065 55280	76°	1,01507	0,006 49729
47°	1,15488	0,062 53838	77°	1,01298	0,005 60048
48°	1,14710	0,059 59977	78°	1,01105	0,004 77060
49°	1,13956	0,056 73652	79°	1,00927	0,004 00754
50°	1,13227	0,053 94809	80°	1,00765	0,003 31121
51°	1,12521	0,051 23407	81°	1,00619	0,002 68148
52°	1,11839	0,048 59395	82°	1,00489	0,002 11827
53°	1,11180	0,046 02736	83°	1,00374	0,001 62152
54°	1,10544	0,043 53383	84°	1,00275	0,001 19114
55°	1,09929	0,041 11293	85°	1,00191	0,000 82709
56°	1,09336	0,038 76448	86°	1,00122	0,000 52927
57°	1,08765	0,036 48790	87°	1,00068	0,000 29770
58°	1,08214	0,034 28285	88°	1,00030	0,000 13230
59°	1,07683	0,032 14903	89°	1,00008	0,000 03308
60°	1,07173	0,030 08610	90°	1,00000	0,000 00000

VALEURS DE *x* ET DE *y*
PROJECTION CYLINDRIQUE INVERSE CONFORME (TERRE SPHÉRIQUE)

VALUES OF *x* AND *y*
CONFORMAL INVERSE CYLINDRICAL PROJECTION (FOR A SPHERE)

ω	0°				5°			
Lat.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	0,00		1888,38	137,22	172,84	13,64	1879,67	136,38
62°	0,00		1751,16	134,69	159,20	13,15	1743,29	133,91
64°	0,00		1616,47	132,41	146,05	12,72	1609,38	131,69
66°	0,00		1484,06	130,38	133,33	12,33	1477,69	129,70
68°	0,00		1353,68	128,54	121,00	11,98	1347,99	127,92
70°	0,00		1225,14	126,92	109,02	11,69	1220,07	126,32
72°	0,00		1098,22	125,49	97,33	11,43	1093,75	124,92
74°	0,00		972,73	124,24	85,90	11,21	968,83	123,69
76°	0,00		848,49	123,17	74,69	11,01	845,12	122 64
78°	0,00		725,32	122,25	63,68	10,85	722,48	121,75
80°	0,00		603,07	121,50	52,83	10,72	600,73	121,02
82°	0,00		481,57	120,91	42,11	10,62	479,71	120,43
84°	0,00		360,66	120,46	31,49	10,54	359,28	120,00
86°	0,00		240,20	120,18	20,95	10,49	239,28	119,71
88°	0,00		120,02	120,02	10,46	10,46	119,57	119,57
90°	0,00		0,00		0,00		0,00	

ω	10°				15°			
Lat.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	343,51	27,00	1853,73	133,89	509,93	39,79	1811,15	129,87
62°	316,51	26,05	1719,84	131,61	470,14	38,46	1681,28	127,87
64°	290,46	25,21	1588,23	129,55	431,68	37,28	1553,41	126,06
66°	265.25	24,46	1458,68	127,70	394,40	36,22	1427,35	124,43
68°	240,79	23,80	1330,98	126,03	358,18	35,29	1302,92	122,95
70°	216,99	23,23	1204,95	124,55	322,89	34,47	1179,97	121,63
72°	193,76	22,73	1080,40	123,23	288,42	33,75	1058,34	120,46
74°	171,03	22,29	957,17	122,09	254,67	33,14	937,88	119,43
76°	148,74	21,91	835,08	121,10	221,53	32,60	818,45	118,54
78°	126,83	21,60	713,98	120,26	188,93	32,15	699,91	117,79
80°	105,23	21,35	593,72	119,56	156,78	31,79	582,12	117,17
82°	83,88	21,14	474,16	119,02	124,99	31,50	464,95	116,67
84°	62,74	21,00	355,14	118,61	93,49	31,28	348,28	116,30
86°	41,74	20,90	236,53	118,33	62,21	31,14	231,98	116,05
88°	20,84	20,84	118.20	118,20	31,07	31,07	115,93	115,93
90°	0,00		0,00		0,00		0,00	

VALUES OF *x* AND *y*
CONFORMAL INVERSE CYLINDRICAL PROJECTION (FOR A SPHERE)

VALEURS DE *x* ET DE *y*
PROJECTION CYLINDRIQUE INVERSE CONFORME (TERRE SPHÉRIQUE)

ω	20°				25°			
LAT.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	670,21	51,80	1752,81	124,46	822,73	62,86	1679,90	117,89
62°	618,41	50,18	1628,35	122,83	759,87	61,05	1562,01	116,65
64°	568,23	48,74	1505,52	121,34	698,82	59,44	1445,36	115,52
66°	519,49	47,44	1384,18	119,98	639,38	58,00	1329,84	114,47
68°	472,05	46,29	1264,20	118,74	581,38	56,70	1215,37	113,51
70°	425,76	45,29	1145,46	117,63	524,68	55,55	1101,86	112,64
72°	380,47	44,40	1027,83	116,65	469,13	54,55	989,22	111,86
74°	336,07	43,62	911,18	115,77	414,58	53,67	877,36	111,18
76°	292,45	42,97	795,41	115,02	360,91	52,92	766,18	110,56
78°	249,48	42,41	680,39	114,37	307,99	52,28	655,62	110,06
80°	207,07	41,95	566,02	113,84	255,71	51,76	545,56	109,62
82°	165,12	41,59	452,18	113,42	203,95	51,35	435,94	109,29
84°	123,53	41,33	338,76	113,09	152,60	51,04	326,65	109,02
86°	82,20	41,14	225,67	112,89	101,56	50,83	217,63	108,86
88°	41,06	41,06	112,78	112,78	50,73	50,73	108,77	108,77
90°	0,00		0,00		0,00		0,00	

ω	30°				35°			
LAT.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	966,13	72,85	1593,73	110,34	1099,35	81,72	1495,77	102,07
62°	893,28	70,98	1483,39	109,53	1017,63	79,89	1393,70	101,64
64°	822,30	69,29	1373,86	108,76	937,74	78.21	1292.06	101,22
66°	753,01	67,76	1265,10	108,03	859,53	76,69	1190,84	100,82
68°	685,25	66,40	1157,07	107,36	782,84	75,32	1090,02	100,42
70°	618,85	65,19	1049,71	106,75	707,52	74,11	989,60	100,06
72°	553,66	64,12	942,96	106,20	633,41	73,02	889,54	99,73
74°	489,54	63,18	836,76	105,69	560,39	72,07	789,81	99,41
76°	426,36	62,37	731,07	105,26	488,32	71,26	690,40	99,15
78°	363,99	61,69	625,81	104,88	417,06	70,56	591,25	98,91
80°	302,30	61,12	520,93	104,56	346,50	69,98	492,34	98,71
82°	241,18	60,68	416,37	104,32	276,52	69,53	393,63	98,55
84°	180,50	60,35	312,05	104,12	206,99	69,18	295,08	98,43
86°	120,15	60,13	207,93	104,00	137,81	68,96	196,65	98,35
88°	60,02	60,02	103,93	103,93	68,85	68,85	98.30	98,30
90°	0,00		0,00		0,00		0,00	

VALUES OF *x* AND *y*
CONFORMAL INVERSE CYLINDRICAL PROJECTION (FOR A SPHERE)

VALEURS DE *x* ET DE *y*
PROJECTION CYLINDRIQUE INVERSE CONFORME (TERRE SPHÉRIQUE)

ω	40°				45°			
LAT.	*x*		*y*		*x*		*y*	
		diff.		diff.		diff.		diff.
60°	1221,63	89,48	1387,46	93,26	1332,46	96,16	1270,24	84,09
62°	1132,15	87,75	1294,20	93,17	1236,30	94,61	1186,15	84,29
64°	1044,40	86,17	1201,03	93,07	1141,69	93,17	1101,86	84,44
66°	958,23	84,73	1107,96	92,94	1048,52	91,87	1017,42	84,56
68°	873,50	83,43	1015,02	92,82	956,65	90,69	932,86	84,66
70°	790,07	82,26	922,20	92,68	865,96	89,60	848,20	84,72
72°	707,81	81,21	829,52	92,55	776,36	88,65	763,48	84,76
74°	626,60	80,29	736,97	92,43	687,71	87,79	678,72	84,80
76°	546,31	79,50	644,54	92,31	599.92	87,06	593,92	84,83
78°	466,81	78,83	552,23	92,21	512,86	86,44	509,09	84,84
80°	387,98	78,26	460,02	92,11	426,42	85,90	424,25	84,84
82°	309,72	77,82	367,91	92,05	340,52	85,50	339,41	84,85
84°	231,90	77,48	275,86	91,98	255,02	85,18	254,56	84,85
86°	154,42	77,27	183,88	91,95	169,84	84,97	169,71	84,86
88°	77,15	77,15	91,93	91,93	84,87	84,87	84,85	84,85
90°	0,00		0,00		0,00		0,00	

ω	50°				55°			
LAT.	*x*		*y*		*x*		*y*	
		diff.		diff.		diff.		diff.
60°	1431,52	101,82	1145,46	74,72	1518,67	106,54	1014,35	65,24
62°	1329,70	100,48	1070,74	75,12	1412,13	105,45	949,11	65,79
64°	1229,22	99,26	995,62	75,48	1306,68	104,43	883,32	66,27
66°	1129,96	98,12	920,14	75,78	1202,25	103,50	817,05	66,70
68°	1031,84	97,08	844,36	76,04	1098,75	102,64	750,35	67,09
70°	934,76	96,14	768,32	76,27	996,11	101,86	683,26	67,43
72°	838,62	95,29	692,05	76,47	894,25	101,14	615,83	67,73
74°	743,33	94,55	615,58	76,62	793,11	100,52	548,10	67,98
76°	648,78	93,89	538,96	76,76	692,59	99,96	480,12	68,20
78°	554,89	93,33	462,20	76,88	592,63	99,50	411,92	68,38
80°	461,56	92,87	385,32	76,96	493,13	99,10	343,54	68,53
82°	368,69	92,50	308,36	77,03	394,03	98,78	275,01	68,65
84°	276,19	92,22	231,33	77,08	295,25	98,55	206,36	68,74
86°	183,97	92,03	154,25	77,12	196,70	98,39	137,62	68,80
88°	91,94	91,94	77,13	77,13	98,31	98,31	68,82	68,82
90°	0,00		0,00		0,00		0,00	

VALUES OF x AND y
CONFORMAL INVERSE CYLINDRICAL PROJECTION (FOR A SPHERE)

VALEURS DE x ET DE y
PROJECTION CYLINDRIQUE INVERSE CONFORME (TERRE SPHÉRIQUE)

ω	60°				65°			
LAT.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	1593,90	110,41	878,05	55,76	1657,27	113,53	737,54	46,30
62°	1483,49	109,57	822,29	56,36	1543,74	112,91	691,24	46,91
64°	1373,92	108,78	765,93	56,92	1430,83	112,34	644,33	47,48
66°	1265,14	108,05	709,01	57,42	1318,49	111,81	596,85	47,98
68°	1157,09	107,37	651,59	57,87	1206,68	111,31	548,87	48,46
70°	1049,72	106,76	593,72	58,27	1095,37	110,86	500,41	48,87
72°	942,96	106,19	535,45	58,63	984,51	110,44	451,54	49,25
74°	836,77	105,70	476,82	58,94	874,07	110,08	402,29	49,58
76°	731,07	105,26	417,88	59,21	763,99	109,75	352,71	49,86
78°	625,81	104,88	358,67	59,44	654,24	109,47	302,85	50,11
80°	520,93	104,56	299,23	59,62	544,77	109,24	252,74	50,31
82°	416,37	104,32	239,61	59,77	435,53	109,05	202,43	50,47
84°	312,05	104,12	179,84	59,89	326,48	108,90	151,96	50,58
86°	207,93	104,00	119,95	59,96	217,58	108,81	101,38	50,67
88°	103,93	103,93	59,99	59,99	108,77	108,77	50,71	50,71
90°	0,00		0,00		0,00		0,00	

ω	70°				75°			
LAT.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	1708,87	115,95	593,72	36,91	1748,85	117,77	447,38	27,59
62°	1592,92	115,55	556,81	37,47	1631,08	117,53	419,79	28,06
64°	1477,37	115,17	519,34	37,99	1513,55	117,32	391,73	28,49
66°	1362,20	114,81	481,35	38,46	1396,23	117,11	363,24	28,88
68°	1247,39	114,49	442,89	38,90	1279,12	116,92	334,36	29,25
70°	1132,90	114,18	403,99	39,29	1162,20	116,74	305,11	29,55
72°	1018,72	113,90	364,70	39,65	1045,46	116,58	275,56	29,89
74°	904,82	113,65	325,05	39,95	928,88	116,43	245,67	30,14
76°	791,17	113,44	285,10	40,23	812,45	116,31	215,53	30,36
78°	677,73	113,25	244,87	40,46	696,14	116,19	185,17	30,56
80°	564,48	113,09	204,41	40,65	579,95	116,10	154,61	30,73
82°	451,39	112,96	163,76	40,81	463,85	116,03	123,88	30,85
84°	338,43	112,86	122,95	40,92	347,82	115,98	93,03	30,96
86°	225,57	112,80	82,03	40,99	231,84	115,93	62,07	31,02
88°	112,77	112,77	41,04	41,04	115,91	115,91	31,05	31,05
90°	0,00		0,00		0 00		0,00	

VALUES OF x AND y CONFORMAL INVERSE CYLINDRICAL PROJECTION (FOR A SPHERE)

VALEURS DE x ET DE y PROJECTION CYLINDRIQUE INVERSE CONFORME (TERRE SPHÉRIQUE)

ω	80°				85°			
Lat.	x		y		x		y	
		diff.		diff.		diff.		diff.
60°	1777,30	119,02	299,23	18,35	1794,33	119,76	149,90	9,16
62°	1658,28	118,92	280,88	18,68	1674,57	119,73	140,74	9,33
64°	1539,36	118,81	262,20	18,99	1554,84	119,70	131,41	9,49
66°	1420,55	118,73	243,21	19,27	1435,14	119,69	121,92	9,64
68°	1301,82	118,64	223,94	19,53	1315,45	119,66	112,28	9,77
70°	1183,18	118,55	204,41	19,76	1195,79	119,64	102,51	9,90
72°	1064,63	118,48	184,65	19,98	1076,15	119,62	92,61	10,01
74°	946,15	118,42	164,67	20,17	956,53	119,60	82,60	10,11
76°	827,73	118,36	144,50	20,33	836,93	119,59	72,49	10,19
78°	709,37	118,31	124,17	20,48	717,34	119,58	62,30	10,27
80°	591,06	118,26	103 69	20,59	597,76	119,57	52,03	10,33
82°	472,80	118,23	83,10	20,69	478,19	119,55	41,70	10,38
84°	354,57	118,20	62,41	20,77	358,64	119,55	31,32	10,42
86°	236,37	118,19	41,64	20,81	239,09	119,55	20,90	10,44
88°	118,18	118,18	20,83	20,83	119,54	119,54	10,46	10,46
90°	0,00		0,00		0,00		0,00	

ω	90°			
Lat.	x		y	
		diff.		diff.
60°	1800,00	120,00	0,00	
62°	1680,00	120,00	0,00	
64°	1560,00	120,00	0,00	
66°	1440,00	120,00	0,00	
68°	1320,00	120,00	0,00	
70°	1200,00	120,00	0,00	
72°	1080,00	120,00	0,00	
74°	960,00	120,00	0,00	
76°	840,00	120,00	0,00	
78°	720,00	120 00	0,00	
80°	600,00	120,00	0,00	
82°	480,00	120,00	0,00	
84°	360,00	120,00	0,00	
86°	240,00	120,00	0,00	
88°	120,00	120,00	0,00	
90°	0,00		0,00	

Lat.	ω 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
60°	180° 0',0	168°29',5	157°12',2	146°18',6	135°54',3	126° 0',3	116°33',9	107°29',7	98°40',9	90°
62°		42,4	35,8	49,2	136°27,5	32,1	117° 0,7	48,9	50,9	
64°		54,0	57,2	147°17,1	58,0	127° 1,4	25,5	108° 6,9	99° 0,3	
66°		169° 4,5	158°16,6	42,4	137°25,9	28,3	48,5	23,5	9,0	
68°		13,9	34,0	148° 5,4	51,3	53,0	118° 9,6	38,9	17,1	
70°		22,3	49,6	26,0	138°14,2	128°15,3	28,9	52,9	24,5	
72°		29,8	159° 3,5	44,4	34,7	35,5	46,3	109° 5,6	31,2	
74°		36,3	15,7	149° 0,6	52,9	53,4	119° 1,8	17,0	37,2	
76°		42,0	26,3	14,8	139° 8,8	129° 9,1	15,4	27,1	42,5	
78°		46,9	35,4	26,9	22,5	22,7	27,3	35,8	47,1	
80°		50,9	43,0	37,1	34,0	34,1	37,3	43,2	51,1	
82°		54,2	49,2	45,4	43,4	43,5	45,5	49,2	54,3	
84°		56,8	53,9	51,8	50,7	50,7	51,8	53,9	56,8	
86°		58,6	57,3	56,4	55,9	55,9	56,4	57,3	58,6	
88°		59,7	59,3	59,1	59,0	59,0	59,1	59,3	59,6	
90°	180° 0',0	170° 0,0	160° 0,0	150° 0,0	140° 0,0	130° 0,0	120° 0,0	110° 0,0	100° 0,0	90°

Lat.	ω 90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
60°	90°	81°19',1	72°32',3	63°26',1	53°59',7	44° 5',7	33°41',4	22°47',8	11°30',5	0°
62°		9,1	11,1	62°59,3	27,9	43°32,5	10,8	24,2	17,6	
64°		80°59,7	71°53,1	34,5	52°58,6	43° 2,0	32°42,9	2,8	6,0	
66°		51,0	36,5	11,5	31,7	42°34,1	32°17,6	21°43,4	10°55,5	
68°		42,9	21,1	61°50,4	7,0	42° 8,7	31°54,6	26,0	46,1	
70°		35,5	7,1	31,1	51°44,7	41°45,8	34,0	10,4	37,7	
72°		28,8	70°54,4	13,7	24,5	25,3	31°15,6	20°56,5	30,2	
74°		22,8	43,0	60°58,2	6,6	41° 7,1	30°59,4	44,3	23,7	
76°		17,5	32,9	44,6	50°50,9	40°51,2	45,2	33,7	18,0	
78°		12,9	24,2	32,7	37,3	37,5	33,1	24,6	13,1	
80°		8,9	16,8	22,7	25,9	26,0	22,9	17,0	9,1	
82°		5,7	10,8	14,5	16,5	16,6	14,6	10,8	5,8	
84°		3,2	6,1	8,2	9,3	9,3	8,2	6,1	3,2	
86°		1,4	2,7	3,6	4,1	4,1	3,6	2,7	1,4	
88°		0,4	0,7	0,9	1,0	1,0	0,9	0,7	0,3	
90°	90°	80° 0,0	70° 0,0	60° 0,0	50° 0,0	40° 0,0	30° 0,0	20° 0,0	10° 0,0	0°