VARIOUS INTERPRETATIONS OF A BALLOT.

by

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Volume XLVIII, 1928, of the *Proceedings of the Royal Society of Edinburgh* contains, on pages 140 to 160, an interesting communication from D. M. Y. SOMMERVILLE on various methods of classing the candidates in a ballot by preferential voting. As the Statutes prescribe such a ballot for the election of the Directing Committee of the International Hydrographic Bureau it is not without interest, perhaps, to compare the system used with various others and with the results of applying SOMMERVILLE's ideas.

I. — Let it be assumed that each elector inserts on his voting-paper the names of *all* the candidates in order of preference (should he have several votes he can be taken to represent a number of electors equal to the number of his votes). Then, having the complete expression of the preferences of the voters, the problem is how best to take these into account.

This problem has been examined by BORDA ; by E.-J. NANSON : "Methods of Election" *Trans. R. Soc. Victoria,* vol. XIX (1882) pages 197 to 240 ; by G. HOGBEN : "Preferential voting in single member constituencies, with special reference to the counting of votes", *Trans. N. Z. Inst.*, vol. XLVI (1913), pages 304 to 308 ; by D. M. Y. SOMMERVILLE : "A problem of voting", *Proc. Edinburgh Math. Soc.* vol. XXVIII (1910) pages 23 & 24 and "Certain hyperspatial partitionings connected with preferential voting", *Proc. London Math. Soc.* ; by J. M. BALDW N : "The Technique of the Nanson preferential majority system of election" *Proc. R. Soc. Victoria* (*N. S.*) Vol. XXXIX (1926) pages 42 to 52.

Let the complete table showing the preferences expressed by the electors be drawn up as follows :-

Count the number of times that candidate A is preferred to candidate B, then the number where B is preferred to A. The difference gives a positive or negative number, N_{AB}, which clearly shows the opinion of the voters as to A & B. If N_{AB} is positive then it shows that A is preferred to B by a majority equal to N_{AB}, but if N_{AB} is negative the number N_{BA} (which is equal to $-N_{AB}$) is positive and shows that B is preferred more frequently than A.

With these numbers the following double-entry table can be drawn up :-

	A	B	C	D	E	F	G
A		$-N_{AB}$	$-N_{AC}$	$-N_{AD}$	$-N_{AE}$	$-N_{AF}$	$-N_{AG}$
B	N_{AB}		$-N_{BC}$	$-N_{BD}$	$-N_{BE}$	$-N_{BF}$	$-N_{BG}$
C	N_{AC}	N_{BC}		$-N_{CD}$	$-N_{CE}$	$-N_{CF}$	$-N_{CG}$
D	N_{AD}	N_{BD}	N_{CD}		$-N_{DE}$	$-N_{DF}$	$-N_{DG}$
E	N_{AE}	N_{BE}	N_{CE}	N_{DE}		$-N_{EF}$	$-N_{EG}$
F	N_{AF}	N_{BF}	N_{CF}	N_{DF}	N_{EF}		$-N_{FG}$
G	N_{AG}	N_{BG}	N_{CG}	N_{DG}	N_{EG}	N_{FG}	

II. — The simplest case is that in which it is possible to class the candidates in such order that the N numbers for each candidate are all positive with reference to those below him. If the above table be constructed in such order all the N numbers below the diagonal will be positive. The ballot is then said to be *consistent.*

Should such order be found, there can be but one, and it appears indisputable that it clearly expresses the wish of the electors.

For example, take the case of 3 candidates.

Let the order	A B C	be recorded on	22 voting-papers
	A C B	—	1 voting-paper
	B A C	—	1 —
	B C A	—	16 voting-papers
	C A B	—	2 —
	C B A	—	2 —

The table will be :

	A	B	C
A		-6	-4
B	6		-34
C	4	34	

The ballot is consistent and, since A is preferred to B & C and B is preferred to C, the order of preference is : A B C.

Take another case :—

Let the order	A B C	be recorded on	7 voting-papers
	A C B	—	3 —
	B A C	—	22 —
	B C A	—	2 —
	C A B	—	24 —
	C B A	—	3 —

Then the table is :—

	A	B	C
A		−7	−3
B	7		−1
C	3	1	

The ballot is still consistent and the order of preference is clearly A B C

The method employed by the *International Hydrographic Bureau* was referred to by both BORDA & NANSON ; it consists in numbering the names of candidates on each voting-paper in the order in which they appear and in adding up the respective numbers for each candidate. Suppose that this numbering commences with the last name on the voting-paper, then the candidate who is preferred is he with the highest total (Of course the numbering might be reversed and then the smallest total would indicate the preferred candidate ; it comes to the same thing).

In the first example given above, A would have a total of 93 points,
B — — 102 — ,
C — — 69 — ,

and the accepted order would be B A C, though A was preferred to B six times more than B to A.

In the second example, A would have a total of 127 points,
B — — 119 — ,
C — — 120 — ,

and the order accepted would be A C B, although B was preferred once in excess over C. Therefore this method does not appear to be perfect.

A second method is based on the consideration of the sum of the N numbers of preferences obtained by each candidate; *i. e.* the algebraical sum (Σ N) of the figures in the vertical columns of the tables given above. This method may be subdivided into several variants :—

(*a*) The first place is given to the candidate for whom Σ N is largest, then the table is reconstructed for the remaining candidates and again the highest Σ N is extracted and so on.

In the first example, B would come first with a total of 28, then A being preferred to C the order :—

B A C

would be adopted.

In the second example A would take the first place and the order would be :—

A B C

(*b*) Here the candidate with the lowest ΣN is extracted and placed last and the table is reconstructed for the others and so on.

In the first example C would be placed last and the adopted order would be :

A B C

In the second example B would be the last and the order would be :—

A C B

(*c*) In this method all condidates for whom ΣN is $< O$ are extracted from the table which is then reconstructed for the remainder. The same operation is carried out until but a single candidate remains ; he is given first place. The whole procedure is repeated to find the second and so on until all candidates have been placed.

This would lead, in the first example, to the order A B C,
and in the second example to A C B.

A third method consists in laying down simply that the order which appears most frequently on the voting-papers shall be that which is to be adopted. This would give for the first example A B C,
and for the second example C A B.

It is obvious, therefore, that these various methods may lead to very different results. The desire of the electors appears undoubtedly to support the order A B C in both of the examples given, yet the methods used above gave for the first example B A C, B A C, A B C, A B C, A B C,
and for the second example A C B, A B C, A C B, A C B, C A B.

None of them appears therefore to lead to a true interpretation of the opinion of the electors.

III. — More often than not the ballot is *inconsistent, i. e.* it is impossible to class the candidates in such order that each one of them is preferred to those below him.

Again assume 3 candidates and the following ballot :—

the order	A B C	appears on	4	voting-papers
—	A C B	—	5	—
—	B A C	—	3	—
—	B C A	—	3	—
—	C A B	—	1	voting-paper
—	C B A	—	5	voting-papers

The table works out thus :—

	A	B	C
A		1	-3
B	-1		1
C	3	-1	
ΣN	2	0	-2

Hence B is preferred to A who is preferred to C; but C is preferred to B, therefore the order is :—

B A C B

The ballot has given no indication as to which candidate should be placed first. It may be said that a consistent ballot classifies the candidates on a straight line whereas an inconsistent ballot (only in the case of 3 candidates) classifies them in a circle.

The system used by the International Hydrographic Bureau would give.:—

A	43 points
B	42 —
C	41 —

and thus the order declared would be A B C, although obviously B was preferred to A, and C to B.

The variants of the second method would give the orders :—

(*a*)	A C B
(*b*)	B A C
(*c*)	B A C

The third method would give A C B or C B A quite impartially.

SOMMERVILLE suggests, for such cases, the adoption of the following rules, the application of which appears to be quite legitimate :—

— *Should all the* N *numbers of a candidate be positive, he should be placed first.*

— *Should all the* N *numbers of a candidate be negative, he should be placed last.*

These rules are applicable only when there are more than 3 candidates, for, if there are but three, it is obvious that the ballot must be consistent if all the N numbers of one of the candidates are of the same sign.

In the case of 4 candidates, should two of them register all their N numbers of the same sign (+ or —), the ballot will be consistent also.

Let the following be the table for four candidates, A, B, C & D.

	A	B	C	D
A		-1	-3	-5
B	1		7	-3
C	3	-7		1
D	5	3	-1	

Evidently A should be classed first since he was preferred to all others, but the ballot is inconsistent for the others, for B is preferred to D who is preferred to C, who in turn is preferred to B.

SOMMERVILLE *suggests that, in an inconsistent ballot for* 3 *candidates, the preference which is indicated, among those which are incompatible, by the least number shall be eliminated.*

In this example :—

B	is preferred to	D	by a majority of	3
D	—	C	—	1
C	—	B	—	7

and, if the indication D preferred to C be eliminated, the order C B D remains and the four candidates will be classed in the order A C B D

The result of the application of this rule will be examined in the two following examples :—

			Example 1	*Example* 2
The order	A B C	is preferred by	14	38 votes
	A C B	—	5	5 —
	B A C	—	1	1 —
	B C A	—	11	11 —
	C A B	—	4	28 —
	C B A	—	9	9 —

The tables will be :—

	A	B	C
A		−2	4
B	2		−8
C	−4	8	
ΣN	−2	+6	−4

	A	B	C
A		−50	4
B	50		−8
C	−4	8	
ΣN	46	−42	−4

and the ballots are inconsistent.

				Example 1	*Example* 2
A	is preferred to	B	by a majority of	2 votes	50 votes
B	—	C	—	8 —	8 —
C	—	A	—	4 —	4 —

In the first example the preference of A over B will be eliminated, and the order is B C A.

In the second example the preference of C over A must be climinated, and the resulting order is A B C.

The system used by the International Hydrographic Bureau would give :—

	Example 1	*Example* 2
to A	87 points	207 points
B	91 —	163 —
C	86 —	182 —
and the orders	B A C	A C B

The variants of the second method would give :—

(*a*)	B C A	and	A B C,
(*b*)	A B C	—	C A B,
(*c*)	B C A	—	A B C,
and the third method,	A B C	—	A B C.

Variant (*b*) is obviously defective in this case whereas variants (*a*) and (*c*) give orders which conform to the rule laid down by the author.

The third method must certainly be rejected.

The above rules do not always suffice to determine the order when there are more than three candidates.

To get over this another rule, which takes into consideration the *weight* of the order of the candidates, is introduced. The weight is the figure of the algebraical sum of the N numbers of all the candidates with reference to those which come below them on the list.

Then the order to be adopted is that with the highest weight.

This rule is an extension of that for an inconsistent ballot for three candidates. and is consistent with the three rules previously laid down.

It is here applied to the last two examples :—

The weights are :—	*Example* 1	*Example* 2
A B C	+ 6	+ 54
A C B	— 10	+ 38
B A C	+ 2	— 46
B C A	+ 10	— 38
C A B	— 2	+ 46
C B A	— 6	— 54
orders adopted	B C A	A B C

which are the solutions already found.

IV. — It has been assumed in the foregoing that the electors insert the names of *all* the candidates on their voting-papers, but this is not done in the election of the Members of the Directing Committee of the International Hydrographic Bureau where the electors enter but three names of candidates on the papers (*i. e.* the number of places to be filled). It can easily be demonstrated that this method has disadvantages.

One of them will be grasped immediately if it be assumed that some of the electors may not have entered on their voting-papers the names of any of the candidates who are elected. Thus they will not have been able to show the order of the elected candidates which they preferred, and their votes are of no effect. Nevertheless, in all probability, they had an opinion on the subject.

Assume that there were six candidates and that three names only appear on the voting-papers. But let the papers be completed with the names of the remaining candidates as they might have been, if the electors had expressed their full preference :—

7	voting-papers	showed	A B D	C E F
5	—	—	A B E	C D F
8	—	—	A B F	C E D

The table would then be :—

	A	B	C	D	E	F
A		-20	-20	-20	-20	-20
B	20		-20	-20	-20	-20
C	20	20		-6	-10	-4
D	20	20	6		6	-4
E	20	20	10	-6		6
F	20	20	4	4	-6	

Candidates A B C, all of whose N numbers below the diagonal are positive would, obviously, be classed on that order.

The remaining table for D E F is inconsistent, for

F	is preferred to	E	by a majority of	6
E	—	D	—	6
D	—	F	—	4

Eliminating the preference of D over F, as having the smallest majority, we get the order :—

A B C F E D

Under the system employed by the International Hydrographic Bureau :—

A	would have had	60	points
B	—	40	—
C	—	0	—
D	—	7	—
E	—	5	—
F	—	8	—

and the order declared would have been A B F D E C.

Candidate C would have been declared to be the sixth, whereas he should have been third, yet his name would not have appeared on any voting-paper if three names only had been entered thereon.

V. — It is clear, therefore, that very different results may be obtained according to the method employed in counting the votes. That proposed by SOMMERVILLE is the least arbitrary possible, it rests on simple principles which seem to be quite sound and thus appears to lead to the closest expression of the wishes of the electors, even when these wishes are somewhat obscure and seem to show incompatibilities. The application of the method might be laborious in practice if there were many candidates, but is fairly simple when they are but few.

Anyway, should these rules be incapable of determining the order of all the candidates, it would be best, after having classed all those for whom this is possible, to recommence a doubtful ballot rather than to strive by adopting arbitrary rules to find a meaning therein which may not, in reality, arise therefrom.