RESEARCHES ON THE DIP OF THE SEA HORIZON

by

H. C. FREIESLEBEN, OF THE DEUTSCHE SEEWARTE.

(Extract from *Der Seewart*, Hamburg, Heft 2, 1934, p. 60).*

The most accurate method of taking a fix at sea is still to measure the altitude of stars above the sea horizon. In this operation, the principal source of error is the reduction of the apparent horizon to the true horizon, i.e. the dip. This value, generally found from a nautical table, is actually subject to considerable variation. Yet one cannot at present escape from the apparent horizon as datum line, for in spite of numerous attempts there does not yet exist any device enabling the true altitude of heavenly bodies to be measured with certainty from a mobile observation spot. Such observations can always be made ashore, because in that case gravity, acting through spirit levels, permits of direct measurements below the true horizon. At sea it has been impossible hitherto to effect by means of gravity (bubble sextant) the same observation above the apparent horizon. And so any works which study the dip of the horizon as a function of the various causes discoverable with the means available on board ship, deserve attention. As a result of the contradictory results obtained by different observers, Rear-Admiral (retd.) Dr. CONRAD has recently undertaken studies of this kind on a very large scale. It will therefore be of interest to the navigator to give here an outline of the whole question.

The rays of light which transmit the image of the horizon to the observer are not, unhappily, rectilinear, for otherwise the dip of the horizon would depend only, in a simple ratio, on the observer's height of eye :

$$(1) \quad Kt = 2R\sqrt{Ah} = 1'.93\sqrt{Ah}.$$

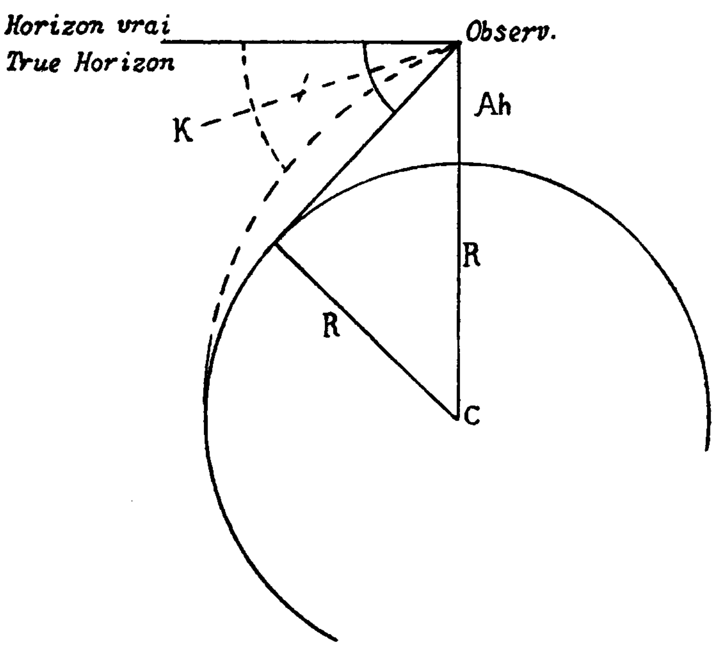

Fig. 1

(*) *See also, on this subject, the articles published in the* Hydrographic Review, *Vol. VIII, No.* 1, *May* 1931, *pp.* 101, 112, 119, 209 *and* 221.

The ratio can be deduced from Fig. 1 (full lines) by simple geometric considerations which may be found in any treatise on navigation. *Kt* is the dip of the horizon, *R* the radius of the earth, *Ah* the height of eye of the observer.

But the rays from the apparent horizon, like any other rays passing through the atmospheric blanket of the earth, undergo certain deviations owing to refraction. Consequently the angle of dip of the horizon appears smaller, for example, when, as shown by the broken lines of Fig. 1, the ray from the apparent horizon describes a path curved towards the earth; in other words, it appears to the observer to be coming from the direction of *K*. The reason for the refraction of the rays is of course the difference in the densities of the individual atmospheric layers. Consequently the curved path of the ray from the apparent horizon will depend, mathematically speaking, on the density of the atmospheric layers which it has to cross in its journey to the observer's eye. In practice this density depends only on the differences of temperature of the atmospheric layers met with. The results of all researches on the dip of the horizon agree that the other quantities which affect the density of the air, namely temperature and humidity, have no influence or only a very slight influence in this respect, so we shall ignore them in the following discussion. Only the differences of temperature of the atmospheric layers which the ray passes through on its way from the apparent horizon have any influence. The lowest layer, which is in direct contact with the water and in which the ray of the apparent horizon has its origin, must be taken to be at the same temperature (warm or cold) as the water. Sometimes, indeed, it will be necessary to imagine this lowest layer as being of very minute depth, but it will be accepted that there is a continuous transition from the temperature of the water to that of the air.

The result of the curvature of the light rays is that the mathematical ratio defining the relationship between the dip and the height of eye does not have the numerical value which would follow from plain geometrical considerations. As a series of experiments has proved, the formula must contain also the difference of temperature between the water and the air; at the same time there is no agreement on which is the determining layer. The relationship would not be that of formula (1), but

$$(2) \qquad Kt = a\sqrt{Ah} + b\,(T_a - T_w)$$

T_a and T_w are the temperature of the air and the water, a and b are numbers whose exact value we shall give presently. Formula (2) becomes formula (1) when $a = 1'.93$ and $b = 0$.

The first fundamental researches into the value of the dip of the horizon were made in the Adriatic by two Austrian officers, Koss and Count THUN-HOHENSTEIN. They observed from shore, with a universal instrument, the zenith distance of the apparent horizon for four different heights of eye, viz. 6, 10, 16 and 42 metres (19.4, 32.8, 52.5 and 139.7 ft.). The observations extended over all times of day and all seasons. During the observations they carefully measured the temperature of the water and air at different heights, sometimes also in the offing in the neighbourhood of the apparent horizon. Their results were analysed by KOHLSCHÜTTER in 1903 in the *Annalen der Hydrographie*; the German nautical tables (e.g. *Nautische Tafeln des Reichsmarineamts*, 4th and 5th editions, Kiel, 1917-1919) give values calculated from these observations. Mathematically speaking, the values of the two numerical quantities of formula (2) were determined by these measurements to be $a = 1'.81$ and $b = 0'.33$, temperature of the air being understood to be that measured at the height of the eye. If, then, there is a difference of temperature of 3° between water and air at the height of the eye, the dip of the horizon varies by 1'. The deductions drawn by KOHLSCHÜTTER for the distance at which a light on the horizon becomes visible are also noteworthy; and the distance of the apparent horizon is a further consequence of the shape of the path followed by the light. For instance, he shows that, according to Koss's observations, there are two values for the distance at which a light 41 metres (134.5 ft.) above sea level becomes visible, differing by 4.3 miles, when the water is 5° above the air or when the air is 5° above the water. In point of fact only a part of the effect has been captured, for as Koss's observations do not go below 6 metres (19.4 ft.) height of eye, calculations can only be made for that part of the ray of light which extends from the light to 6 metres above the water. As far as the remainder of its path is concerned, the length of its trajectory cannot be worked out; yet that is exactly where one would expect to find an even greater influence of the thermal stratification on the curvature, and consequently the length, of the trajectory. Experience has further revealed even greater irregularities in the distance of a light at the apparent horizon.

It was naturally desirable to obtain confirmation or correction of the results of Koss and Kohlschütter in the open sea. For this purpose it was necessary that an instrument should be constructed to enable the dip of the horizon to be measured on board a ship under way. On the other hand, if one possessed a reliable instrument of this kind, the calculation of the dip from formulae and tables would become superfluous, a direct determination being preferable particularly when one expects to find values differing from the average.

An instrument for determining the dip of the horizon has been invented by Pulfrich. This instrument has a fixed reflecting prism and a movable prism, both of glass. The latter prism is rotated by means of a micrometer drum. The method of observation is such that the reflected images of two opposite horizons are brought into coincidence. These appear in the field of vision as two vertical lines running parallel to each other when the axis of the glass is horizontal, a condition precedent to taking a reading. If, for example, in the position of the apparatus shown in Fig. 2, i.e. drum downwards, the horizons have been brought together, the apparatus is then made to pivot 180° round the axis of the glass (i.e. drum upwards), so that the image of the horizon first observed with the fixed prism now comes into the field of view by way of the movable prism. But the latter has just been set at a certain angle with respect to the fixed prism. The horizons are now seen in the field of view at a distance apart of four times the dip of the horizon, so one of the rays from the horizon must be deflected and brought

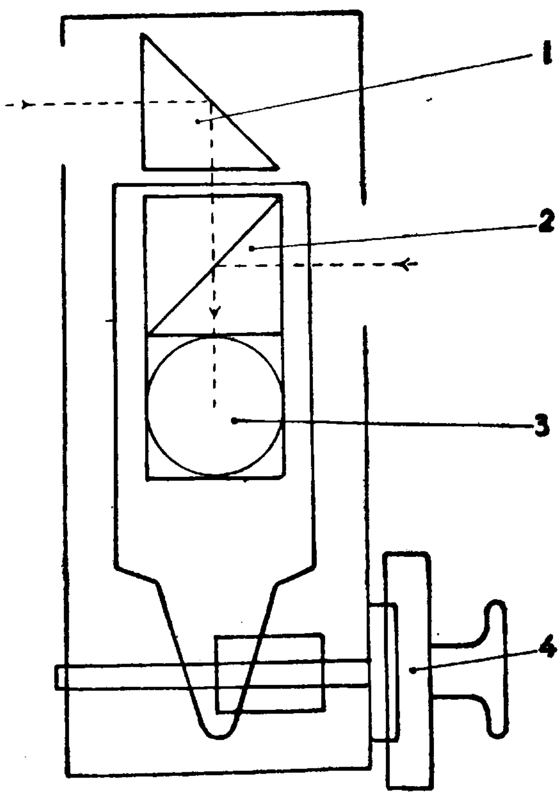

Fig. 2

back with the aid of the movable prism so that in the second position of the instrument the two images will once more be in coincidence. During this time the prism will have turned one half of this deflection, as shown by the micrometer drum. Thus, from the difference of the drum settings in the two positions for measuring the dip, is obtained the sum of the two dips of the working horizon and of the opposite horizon

180° away. To be able to use the results of the measurement in practice, it must be assumed that the two horizons are at the same height below the true horizon, so that the dip of one of them may be taken as half the value of the difference between two settings.

Numerous series of measurements have been made with these instruments, chiefly on board ships of the Navy of the Reich. At the same time, measurements of temperature were made as controls for the formula (2) quoted above; but only in the ship and not at the horizon. In the open sea it should as a rule be sufficient to deduce the temperature conditions for two directions differing by 180°, from the values measured at the middle, on board the ship; besides, it is only what can be obtained on board the ship that is of any practical value.

The results of these series of measurements have been most contradictory. In more cases than not, smaller numerical values for a and b were found than those given by KOHLSCHÜTTER, as far as one can really talk about conformation with any law. The deduction of numerical values for a and b from a great number of observations is carried out precisely by means of a sort of compensation which may be made graphically or by calculation. For two unknowns can already be determined with two equations; if there are more equations — each observation furnishes one — those values should be sought which best adapt themselves to the aggregate of all the observations. For, even if all the observations were to satisfy the equation (2) with mathematical rigour, one could still not determine its accuracy. Rather does one find differences between the observed values and those worked out from formula (2) by means of appropriate values of temperature and of height of eye. When these differences, taken all together, are as small as possible, the "representation" of the observations by the formula deduced from them is declared to be good. But the "representation" can also be so bad as to be, for instance, immaterial for the whole of the differences of dip observed and calculated, whether a formula of form (1) has been adopted for the compensation or one of form (2).* If this is so, it is an indication that the adoption of a formula of the type of (2) has no meaning. That is to say, the determination of b is but a result of calculation and has not the significance of a natural law. (For the establishment of a formula or a table for the dip of the horizon is nevertheless certainly a question of determining a natural law).

With the results obtained with the PULFRICH dip-measuring instrument, the representation of the observations was for the greater part of the time so bad that one could have done just as well with formula (1): which is as much as to say that it had merely been possible to note a dependence between the dip of the horizon and the height of eye; the other oscillations of the dip of the horizon seem to follow an absolutely irregular course.

The representation of Koss's observations in the Adriatic by KOHLSCHÜTTER's formula had however been excellent. For that reason the observations made with the dip-measuring instrument were at first distrusted. Then it was decided to delete from the nautical tables the relationship, which had become doubtful, between the dip of the horizon and the difference in the temperatures of the air and the water.

As we have already said at the beginning, Rear-Admiral CONRAD took up the question again and carried out observations of the most varied kinds on a very large scale. He showed that it is also possible to deduce a formula from the results furnished by measurements of dip of the horizon, which approaches closely to that of KOHLSCHÜTTER. At the same time it is necessary that the instruments should be carefully checked, for they might easily undergo changes in the long run. During the observations it is also necessary carefully to eliminate days when the sea is high. For even a moderate sea is sufficient to cause a raising of the apparent horizon. In isolated practical measurements, more often than not, this raising is indeed unimportant, but it is sufficient to render observations of the dip so heterogeneous that results of general validity cannot be drawn from them. Another reason for the failure of the measurements of dip made hitherto for setting up a general law lies in the temperatures determined at the same time. These temperatures are certainly often inaccurately measured by the ship when under way, for the formation of eddies around the ship produces unforeseeable temperature conditions. CONRAD took his readings with the very greatest care on board ships not having way

(*) *In isolated observations the differences between observation and calculation naturally vary; some will be smaller, others, on the other hand, greater.*

upon them. He was also able occasionally to intersperse measurements of the dip which agreed well, in addition, with the value obtained in the ship.

Conjointly with these observations, sextant observations of the altitudes of heavenly bodies were made in positions of the ship exactly fixed by bearings of shore objects. Under such conditions, the difference between the altitude observed without reduction to the true horizon and the calculated altitude constitute a second series of determinations of the dip of the horizon. The two series of observations conducted separately give as the best formula for the dip of the horizon,

$$(3) \qquad Kt = 1'.92\sqrt{Ah} - 0'.35\,(T_a - T_w)$$

The temperature of the air may be taken practically as that prevailing at the height of the eye. In considering the other unsuccessful experiments made with the dip measuring instruments, CONRAD recommends that in cases where there is a specially large difference between the water and air temperatures it should be measured and used in the calculation of the dip.

But his investigations are not yet concluded. By making repeated measurements of zenith distance of the apparent horizon from ashore with universal instruments, in conjunction with observations made with dip measuring instruments, it will be possible to reach a point where new light can be thrown on the whole question. The nautical tables will have to be adapted to these new results. It will also be necessary to have an arrangement making it possible to supervise the dip-measuring instrument on board, or that this instrument should be improved to render its use on board of practical utility.