THE TIDES OF THE ARCTIC OCEAN

by

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My charts showing the cotidal lines of the world (*see Annalen der Hydr. u Mar. Met. Heft. III* 1922, plates 8 and 9) have not so far included the Arctic Ocean. Thanks to the kind collaboration of Dr. J. FELL, Captain on the Reserve List, Graz, I am now in a position to extend the chart of Cotidal lines hypothetically to this area, at any rate as far as concerns semi-diurnal tides. At the same time I shall take the opportunity to examine the semi-diurnal tides M_2 and the diurnal tides K_1 of the Arctic Ocean from a theoretical point of view. (1)

I. *EXISTING COTIDAL CHARTS.*

Until now, only two different representations of the cotidal lines of the Arctic Ocean were available. The first was sketched by R.A. HARRIS in his *Manual of Tides* part IVB, 1904, but was reproduced with rather considerable modifications in his book *Arctic Tides,* which was published in 1911. The second was produced by J. E. FJELDSTAD (2) and was drawn up by making use of the data relative to tides on the Siberian plateau which were brought back by the ***Maud*** expedition. We are unable to accept either of these two representations. Both of them admit the general principle of tidal waves of translation. This hypothesis is doubtless correct for the regions of the plateau on the coastal shelf, which extends over a certain width, for the results of the observations of the *Maud* expedition (3) published by H. U. SVERDRUP show that the cotidal lines in the locality of the Siberian plateau run nearly parallel to the coast and, therefore, the tidal wave here appears as a wave which has a movement of translation. However, it by no means follows that we must assume the same phenomenon for the interior of the Arctic Ocean.

Whereas HARRIS's chart omits entirely a very considerable area and thus is insufficient to explain, even hypothetically, the correlation which exists between the various observed data, FJELDSTAD attempts to show at any rate, a complete system of cotidal lines for the whole of the Arctic Ocean. He assumes a wave of translation, which, coming from the Atlantic Ocean between Greenland and Spitsbergen, spreads in the same direction as far as the coast of East Siberia at a speed which is a function of the prevailing depth. As far as I can see, this hypothesis in no way agrees with the fairly accurate observed data obtained on the North Coast of Greenland and of Grant's Land, where the two stations, Point Aldrich (Cape Columbia) and Cape Morris Jessup, have, so far as M_2 is concerned, establishments of 0 hours and 0.3 hours (Greenwich) respectively; whereas, according to FJELDSTAD's chart, the establishments in this vicinity should be between 2 hours and 3 hours. It does not seem possible that such an advance could be explained away here. Besides, FJELDSTAD does not extend his cotidal lines

(1) *With the consent of Dr. Fell, I am here making partial use of the results which he obtained.*

(2) *J. E. Fjeldstad's* "Literature on the Tides of the Arctic Ocean," *Naturen, 47th year, pages* 161 *to* 175, *Bergen and Copenhagen,* 1923.

(3) *H. U. Sverdrup,* "Dynamics of Tides on the North Siberian Shelf" *Geophysic Publications, Vol.* VI. *N°* 5, *issued by the Norwegian Academy of Sciences, Oslo,* 1926.

as far as the American coast; his chart, likewise, shows a fairly wide "blank band". In my opinion it would be well to abandon the hypothesis of a tidal wave of translation in the Arctic Ocean, at any rate insofar as its central very deep portion is concerned, and to assume instead stationary oscillations.

II. *DISTRIBUTION OF DEPTH.*

At all places where, within recent times, deep sea soundings have been taken in fairly large numbers, it has been seen that the wide plateau which is attached to the coast and has depths less than 200 metres and even 50 metres, passes almost immediately into a basin of from 2.000 metres to 3.000 metres deep. It may be assumed with considerable probability and as has already been done by F. NANSEN in drawing up his bathymetric chart of the Arctic Ocean (*Geographical Journal,* 1919), that the depth contours of 200 metres, 1,000 metres and 2,000 metres run so closely together nearly everywhere, that two totally dissimilar regions must be recognised in this ocean, viz : a central basin which is very deep and a bordering zone of shallow water (which appears to be absent only over a very short stretch of the American coast). This characteristic is very clearly shown in the "*Hand Atlas*" by STIELER, 10th edition, Chart N° 3. The depth contour of 1,000 metres is shown in pecked lines on the attached plate.

In opposition to H. U. SVERDRUP, who chose the shallow border zone and particularly the region of the Siberian plateau as the area of his research, which allowed him to give a theoretical explanation based on tidal waves of translation, we will try to obtain some information on the phenomena of the tide in the deep central basin.

III. *STATIC TIDES.*

It may be presumed that the central deep area of the Arctic Ocean will likewise have static tides. They will be calculated by approximation and it will be assumed that the basin is entirely enclosed. This method of procedure approaches more closely to the real conditions than might be supposed at first glance, for the deep central basin, as a rapid calculation will prove, contains more than 97 % of the total mass of water of the Arctic Ocean, and also because the region of the plateau which is close to it, with the considerable influence which it exercises on account of the friction to which it gives rise, would be an obstacle to the production of uniform oscillation phenomena throughout the Arctic Ocean.

It was assumed as a close approximation that the dividing line lies along the depth contour of 200 metres. The basin, as thus defined, has an elongated form, the medial line of which has a fairly sharp curve and a length which is nearly double the greatest transversal distance of the basin. The medial line was divided into sixteen equal parts and sections of the basin perpendicular to this line have been drawn at the points thus obtained and, in so far as soundings permitted, these sectoins were measured, or rather evaluated approximately as to width and area. Then by means of Merian's (4) formula, subjected to the corrections introduced by the Japanese for width and volume, the proper period of a longitudinal oscillation of the basin was calculated and this gave 9.25 hours. In a transversal direction and for the medial sections, the proper period is about 5.82 hours. The calculation is obviously only an approximate evaluation, but it clearly brings out that the proper period in the transversal direction is incontestably much smaller than the tidal period and so much so that we may safely assume the processus of the movement to be the result of the longitudinal and transversal oscillations taken together.

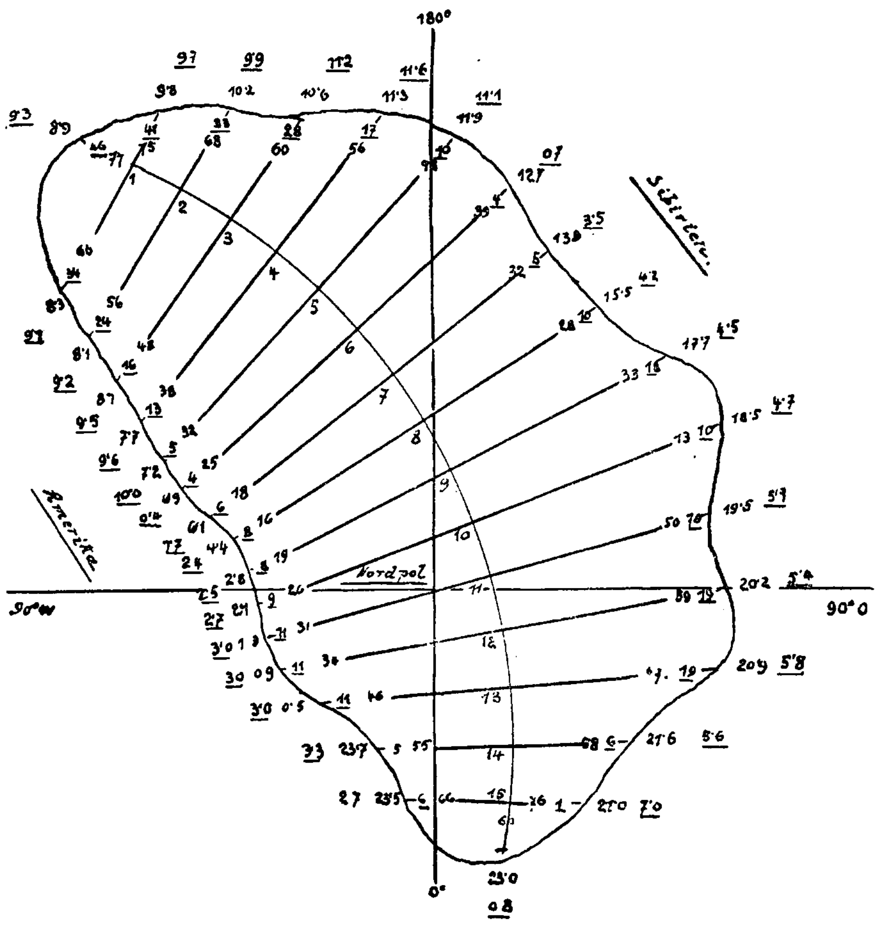
I undertook the calculation of the part arising from the static tide in the deep central basin, both for the partial semi-diurnal waves M_2 and for the diurnal waves K_1, taking as a basis the *Theory of the Analysis of the Tides in a Channel* which I set out in the *Zeitschrift für Geophysik,* N° 2, pages 319 to 326. In order to do this, both the longitudinal oscillations and the transversal oscillations resulting from the influence of the rotation of the earth and the inclination of the level surface, were reduced to time components of the epochs 0 hours and 3 hours (Greenwich) and for the diurnal tides 0 hours and 6 hours. These two components were calculated separately, then they were combined in exactly the same way as I carried out

(4) *K. Honda, T. Terada, etc.* An investigation on the Secondary Undulations of Oceanic Tides. *Tokyo,* 1908.

the calculation of the tides of the Black Sea (*Annalen der Hydr. etc.*" 1926, pages 289 to 296). Further, since the numerical integrations could not be made with accuracy, owing to the absence of accurate knowledge of the depth of the sea, the longitudinal oscillations were simply calculated in accordance with the inclination of the level surface from one section to the other and then multiplying by the factor of inertia. This makes the times of the commencement of the tide somewhat uncertain, principally in so far as M_2 is concerned.

Instead of collecting the results of the fairly long calculations in a table Dr. FELL represented them by the annexed figure.

The edge of the figure corresponds approximately to the depth contour of 200 metres. At the extremities of each section will be found the theoretical times of the commencement of the tides and the amplitudes for both M_2 and K_1 ; these are, outside the curve, the times of the tide referred to the culmination of the fictitious body for Greenwich and, inside the curves, the amplitudes in millimetres. In both cases the underlined figures refer to M_2 and those which are not underlined to K_1.



The theoretical times of the static tides, as may be seen in the figure, do not show for M_2 a very pronounced circulation of high tide in a definite direction; for K_1, on the other hand, there is an amphidrome turning right-handed at an almost constant angular speed. The mean of the amplitudes of all the points on the edge of the figure has a value of 1.4 centimetres for M_2, but for K_1 it reaches the much higher figure of 4.8 centimetres.

These theoretical mean values for the amplitudes found for the tides must now be compared with the observed data. The table below contains the observed amplitudes for M_2 and K_1 (in centimetres) for five stations in the vicinity of the central basin (5).

Stations	μ	λ	H_{M_2}	H_{K_1}
Flaxman Island	70,2° *N*	145,8° *W*	6,7 cm	2,4 cm
Point Aldrich	83,1° *N*	69,6° *W*	11,6 »	5,2 »
Treurenberg	80,0° *N*	16,9° *E*	28,0 »	7,3 »
Teplitz Bay	81,8° *N*	57,9° *E*	15,5 »	2,7 »
Cap Tchelyuskin	77,6° *N*	105,7° *E*	9,9 »	4,8 »
		Mean	14,3 cm	4,5 cm

It will be seen, therefore, that the theoretical amplitudes of the static tides of the deep central basin are about one tenth of the observed values for M_2, whereas for K_1 the mean of the theoretical amplitudes for these tides agrees very closely with the mean of the observed amplitudes. From this we are able to deduce that the semi-diurnal tides of the Arctic are mainly dependent on the oscillatory tides of the Atlantic Ocean (*i. e.* that they oscillate in resonance with the Atlantic Ocean). The diurnal tides, however, are formed mainly by the proper motion of the tides of the deep central basin.

This result is not surprising in itself, for the intensity of the semi-diurnal tides near the Pole is extremely small, whereas the diurnal tides reach their maximum intensity in this area. With reference to the oscillation in resonance with the Atlantic Ocean, we find that at Jan Mayen Island, which is the nearest station (φ = 71.0° *N*, λ = 8.5° *W*.) M_2 has a large amplitude, namely 40.2 cm., whereas K_1 has only 3.4 cm. The observations alone make it possible to conclude that the resonance oscillation of the Arctic Ocean with the Atlantic Ocean would be considerable for M_2 and that it would be infinitely small for K_1; this is in absolute agreement with the theoretical results of the calculations.

IV. *OSCILLATING TIDE FOR* M_2.

Whereas the theoretical times of the commencement of high water for M_2, which times, as we saw above, depend on the spontaneous components of the deep central basin, do not allow us to distinguish any very pronounced rotary character, the oscillating tide should, and mainly for M_2, give rise to an amphidrome turning to the left, ; for this manifests itself principally by longitudinal oscillations the nodal line of which (this nodal line as a rough calculation will show, should coincide with the section 8 or 9 or at any rate should be close to the middle of the deep central basin) will lie transversally across the latter (6). But the action of the earth's rotation creates a transversal oscillation which, by combining with the longitudinal oscillation, gives rise to a left-handed amphidrome. The relation between the amplitude of this transversal oscillation and that of the longitudinal oscillation depends only on the period of oscillation T and the geographical latitude, as I have already demonstrated (*Wiener Ber.* IIa, volume 124, 1915, pages 972 and 973). I will repeat here the very simple manner of deducing the formula in question. The longitudinal oscillation depends on the differential formula :

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(5) *Bennett Island has been omitted therefrom* (φ = 76.7°, *N*. λ = 149.1° *E*.) *where the extraordinary amplitude* H_{M_2} = 37.5 cm. (*?*) *can be due only to a local perturbation arising from the the configuration of the sea bottom.*

(6) *It is absolutely impossible to calculate the longitudinal oscillations of the deep basin and those of the plateau area in a uniform manner because the necessary hypothesis that the displacements of the water in a single section are the same everywhere, is not fulfilled here even approximately. I note this in connection with an article by A. Defant* (Ann. d. Hydr. etc., 1924, *pages* 153 *to* 166 *and* 177 *to* 184) *in which the calculations for such phenomena lead to a distribution of the hourly lines of advance of tides in the Arctic Ocean which is in absolute disagreement with the data obtained by observation and which, therefore, it is unnecessary to take into further consideration.*

$$\frac{\delta^2 \xi}{\delta t^2} \quad -g \frac{\delta \eta}{\delta x}$$

(H. LAMB, *Hydrodynamik*, paragraph 168) when direction x coincides with the medial line of the channel and ξ and η represent the horizontal and vertical displacement of the particles of water at the point x at the time t, both of which, if T be the period of oscillation, are proportional to $(\frac{2\pi}{T} t + \varepsilon)$. If the double differentiation according to t be developed and if, after having omitted the time factor, the maximum resulting elongations at the point x be designated by ξ and η these will obviously satisfy the equation.

$$\frac{d\eta}{dx} = \frac{4\pi^2}{g T^2} \xi$$

The earth's rotation causes deviation of the moving particles of water and creates a transversal oscillation the phase of which is less by a quarter of a period in relation to the longitudinal oscillation and the combination of which with the longitudinal oscillation gives rise to a left-handed amphidrome. Whereas, for example, on account of the longitudinal oscillation the flow of water takes place at a maximum speed towards the North, under the influence of the earth's rotation, its maximum will be to the right, in other words, deviating towards the East. For this reason, high water occurs on the East a quarter of a period earlier than on the North, and it is this fact which produces a left-hand turning amphidrome.

The mean speed of displacement of the particles of water is $\frac{4\xi}{T}$ and thus the maximum speed is $v = \frac{4\xi}{T} \cdot \frac{\pi}{2}$. The deviating force acting to the right which is due to the earth's rotation (the corresponding acceleration of which is $2\,\omega \sin\varphi\, v$, where $\omega = 0{,}0000729212$ is the angular speed of the earth per second and is the geographical latitude) acts on the particles of water which are in movement with the above-mentioned maximum speed. This deviating force combines with the earth's gravity into a result to which the level surface in a transversal direction is perpendicular, *i. e.* it is inclined to the horizontal at an angle α for which $\tan\alpha = \frac{2\,\omega \sin\varphi}{g} \cdot v$.

If the value of v be introduced, the amplitude of the transversal oscillation will be

$$\tan\alpha = \frac{2\,\omega \sin\varphi}{g} \cdot \frac{4\xi}{T} \cdot \frac{\pi}{2}$$

If β be the amplitude of the longitudinal oscillation on the nodal line, then

$$\tan\beta = \frac{d\eta}{dx} = \frac{4\pi^2}{g T^2} \cdot \xi.$$

Hence the value of the relation between the amplitudes of the transversal oscillation and the longitudinal oscillation will be

$$\frac{\tan\alpha}{\tan\beta} = \frac{T\omega}{\pi} \cdot \sin\varphi.$$

which, as may be seen, really depends on T and φ only. In the case under consideration, $T = 12{,}4206 \times 3600$, which is the period of M_2 in seconds and φ is 85^0 which is the geographical latitude of the centre of the amphidrome. Calculation then gives the value of $\frac{\tan\alpha}{\tan\beta} = 1.0339$, which shows that the longitudinal and transversal oscillations, which arise from the synchronous oscillation, will have nearly equal amplitudes. As a general result there will be, consequently, a very distinct amphidrome turning left-handed for M_2, even when the very small part of the static tide comes into play (7).

V. *STATIC TIDE FOR K_1.*

Conditions are quite different for K_1. As was determined above, the component of the static tide comes mainly into play here and a right-handed amphidrome of almost constant angular speed corresponds thereto, at any rate in the deep central basin. Here we desire only to determine this theoretical result, even though we are not able to check it with any accuracy from observed data ; for the data at present available as to the diurnal tides in the Arctic Ocean are mostly extremely uncertain, which may be seen from the great contradiction between the Kappas of K_1 and O (*see* the table in "*Arctic Tides*" by HARRIS pages 40 to 43). Some of the data as to K_1 seem, however, to be quite reconcilable with a right-handed amphidrome; for example, we find on the coast of Grant's Land about 0 hours, on the north coast of Alaska about 10 hours, at Bennett's Island 17 hours and at Franz Josef Land 22 hours as the observed times for the commencement of the high tide of K_1 with reference to the culmination of the fictitions body at Greenwich, which agrees well enough with the theoretical times of commencement (the underlined numbers on the outside of the figure). But all this cannot be considered sufficient to make the existence of such amphidrome certain.

VI. *NEW CHARTS OF COTIDAL LINES FOR M_2.*

Consequently, we will confine ourselves to the semi-diurnal tide M_2, for which we will try to outline a new cotidal chart, even though it be only diagrammatically. Dr. FELL has collected, as far as possible, all the bibliographical data which exist (8) ; the stations are represented by points on the annexed chart. The numbers inserted represent the lag of the high water relative to M_2 with reference to the lunar culmination at Greenwich ; these numbers were calculated by the formula :

$$\frac{\varkappa\ M_2 - 2\,\lambda}{30} \cdot \frac{12.42}{12.00}$$

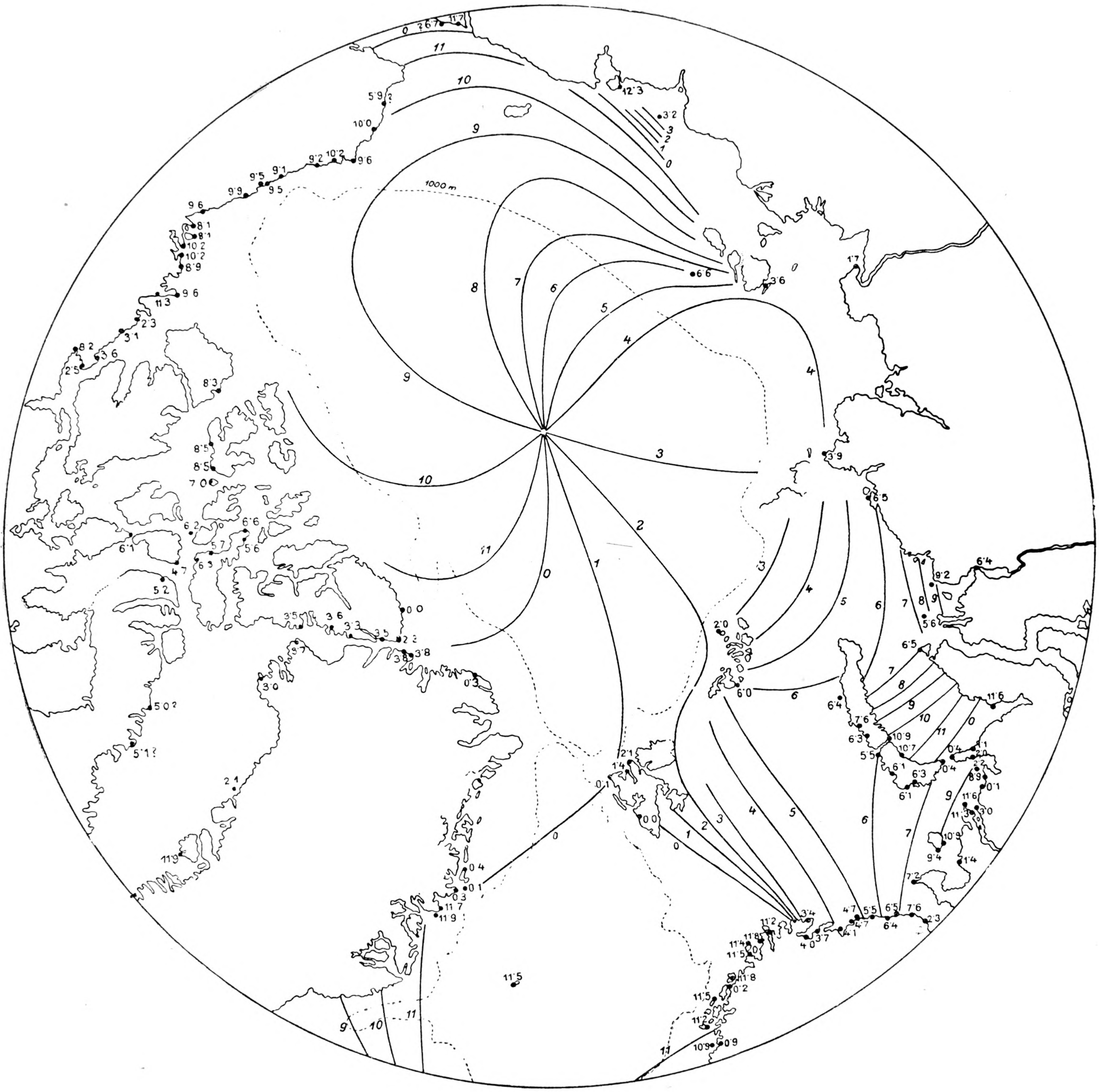
where $\varkappa\ M_2$ is the Kappa relating to M_2 and λ the E. longitude of the place of observation. Therefore, the computation of these lags goes from 0 hours to 12.24 hours, period of M_2. For stations where harmonic analysis was not available, HARRIS employed a method of reduction to connect the observed times of the establishments of ports with M_2.

The cotidal lines are thus drawn on our chart assuming that, in the interior of the Arctic Ocean, there exists a left-handed amphidrome, while the engendered wave spreads towards the coast in the form of a wave of translation. H.U. SVERDRUP has verified this latter in an incontestable manner and also explained it theoretically. We adhere to these Cotidal Lines without reserve for the region of the Siberian plateau. The simultaneous presence of an amphidrome in the interior and of a wave of translation in the bordering zone leads to a characteristic configuration of the Cotidal Lines which would probably not be found in any other part of the Ocean. They take the form of a sort of spoked-wheel.

The lie of the Cotidal Lines agrees very satisfactorily with the observations, taking into account the lag caused by the flood-tide when entering the plateau zone. At one point, particularly at the mouth of the Banks Strait, the establishment of the port seems to be a little smaller than our theory would have led us to expect. But it should be noted that here an

(7) *The influence of the static component makes itself apparent perhaps in that, whereas the establishment at Spitsbergen is approximately* 1 *hour* (*Greenwich*), *at the other end observation does not give* 7 *hours but* 9 *hours, a delay which can be explained by the interference of the static tide* (*compare the preceding figure*).

(8) *Rollin A. Harris,* Arctic Tides, *Washington* 1911. - Tide Tables, *published by the Reichs-Marine-Amt, Berlin.* - *P. Schureman,* A Manual of the Harmonic Analysis and Prediction of Tides, *Washington,* 1924, *pages* 319 *to* 321. - Tide Tables of the Arctic Ocean and of the White Sea for 1925 (*in Russian*), *Leningrad* 1924. - International Hydrographic Bureau Tables *Suppl. to Special Publication* N° 12, *Monaco* 1926. - *H. U. Sverdrup, Loc. cit.* 1926.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interference takes place with the flood-wave which, coming from Baffin Bay, passes Barrow's Strait and enters Banks Strait; on the one hand it has a fairly large amplitude and on the other it exhibits times of high tide which are very much smaller than those which correspond to the lie for the Arctic Ocean. Consequently, this must result in an advance of the time of the commencement of high tide in the vicinity of the estuary.

A second anomaly consists in the fact that at Flaxman Island, which is situated at almost the other extremity of the deep basin, an amplitude is observed for M_2 of barely one half the value (in this case 5.7 cm.) of that which it should be according to the theory of oscillation. Judging by this difference, we are no doubt faced by the influence exercised by friction in the vicinity of the plateau, which is fairly near, which friction absorbs a part of the energy of the tide because the tidal wave, which at that point spreads toward the coast, must overcome a very considerable frictional resistance. The anomaly indicated cannot in any way give rise to the assumption that a vast stretch of land exists at some place in the Arctic Ocean which is not yet explored, as HARRIS seems inclined to assume.

The observation data, on which our chart of cotidal lines depends, are, on account of the nature of the thing itself, somewhat sparsely scattered over great coastal areas and some of them are not entirely reliable. Observations of ebb and flow are exceedingly difficult to make and only possible on rare occasions over the greater part of the explored portion of the coasts of the Polar Sea; therefore, we cannot hope to have empiric material of any importance at our disposal in the near future. New isolated observations along the Siberian Coast could not, in any case, tell us much, as owing to the large plateau which extends in front of it, the times of commencement appear with a very great lag to the observer there; the evaluation of this lag is almost impossible. New isolated data concerning the outer edge of the North American Archipelago would be far more important for checking our theory for, so far as we are able to judge at present, this would be found to be much nearer the central deep basin.

In any case, for a long time to come, the phenomenon of the tides of the Arctic Ocean will remain in the domain of hypothesis. However, the data which we possess at present, combined with theoretical reasoning, permit the course of the hourly advance of the flood tide to be assumed with a certain degree of probability; in general, it should agree with that represented on the annexed chart.

SUMMARY.

In the researches made here on the Tides of the Arctic Ocean, it is assumed, in the first place, that in the central area, bounded by the 200 m. depth line, we have to deal with stationary oscillations. For M_2, as a theoretical calculation makes evident, they are mainly influenced by the oscillation caused by the tidal movements in the Atlantic, but for K_1 they are dependent on the movements of the static tide.

This determines that, for M_2, we must assume a left-handed amphidrome in the deep central basin, but for K_1 a right-handed amphidrome.

This tidal wave spreads from the deep central basin to the plateau. On account of the sudden diminution of depth and the great frictional resistance, it takes the form of a translation wave. Thus, especially for M_2, the appearance of the cotidal lines as shown on chart is produced and, although this is, so far, but hypothesis, it must be fairly near the truth in its principal features.