ON COTIDAL MAPS

by

H. A. MARMER

U. S. Coast and Geodetic Survey

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On maps delineating the features of the earth, the sea is generally pictured as a vast plain undistinguished by any marks of whatever kind. Even when contour lines or soundings are shown in ocean areas it is to the land that they refer—to the land beneath the sea. The surface of the sea, however, is not a monotonous plane stretching from horizon to horizon. For altogether apart from the swells and waves which are brought about in the surface layers of the water by the play of wind and weather, the sea sustains periodic changes in response to the mighty pulse of the tide-producing forces. Twice daily, in rhythmic fashion, the tide-producing forces of sun and moon stir the sea to its depths and give rise to the phenomena which for short are called the tide.

In so far as our knowledge of the tide is based on observed data, it may be said to be confined wholly to the coast line. The difficulties involved in measuring the rise and fall of the tide in the open sea are obvious, and as yet there are no such observations at hand. To be sure the mid-ocean islands interrupt, here and there, the wide expanse of tidally unknown seas; but these islands are few in number. Moreover, the tide rising and falling on the island shores is no longer the unaffected open ocean tide; for in breaking the uniformity of the ocean depths, the islands by the same token modify the tide.

It may perhaps not be out of place here to emphasize the fact that the tide has its birth in the open ocean basins. It is the action of the tide-producing forces of sun and moon on the waters of these huge basins that makes possible the rise and fall along the coast. What, geographically, is the course of the tide across the oceans? How can the progress of the tide from place to place be pictured? It is these questions that cotidal maps attempt to answer.

COTIDAL LINES AND COTIDAL HOURS.

The term “cotidal” appears to have been coined by Sir John Lubbock in 1831. In his paper "On the Tides in the Port of London" he speaks of "cotidal lines," adding in parenthesis "I mean the series of points at which it is high water at the same instant." (1) A cotidal line is thus a line connecting points on the ocean at which high water occurs simultaneously.

To show the progress of the tide from place to place the cotidal lines must be numbered in some way. This means that some zero time of tide must be established. Since, as it is colloquially put, "the tide follows the moon" it is of advantage to take the time of the moon’s passage across some meridian as the zero hour, and for this purpose the meridian of Greenwich naturally suggests itself.

Now the average period of the tide (the time interval from one high water to the succeeding high water, or from one low water to the next low water) is 12 hours and 25 minutes, or half a lunar day. Obviously, it will be of advantage to have this interval divided into equal parts, which solar hours will not do. But it may be done very conveniently by dividing it into 12 lunar hours, which means that each lunar hour is 1.035 solar hours. Cotidal maps, therefore, are generally drawn with cotidal lines numbered from 1 to 12 (12 being used in preference to 0),

the time being lunar time referred to the moon's transit across the meridian of Greenwich as the zero hour.

It has been customary for many years to define the time of tide at a place by its relation to the time of the moon's local transit. For the high water this interval is known as the high water lunitidal interval and gives the hours and minutes (in solar time) by which, on the average, high water at that place follows the moon's local meridian passage. It is important to note that the times shown on cotidal maps are not the lunitidal intervals; and, to distinguish between the two, the time of high water at any place as determined in Greenwich lunar time is called the cotidal hour.

The relation subsisting between the lunitidal interval and the cotidal hour at any place can be readily determined so that they may be converted one into the other. Stated briefly, the high water lunitidal interval at any place may be reduced to the cotidal hour by dividing by 1.035 and adding one-fifteenth of the longitude if west, or subtracting if east. Thus Atlantic City, N. J., in longitude 74° 26' W., has a high water lunitidal interval of 7 hours 13 minutes. Its cotidal hour therefore is \((7.217/1.035) + (74.43/15) = 11.93\) hours. The advantage of using cotidal hours is that it makes the tide at all places directly comparable in time, which is not the case with lunitidal intervals.

**EARLIEST COTIDAL MAP.**

Some twenty years before Lubbock's use of the term "cotidal lines" the idea of representing the progress of the tide by a system of lines was put forward by the English physicist, Dr. Thomas Young; and in his "Natural Philosophy", published in 1807, he illustrates the progress of the tide around the British Isles by such a system of lines. His map, reproduced in Figure 1, is the first cotidal map known, but it is clear that Young intended it more in the nature of an illustration than as a definite attempt to depict the precise course of the tide.

![Fig. 1, — Thomas Young's cotidal map of the British Isles, 1807.](image)

This is the earliest example of a cotidal map known.

Lubbock's paper "On the Tides in the Port of London," to which reference has already been made, was illustrated with two maps, "the one showing the time of high water on the coast of Great Britain, the other throughout the world, or at least in as many places as it has been ascertained." He makes no attempt to draw cotidal lines but contents himself with noting the cotidal hours at a number of places along the various coasts. With regard to these cotidal hours, however, he remarks, "It will be seen that the continents alter the direction of the cotidal lines............ and that the progress of the tide is not always from east to west: in the Atlantic it is from south to north; so that it is high water at nearly the same time on the coast of Portugal and on the opposite coast of America."
In explanation of Lubbock's remark that "the progress of the tide is not always from east to west" it is to be observed that the notion of a westward movement of the tide across the seas was one of long years' standing. For it was argued that since the tide follows the moon, the progress of the tide must be from east to west; and therefore the tide was conceived as a wave moving westward following the moon.

It should be noted that Young in his cotidal map joined two points on opposite shores having the same cotidal hour by a straight line. In narrow seas this procedure appears reasonable as a first approximation. But, when it comes to ocean areas, such procedure is altogether too simple to fit the facts. In other words, across the open sea cotidal lines can be drawn only in accordance with certain assumptions as to the nature of wave movement or in consonance with some theory of the tide.

WHEWELL'S COTIDAL MAP.

The first comprehensive cotidal map of the world is due to William Whewell. In 1833 he presented before the Royal Society of London a paper entitled "Essay towards a First Approximation to a Map of Cotidal Lines." Two cotidal maps accompany the paper, one for the British Isles and the other for the world at large between the latitudes 60° N. and 52° S. In the latter of these maps the cotidal lines are drawn across the wide expanses of the Atlantic and Indian Oceans from shore to shore; but in the Pacific Ocean the cotidal lines, because of the meager information at hand at that time, go out but a short distance from the western coast of the United States. Note may be made, however, of a number of cotidal lines across the Southern Pacific from Chile to the Friendly Islands. In Figure 2 the cotidal lines for the Atlantic Ocean are reproduced from Whewell's map.

Fig. 2. — Whewell's cotidal map of the Atlantic Ocean.
As a preliminary to drawing his maps Whewell considers the form cotidal lines must have in virtue of the laws of fluid motion. He shows that they must be convex in the direction of wave progression, because the tide wave will progress faster in the deeper central part of a channel than in the shallower areas near the shores. He considers also the effects of islands and shoals, concluding that these must deflect the course of the cotidal lines in their vicinity.

Whewell expressly states that he endeavored "to trace the course of the cotidal lines according to which the tide is actually propagated in the Ocean." In other words, he makes it plain that he was basing his map on observations so far as possible rather than on an a priori theory of the tide. He was fully aware of the difficulties involved in securing accurate tide observations and the further difficulties of correcting the data derived from short series of observations to mean values; and he is scrupulously careful to make it clear that he is endeavoring "to draw a first approximation to the course of the cotidal lines, begging the reader to bear in mind that from the nature of our materials it must be imperfect." Finally, in the conclusion to his memoir he adds, "I shall be neither surprised nor mortified if the lines which I have drawn shall turn out to be in many instances widely erroneous; I offer them only as the simplest mode which I can now discover of grouping the facts which we possess."

If there were sufficient tide observations in the open sea the construction of a cotidal map would offer no more difficulty than the drawing of a contour map of a well-surveyed area; but tide observations are confined wholly to the coast line. In Whewell's case the difficulty was aggravated by the absence or meagerness of accurate observations over wide stretches of the coast. Under such conditions some theory of the tide must, obviously, underlie the construction of a cotidal map.

Parenthetically it may be observed that in speaking of a "theory" of the tide no question is raised thereby as to the moon and sun being the agencies that give rise to the tide. Nor is there any question as to the tide-producing forces brought about by the attraction of moon and sun, for these are well known and can be readily determined from known astronomical data. But taking these tide-producing forces for granted, what is the precise nature of the movement of the ocean waters in response to them?

PROGRESSIVE-WAVE THEORY.

Underlying Whewell's cotidal map is the so-called "progressive wave" or "Southern Ocean" theory of the tide. This theory considers the tides of the world as due primarily to the action of the tidal forces of moon and sun on the broad and deep waters of the Southern Ocean, the belt of water that completely encircles the earth south of the great land masses. Here it is argued, the tidal forces have almost uninterrupted sway, and here these forces raise two tidal waves 180° apart in longitude, which travel from east to west. As each of these waves sweeps past the Cape of Good Hope it generates a wave which travels up the Atlantic Ocean, and it is this secondary wave generated by the primary tidal wave of the Southern Ocean which determines the tides on the shores washed by the Atlantic.

It is important to note carefully the distinction between the primary tidal wave and the secondary wave which, according to this theory, sweeps up the Atlantic Ocean. The secondary wave travels freely, according to depth. But the primary tidal wave in the Southern Ocean keeps step with the moon; it travels not as a "free" wave, but as a "forced" wave, compelled by the moon to keep in time with its own movement.

At Whewell's instance simultaneous tide observations were made in 1835 at a number of places both in Europe and in America. In reporting on the results of these observations he states, "It is not surprising, therefore, that the differences between the form of the [cotidal] lines now obtained and my former maps should be considerable. At the same time I may observe, that all my views of the general course of the tide-wave have been confirmed by the present examination." As a result of these newer observations he presents a revised cotidal map of the waters around the British Isles, but not for the world at large. It is of interest to note, however, that from the study of these more recent observations Whewell finds a number of difficulties in the notion of the progress of the tide wave from south to north in the Atlantic Ocean.
The progressive-wave theory is very plausible and explains a number of features of the tide as they manifest themselves in the seven seas. To the end of the nineteenth century the greater number of tidal investigators, and in turn the geographers, lent the weight of their authority to the progressive-wave theory and to Whewell's cotidal map. In his article on "Tides and Waves" Sir George Airy, whose contribution to the mathematical development of tidal theory was considerable, directs attention to the fact that the cotidal lines in the other seas cannot be considered as known with great certainty; but with regard to the North Atlantic he remarks, "We conceive therefore (recognizing also the justness of the principles on which Mr. Whewell has generally drawn his curves) that the cotidal lines of the Northern Atlantic are now drawn with very great accuracy." He reproduces Whewell's cotidal map of the world with the corrections indicated by Whewell in his memoir of 1836. Five years later Heinrich Berghaus reproduces Whewell's map of 1833 with no corrections; he adds, however, cotidal lines for the Pacific Ocean (2). These latter he draws in dotted lines for distinction from those laid down by Whewell, which are shown in full lines, explaining in a legend that the full lines are certain while the dotted lines are uncertain. Even so late as 1911 Airy's cotidal map is reproduced by Darwin in the third edition of his well-known book on the tides (3), although attention is directed to the assumptions underlying it and to the results of later investigations which call some of these assumptions into question. Darwin goes on to state, however, that "the tide-wave is principally progressive in the ocean."

STATIONARY-WAVE THEORY.

A body of water disturbed from its condition of rest tends to set up waves. This is a familiar fact illustrated not only by the ocean waves and swells but in a variety of ways—by the pebble dropped in the pond, the wind disturbing the surface of the lake, the boat plowing through the water. And in the same way the tide-producing forces of sun and moon acting on the waters of the sea bring about the mighty wave which manifests itself as the tide.

It is to be noted, however, that a body of water is capable of sustaining two different types of wave movement. If we take a rectangular tank, say about 20 feet long filled with water to a depth of about a foot, we can start a wave by agitating the water at one end with a paddle. This wave will progress from one end of the tank to the other, the outline of the wave being the sinuous curve which we generally associate with wave movement. This wave is technically a progressive wave.

But we may also set up in the tank of water a wave movement of an entirely different kind by raising and then immediately lowering one end of the tank. The water will now no longer progress from one end to the other, but will oscillate or swash about an axis in the middle of the tank, so that it will be high water for half the tank at the same instant that it is low water in the other half. This type of wave is known as the stationary wave.

On the basis of the progressive-wave theory the rise and fall of the tide was conceived as a world phenomenon dominated by the forced tide wave of the Southern Ocean. But a study of the results of the more numerous tide observations since Whewell's time brings out the fact that, while the characteristic features of the tide in the various oceans differ considerably, for any one ocean basin these characteristic features are much the same. Is it not, therefore, more probable that the tides in the various oceans are brought about by the action of the tide-producing forces on the waters of the individual oceanic make-up? Furthermore, may not the rise and fall of the tide in the open ocean be due to a stationary wave rather than to a progressive wave?

From time to time various investigators have put forward the suggestion that certain features of the tide could best be understood on the assumption that the response of the waters to the tidal forces of sun and moon was of a stationary-wave character. Indeed, Whewell himself, some fifteen years after the publication of his cotidal lines thinks it unlikely that the progressive

wave in the Atlantic and Pacific Oceans which his scheme of cotidal lines assumes "rightly represents the mode in which the waters of the Atlantic and Pacific obey the action of the sun and moon" and adds, "If it be asked what other mode of operation of the lunar and solar forces upon the ocean can be conceived, different from this progressive wave which is expressed by means of cotidal lines, an answer immediately suggests itself, that a stationary undulation...... is a possible mode of motion for a fluid under such circumstances."

As a result of his later researches Whewell is "disposed to retract parts of what I have said with regard to the form of the cotidal lines of the Atlantic in my 'Essay.' I do not think it likely that the course of the tide can be rightly represented as a wave travelling from south to north between Africa and America." (4) For the wide expanses of the open sea Whewell definitely discards the cotidal lines previously laid down by him. It is a curious fact, therefore, that despite this his cotidal map was reproduced in various books for many years thereafter.

Stationary-wave movements as related to tides received considerable mathematical discussion in the tidal researches of William Ferrel. He concluded that the tide in the North Atlantic Ocean was due primarily to an east-and-west stationary-wave oscillation. He discarded completely the notion of a progressive wave sweeping up the Atlantic from the Southern Ocean and emphasized his views by the statement, "If there were a dike extending from the Cape of Good Hope to the coast of South America, the tides of the North Atlantic Ocean would most probably be very nearly the same."

OCEANIC OSCILLATING SYSTEMS.

It was, however, not till the beginning of the present century that the stationary wave was made the basis of a full-fledged theory of the tide. In 1901 Harris published his "Outlines of Tidal Theory." This is primarily a mathematical discussion of the modes of oscillation of bodies of water. Harris' thesis is that the tide in the open sea is brought about by stationary-wave movements of various oceanic areas which he denominates as "oscillating systems," these systems having free periods of oscillation which are approximately the same as the periods of the tide-producing forces.

The period of oscillation of a body of water depends on its length and depth. Since the principal tidal forces are those having a period of half a lunar day, Harris separated out oceanic areas which have the requisite length and depth to support stationary waves with a period of half a lunar day. These oceanic areas constitute the "oscillating systems" and are illustrated in Figure 3.

Fig. 3. — Oceanic oscillating systems for the semi-daily tide-producing forces, according to Harris.

The Roman numerals on the oceanic oscillating systems of Figure 3 give the cotidal hours over the areas indicated. Several of the oscillating systems overlap, and these are distinguished by different kinds of shading. The nodal lines of the various systems are indicated by dashed lines across the shaded areas. The unshaded parts of the ocean are areas of such size and depth that the action of the tide-producing forces on these areas can produce but little tide. In such regions, therefore, the tides that occur are due to the tides produced in the neighboring oscillating areas.

According to the stationary-wave theory, the tide-producing forces of sun and moon as they sweep over the oceans maintain a stationary-wave oscillation in these open ocean systems, the rise and fall being greatest at the ends and least at the nodal lines. These oscillations give rise to the dominant semidaily tides of the world. Openings in the coast line and irregularities in depth cause the stationary-wave tides generated in these ocean basins to send off progressive waves into coastal tidal waters, or into other parts of the sea.

South of Australia, Figure 3 shows an oscillating area in which the cotidal hours are enclosed in parentheses. This area lying between Australia and the Antarctic Continent is of such length and depth that its period of oscillation is more nearly that of the solar tide-producing forces than of the lunar. Consequently, the tides here should show a relatively large solar constituent. And the results of analyses of tide observations made on the southern coast of Australia prove this to be the case.

A number of puzzling features of the tide in various regions are explained nicely by the stationary-wave theory. Such diverse manifestations of the tide as the relatively large rise and fall on the Atlantic coast of the United States as compared with the small range on the Bahamas and Porto Rico, the small diurnal inequality in the Atlantic Ocean tides, the surprisingly large ratio of the solar to the lunar tide at Tahiti—these phenomena find ready explanation under the stationary-wave theory and Harris' conception of oceanic oscillating systems.

Criticisms of Harris' theory have not been wanting. G. H. Darwin, reviewing the "Outlines of Tidal Theory," dissented absolutely from Harris' views and found himself constrained to criticize it adversely. In his "Handbuch der Ozeanographie" Krümmel discusses the oscillating systems and concludes, "Such a conception is mathematically possible, but from our geographic point of view we are compelled to reject these oscillating systems of Harris decisively."

In this connection it is to be observed that the mathematical basis of the theory deals with difficult hydrodynamical questions. The criticism of so well-known an authority as Darwin carried great weight, and his adverse criticism undoubtedly did more than any other cause to bring Harris' theory into disfavor. But in 1910 Poincaré published the third volume of his "Leçons de Mécanique céleste," devoted to the tides. After subjecting the various tidal theories to searching mathematical analyses the great master states "It is probable that the final theory will have to borrow from that of Harris a notable part of its principal features." Poincaré's exposition of Harris' theory has done much to bring it to the fore again.

HARRIS' COTIDAL MAP

On the basis of a careful synthesis of the results of the tide observations made to the beginning of the present century, Harris constructed a cotidal map of the world, in which the course of the cotidal lines in the oceanic areas was drawn in consonance with his tidal theory. This map differs radically from Whewell's, as may be seen on comparing the cotidal lines of the Atlantic Ocean in Figures 2 and 4, the former illustrating the cotidal lines according to Whewell and the latter according to Harris.

Along the coast the differences between Whewell's and Harris' cotidal maps arise from the fact that Harris had at his command the results of more numerous and more accurate tide observations. But the most striking differences are not near the coast but in the open ocean. Taking the North Atlantic Ocean, on Whewell's map the tide is shown progressing in a simple manner. In Harris' map there is no such simple progression; instead, there is a point about which the cotidal lines radiate. Such points Harris designated by the name of "amphidromic points."

As to the causes that bring about amphidromic points, or points of no tide, it is to be remembered that in a stationary wave no rise and fall takes place along the nodal line; hence
in a simple stationary-wave movement there is a line of no tide. Now suppose that in a body of water there are two simple stationary waves, one in an east-west direction, and the other in a north-and-south direction. The first wave gives rise to a line of no tide in the north-and-south direction while in the other wave the line of no tide runs in an east-and-west direction. Where the two lines cross there will manifestly be a point of no tide; and a little consideration will show that from this point the cotidal lines will radiate.

In the interest of historical accuracy it is to be mentioned that the concept of a no-tide point had been put forward by Whewell in 1836 to explain the behavior of the tides in the North Sea. However, this was done not on the basis of dynamic principles or in consonance with a particular theory of the tide but to give the most likely interpretation of the observed facts. Later, in discussing types of tidal movements, he mentions the possibility of an oceanic tidal movement with "cotidal lines revolving round a fixed center" (5).

On his cotidal map of the world, Harris places two amphidromes in the North Atlantic (shown in Figure 4), one in the North Pacific, two in the South Pacific, and one in the Indian Ocean. Because of the difficulties involved in the observation of tides in the open sea there are no practical means at the present time for proving or disproving the existence of these oceanic amphidromic points. But on theoretical grounds the existence of these no-tide points appears unquestioned.

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Fig. 4. — Harris' cotidal map of the Atlantic Ocean.

(5) Article cited in footnote 4.
Harris' name is associated with another tidal question which is altogether separate from his cotidal map and the theory on which it is based. This is the question of land in the unexplored regions of the Arctic. From his study of Arctic tides Harris was led to conclude that a large tract of land existed in the unexplored region of the Arctic. Recently, however, the features of the Arctic tides on which Harris based his inference of land have been shown to be due to other causes, so that the existence of a large land mass in the Arctic appears very doubtful and can no longer be said to be indicated by the tides. It must be emphasized, however, that this inference of land in the Arctic was in no way based on his stationary-wave theory of the tide in the open ocean, and the failure to find an Arctic land mass would in no way invalidate that theory nor the cotidal map based thereon.

RECENT COTIDAL MAPS.

A new cotidal map of the world was published by Dr. Robert Sterneck in 1920. It was based on the fact that an oscillation may be conceived as arising from the interference to two components with phase differences of one-quarter period. This greatly simplifies the mathematical treatment, and on the basis of the actually observed tides along the various coasts and islands Sterneck constructs his map in consonance with hydrodynamic principles. He finds it necessary to place two amphidromes in the Atlantic Ocean, six in the Pacific Ocean, and four in the Indian Ocean. In other words, he has the same number of amphidromic points as Harris in the Atlantic, three more in the Pacific, and three more in the Indian Ocean. Sterneck's cotidal map for the Atlantic is reproduced in Figure 5.

Fig. 5. — Sterneck's cotidal map of the Atlantic Ocean.

More recently Prof. Defant has drawn a cotidal map for the Atlantic and Arctic Oceans. Starting from the consideration that the Atlantic and Arctic Oceans constitute an enormous


canal open only at its southern end—since, in comparison, the narrow and shallow Bering Strait on the north may be totally disregarded—Defant studied mathematically the various oscillations which such a canal can sustain and the relative importance of these various oscillations in bringing about the actual tide. He found that it is the north-and-south stationary-wave oscillation that determines the tide in the Atlantic Ocean.

Defant's mathematical analysis indicates the existence of six primary nodal lines in the canal made up by the Atlantic and Arctic Oceans. The effect of the earth's rotation and of cross oscillations is to eliminate two of these nodal lines and to convert the other four into amphidromic points as shown in Figure 6, which is a reproduction of Defant's map. For the greater part of the Atlantic Ocean the tide behaves as if it were a progressive wave coming from the Southern Ocean; but Defant emphasizes the fact that it is only an appearance, due to the interaction of north-and-south and east-and-west oscillations and not to an actual progressive wave sweeping northward from the Southern Ocean.

Comparing Defant's cotidal map of the Atlantic with those of Harris and Sterneck shown in Figures 4 and 5, a close family resemblance appears, especially in the recognition of the existence of amphidromic points. This is all the more striking since each of these three investigators starts from different theoretical considerations. It may therefore be taken for granted that these cotidal maps represent, in the large at least, the movement of the tide across the open sea.

Fig. 6. — Defant's cotidal map of the Atlantic Ocean.
ON COTIDAL MAPS.

The latest cotidal map of the world to date is by Dr. T. J. J. See, who in a recent publication devoted to tides puts forward "a revised cotidal map of the oceans." (8) This a revision of Whewell's cotidal map and, like the latter, pictures the tide as progressive waves traversing the open sea. Neither Whewell's own objections to this conception nor the criticisms of later investigators appear to have any weight with Dr. See.

Brushing aside the work of Ferrel, Harris, Poincaré, Sterneck, Defant, and other tidal students, Dr. See sets up a new "wave theory" of the tide; but, although making use of considerable mathematical scaffolding, it is unconvincing. Indeed, it has been very severely criticized in a review by Dr. J. Proudman (9) who speaks as a tidal mathematician of recognized authority. Dr. See's cotidal map runs so completely counter to the results of all other tidal investigators of the present time that the author's reputation in other fields cannot be taken as surety for the validity of the mathematical reasoning he advances in support of his thesis.

SEMIDAILY VERSUS DAILY TIDES.

The principal tide-producing forces are those having a period of half a day; and it is for this reason that the tide, the world over with but few exceptions, has a period of half a day. Cotidal maps picturing the tide as it actually takes place, or those based wholly on the semidaily constituents of the tide are practically indistinguishable.

For certain purposes, however, it is important to have cotidal maps of the daily tide. By means of the mathematical process known as harmonic analysis the observed tide at any place may be resolved into its simple constituents. And it is on the results of harmonic analysis that cotidal maps for the daily tide are based. Sterneck has published a cotidal map of the daily tide for the world at large (10), and Defant one for the Atlantic and Arctic Ocean (11). In both of these maps amphidromic points constitute the basic features.

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