NAVIGATION BY GREAT CIRCLE SAILING AND RADIOGONIOMETRIC BEARINGS.

by

INGÉNIEUR-HYDROGRAPHE GÉNÉRAL P. DE VANSSAY DE BLAVOUS,

Director.

I. For nearly a century mariners have used the method of navigating on the arc of a Great Circle, and the works, tables and diagrams designed to help them in the art are innumerable.

Now, aerial navigation over the great routes has revived interest in the subject.

Also, the more and more frequent use, over ever-increasing distances, of radiogoniometric bearings, raises similar problems which have likewise given rise to numerous articles during the last 20 years; among those that have quite recently appeared are:

S. H. LONG. Wireless Navigation. London 1927.

Navegaçāo radiogoniometrica. Curvas e rectas do azimute, by A. FONTOURA DA COSTA, Lisbon 1927.

De Nautica Astronomica, by Capitaine de Corvette Rafaël ESTRADA. Article which appeared in the *Revista General de Marina,* March 1928.

Rivista Marittima, March 1928, page 753. Article by Professor E. MOLFINO on "the determination of the ship's position by radiogoniometric bearings."

Rapid construction, on the chart, of lines of constant bearings for use with radiogoniometric bearings, near shore. Note by Mr. O. GERNEZ, presented by Mr. CHARCOT. *Comptes-rendus de l'Académie des Sciences de Paris,* May 7. 1928, page 1278.

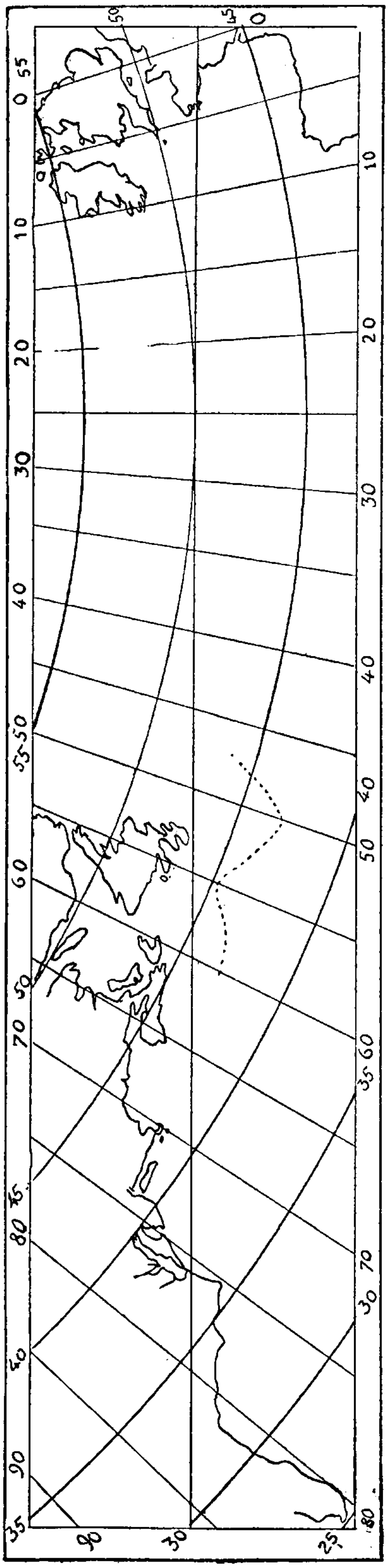
These show that the interest of the subject is not exhausted.

II. Publication N° 8 (20th February 1928) of the records of the sittings of the PARIS ACADEMY OF SCIENCES, contains a note by Mr. Louis KAHN, presented at the sitting of 13th February 1928, by M. P. PAINLEVÉ (*).

The author describes as follows a conformal chart for use as an orthodromic chart for the great routes: —

" the principle consists in developing a cylinder encircling a geographical " sphere, along the length of the route to be followed, or, more generally, " along the length of a great circle which traverses the whole area to be " represented; and in adopting, in order to obtain a conformal projection, " the same law of correspondence between the points of the cylinder and those " of the sphere, as is used in the well known Mercator's Projection.

(*) Since the drawing up of this note an article on the same question has been published by Mr. L. KAHN in the *Revue Maritime,* Paris, September, 1928.



" The axis of the chart, which is the " terrestrial equator in the Mercator's Project- " ion, is here represented by the particular " great circle chosen as the circle of contact.

" The essential properties of this pro- " jection, which does not appear to have " been employed up to now, are that, first, " it is conformal, as is Mercator's Projection, " and second, that the representations of " the great circles exhibit a point of inflexion " on the axis. They can be assumed to " coincide with their tangent of inflexion.

" The chart is thus approximately ortho- " dromic from the axis to a distance limited " by the degree of accuracy desired. "

It appears that the chart which M. L. KAHN proposes, relates only to a zone of the sphere adjoining a great circle chosen as a base. In this zone the outlines are no more distorted than are those in the neighbourhood of the Equator on charts drawn on Mercator's projection. There is practically no distortion either of the angles or of the distances. (*).

The meridians and parallels are represented on it by curves, the study of which, made for the first time in the *Annales Maritimes* by J. BARTHET, in 1847, was often renewed, and has since been completed (see E. GUYOT, *Annales Hydrographiques* 1901, page 60, and *Admiralty Manual of Navigation*, 1922, Vol. II. page 25).

Throughout the extent of the zone under consideration, the parallels will differ very little from circles, and the meridians will be practically straight lines, thanks to the property of curves which represent them of having a point of inflection on the great circle chosen as a base. The azimuth may thus be obtained almost as easily as on the ordinary marine chart.

M. L. KAHN points out that in substituting, on his chart, the orthodromic course

(1) On the scale of a route chart of this kind, obviously the ellipticity of the earth is not taken into account.

by a straight line, the maximum error will occur when the course is parallel to the great circle chosen as base. A straight line 2700 (45°) miles long, 6° distant from the axis, will give a course less than 2 miles longer than the arc of the great circle. The chart can, therefore, as regards scale, be treated practically as a plane.

A radio-goniometric bearing is the arc of a great circle on the sphere ; it will be represented on this chart by a curve, having a point of inflection on the axis, and consequently differing very little from the chord which joins the point of emission to the point of reception. If the point of reception is on the axis, and the point of emission at a distance of 300 miles, the angle between the chord and the observed direction will be 3' at most ; for a distance of 600 miles it would be 8.5' and for a distance of 900 miles, 18'. It is, therefore, always negligible as regards the approximate position obtained from a radiogoniometric bearing. If the bearing of a point of known position is taken, the straight line representing it can be drawn at once. If the bearing is taken of a point of which it is desired to determine the position, this point will be on a geometrical locus ; the theoretical form of this locus has been studied by W. IMMLER, in the *Annalen der Hydrographie*, 1917, page 273. This author gives (diagram 14) the curves of equal difference of azimuth between two points of the sphere $S1$, $S2$, for the ordinary Mercator's Projection. In the projection of M. L. KAHN, the point $S2$ will be the pole, the point $S1$ the radio beacon—the determination of the position by two radiogoniometric bearings of this kind is *Pothenot's* theorem on the sphere, which theorem has also been described by W. IMMLER, in the same article.

In practice, the determination of the position by this process does not require any calculation ; straight lines are drawn in the observed direction, with reference to the meridian of estimated longitude, and passing respectively through the two wireless stations. The point of intersection of these two straight lines will give a new approximate position with regard to the meridian, from which the construction is recommenced, if it appears necessary. Also, the locus of the points of equal azimuth can be constructed, for each radio beacon, by means of a protractor or a station-pointer by determining a point on some of the meridians adjacent to the estimated position ; then joining, by a curve, the points obtained. The construction is easy because the meridians practically coincide with straight lines.

Another method consists, after having fixed a point of the geometrical locus, of replacing this curve by its tangent at this point, which is known to make, with the great circle through the point of emission or, in this case, with the straight line passing through practically this point, an angle equal to double the Givry correction (*).

The projection advocated by M. L. KAHN appears, therefore, most practical ; it deserves to be used to draw up charts of the principal maritime or aerial routes.

A diagram is given which is drawn according to this system, and represents a zone of the globe in the region of the maritime routes between

(*) Difference between the orthodromic and the loxodromic azimuths.

EUROPE and NORTH AMERICA. The great circle taken as a base has its vertex in 50° North Lat., 25° West Longitude.

For aeronautical requirements between EUROPE and NORTH AMERICA, it would, doubtless, be of interest to extend, slightly, the limits of the chart, taking, as a base, an arc of a great circle, the vertex of which would be in 50° North Lat., and 40° West Longitude. The zone represented could be extended 700 miles to the North and the South, and 1500 miles to the East and to the West of this point; thus including all the region where an airman might go during these flights and yet keeping, with sufficient accuracy, the property of allowing for the representation of the great circles by straight lines.

In Section 5 (30th July 1928) of the *Comptes-Rendus des Séances de l'Académie des Sciences*, Mr. Louis KAHN points out that the conformal chart which he proposed may be used for fixing the position, particularly in aircraft, by observing the altitudes of heavenly bodies.

If care be taken to observe a star, the position of which may be plotted on the chart and the zenith distance of which is not too great, it will be sufficient to plot this distance, from the star, along the line joining it to the assumed position. The locus of position will be an arc of a circle near the assumed position, with a certain error ε due to plotting the zenith distance along a line which is not strictly a great circle.

In very unfavourable cases, when the star and the observed position are 15° from the axis of the chart, the values reached by ε are given in the table below. These values are quite negligible for fixing the position in aircraft:

Zenith distance	5°	15°	25°	35°
ε	1.3"	14.5"	53.2"	2'26"

III. In order to be able to compare this method, it would, perhaps, be interesting to epitomise briefly some of the graphic means employed, up to now, to solve the problems of navigation on the arc of a great circle and radiogoniometric bearings.

a) *MERCATOR'S PROJECTION.*

In 1857, A. H. DEICHMAN brought out a diagram giving a representation of great circles on the *Mercator's Projection* system, and J. CARSON BREVOORT, in 1887, follows the same idea, tracing great circles on transparent paper, which can be moved over a chart on the same scale, on Mercator's Projection.

In 1922, Ingénieur Hydrographe FAVÉ perfected this method, and compiled tracings N° 5603 *bis* and 2 *bis*, which enable, on the planispheres on Mercator's Projection N° 5603 and 2 (A, B, C,) respectively, the orthodromic courses, their lengths and the course angles, to be determined; they enable also the drawing of the arc of a great circle corresponding to an azimuth observed from a known point. Moreover, by a very ingenious process, the same tracing enables the curve of equal azimuth to be fixed by points, when the azimuth of a known point has been taken by wireless.

The tracing, N° 2 *bis*, laid on a planisphere of a scale of 5 $^{m}/_{m}$ to 1° longitude, enables the operations to be carried out with quite sufficient accuracy. These tracings and planispheres are published by the FRENCH HYDROGRAPHIC OFFICE.

Sir George AIRY, in order to draw the arc of a great circle directly on any Mercator's chart whatever, replaces the curve, which represents this arc of the great circle, by an arc of a circle passing through the points of departure and arrival, the radius of which is the radius of curvature of the curve at the middle latitude.

This process amounts to drawing a circle passing through the points of departure and arrival, and touching, at these points, straight lines which make angles equal to the Givry correction with the chord. It is easy to have as many points of this circle as desired, without fixing the centre, which may be off the chart.

As G. FISHER suggested in 1864, it is possible, by calculation, to determine a third point, conveniently chosen, of the great circle which it is desîred to follow, and to make an arc of a circle pass, on the Mercator projection, through the points of departure and arrival, and through this third point.

A. WEDEMEYER, in the *Annalen der Hydrographie* of 1927, page 505, proposed, in order to draw the arc of the great circle between two points on a Mercator's Projection, to use the following method: —

Let φ be the latitude of the vertex of the great circle. Let a small circle, passing through the pole, having its centre at a distance φ from the pole, and on the same meridian as the vertex of the great circle, be considered. M and m being a point of the great circle and a point of the small circle on the same meridian, the latitude of M is half the distance to the pole of m. If, therefore, the curve representing the small circle is drawn on Mercator's Projection, it will be easy to deduce from it, by points, the representation of the great circle. As the small circles which pass through the pole are all similarly represented (Curves of the third class of Guyot), on a Mercator's planisphere on a given scale, it is sufficient to cut out, once for all, this curve in cardboard, from which all the great circles that it is desired to draw on the same planisphere, may be deduced.

As regards the use, on Mercator's projection, of radiogoniometric bearings taken from a known point, the above methods may be used to draw the arc of the great circle, on which the required point is found. More often, when the point is not more than 100 miles away, it is sufficient to apply the Givry correction to the observed direction, calculated according to the assumed position of the required point, and to trace a straight line in this direction.

If the bearing is taken of the point, the position of which it is desired to determine, it defines a geometrical locus, line of equal azimuth, on which the observer is situated. These lines, which are on the sphere curves of the 4th order, and which can also be considered as the intersection of two cylinders of the 2nd order, have been very carefully studied.

An article published by A. WEDEMEYER on the subject should be consulted in the *Annalen der Hydrographie*, 1910, page 417, wherein these curves are examined on different systems of projection: also articles by W. IMMLER, 1917, page 381, 1921, page 182 and 1925, page 127; those of S. von KOBBE, 1925, pages 73, 187 and 333; those of Prof. Dr. Ernst WENDT, 1925, pages 97 and 157, and that of H. MAHNKOPF, 1925, page 353; see also, in the *Rivista Marittima*, the studies of E. MODENA, February 1919, of Prof.-Doct. Guiseppe SIMEON, January 1926, and of Prof. O. GILIBERTO, January 1927; likewise the study already referred to of A. FONTOURA DA COSTA.

In view of the relative inaccuracy of the radiogoniometric bearing, it generally suffices to use the graphic method, which consists in applying the Givry correction to the observed direction, calculated for the assumed position, and in replacing the great circle by the loxodromic line.

If a considerable error has been made in the assumed position the graphic construction is made again with a new value of the Givry correction.

If φ_o and φ_v denote the latitudes of the point of observation and of the point from which the bearing is taken, and Δl their difference of longitude, then the expression adopted for this correction is:

$$u = \frac{\Delta l}{2} \sin \frac{\varphi_o + \varphi_v}{2}$$

or

$$u = \frac{\Delta l}{2} \frac{\sin \frac{\varphi_o + \varphi_v}{2}}{\cos \frac{\varphi_v - \varphi_o}{2}}$$

It should be noted that, at the point of observation the locus of equal azimuth makes an angle c with the great circle, given by the formula

$$\tan c = \tan \Delta l \sin \varphi_o$$

So that by neglecting infinitely small quantities of an order superior to Δl, it may be said, in comparing the values of c and u, that the loxodromic line bisects, at the point of observation, the angle formed by the great circle and the locus of the points of equal azimuth.

This property remains true for all conformal projections.

The BRITISH ADMIRALTY in the "List of Wireless Signals" 1927, gives (page 5) a graphic scale on which is read in minutes the convergence of the meridians (double the Givry correction) for 10' difference of longitude, as well as a table (page 9) which gives the Givry correction for middle latitudes for each degree and differences of longitudes for each half degree (See also *Admiralty Manual of Navigation*, London 1922, Vol. II, page 20 to 38).

The HYDROGRAPHIC OFFICE OF THE UNITED STATES OF AMERICA in the "Radio Aids to Navigation" of 1927, gives the same table, page 6. It publishes also the Radio bearing conversion diagram, which first appeared on the Pilot Charts in 1923 and is now chart N° 5193. This diagram, which gives directly the corrected azimuth, is based on the latitude of the point of observation, the azimuth observed and the distance from the point of which the bearing is taken.

The FRENCH HYDROGRAPHIC OFFICE has reproduced this diagram at the end of the List of Radiotelegraphic Signal Stations (Liste des Stations de signaux radiotélégraphiques 1927).

The GERMAN ADMIRALTY in the *Nautischer Funkdienst*, 1928, page 29, gives a graphic scale, off which is read, in tenths of degrees, the Givry correction for a difference of Longitude of 1°. A diagram, page 30, gives with more accuracy the angle between the loxodromic line and the great circle, according to the latitudes of the two extremities ; Tables I and II, pages 36 and 43 may also be used.

In 1921 the U. S. COAST AND GEODETIC SURVEY published, with the same object, the graphs *R*1, *R*2, *R*3. The first two, entitled : " Graphic Chart semi-convergence of meridians" give the correction u according to the formula :

$$\tan u = \tan \frac{\Delta l}{2} \; \frac{\sin \dfrac{\varphi_o + \varphi_v}{2}}{\cos \dfrac{\varphi_v - \varphi_o}{2}}$$

The third, entitled "Graphic Chart, second reduction term, true bearing to Mercator bearing" gives a correction which must be added to, or subtracted from, the correction u in order to obtain, with more accuracy, the angle between the arc of the great circle and the loxodromic line at the point of observation, or at the point of which the bearing is taken. It is expressed thus :

$$v = \frac{1}{12} (Y_v - Y_o) \; \Delta l$$

Y_v and Y_o being the meridian parts corresponding to φ_v and φ_o. (See *Radio Compass Bearings* by Oscar S. ADAMS, *Special Publication*, N° 75, 1921).

In 1923, the SWEDISH HYDROGRAPHIC SERVICE published Chart N° 16 of the Skaggerak and the Kattegatt, which is meant to facilitate the use of bearings received from three stations provided with similar radiogoniometric apparatus. On this chart, which is on Mercator's projection, curves are drawn which represent arcs of great circles corresponding to the bearings sent out from these stations ; this enables the bearings received to be used with great speed and accuracy. If the bearings are taken from the ship, it is necessary to apply to them a correction equal to the convergence of the meridians, that is to say to double the Givry correction, in order to obtain the figure corresponding to the great circle on which the position should be, and consequently successive approximations can be plotted on it. (See *Hydrographic Review*, May 1924, page 117).

b) *PLANE CHART PROJECTION.*

John T. TOWSON appears to be first to have proposed the solving of the Spherical Triangle by resolving it into two rectangular triangles. This system has been used many times since, because it has the advantage of enabling tables or diagrams to be drawn up with only two entries. In 1848, the BRITISH ADMIRALTY published "Tables for Great Circle Sailing" by John Thomas TOWSON. To find the vertex of the great circle, a Linear Index is used, drawn up on the system of cylindrical projection of the plane chart; taking the Equator for the horizontal axis, a meridian for the vertical axis, the parallels become equidistant horizontals (the scale is not the same on the 2 axes); the curves represent great circle arcs passing through two fixed points on the Equator; the numbers of the curves and the straight lines representing the parallels, must be reckoned from the pole instead of from the Equator (1).

In 1858, Hugh S. GODFREY published a diagram for the determination of the course angle and distance to the vertex (course and distance diagram). It is a plane chart projection with reference to a meridian which is taken for the horizontal axis. The verticals represent great circles which all pass through two points of the Equator, situated on the perpendicular to the base meridian; they all have their vertex on this meridian and the distance from one of their points to the vertex is measured directly (but on a scale different from that which is adopted for the meridian.) Curves are also drawn representing the parallels, and curves of equal course angle, which are none other than curves of equal azimuth for the points of the Equator common to all great circles perpendicular to the base meridian. This diagram enables all right angled spherical triangles to be solved.

The Ingénieurs Hydrographes FAVÉ and ROLLET DE L'ISLE described in the *Annales Hydrographiques* of 1892, page 159, a diagram which they had engraved, which is derived either from the *transverse plane chart* or *Cassini Soldner's projection*, and which shews meridians and parallels for every 10'. This diagram enables any right angled spherical triangle to be accurately solved to 1' and in consequence any spherical triangle by means of a simple addition or subtraction.

c) *STEREOGRAPHIC PROJECTION.*

In 1857, W. C. BERGAN published tables for the general solution of problems of nautical astronomy, and a diagram which represents the *stereographic projection* of a quarter of the sphere on a plane parallel to the line of the poles. The meridians and parallels are drawn on it for each degree, as well as the arcs of great circles converging at the Eastern end of the Equator. This diagram had for its object the fixing of the latitude and longitude of the vertex of the great circle passing through two points; knowledge of the position of this vertex was necessary for using the tables.

CHAUVENET has likewise used (Chauvenet's great circle protractor) the *stereographic projection* on a meridian plane. Two identical reticulations formed of circles representing the meridians and parallels in this system of projection, and capable of being turned about the centre of the figure, are sufficient to solve all problems concerning the course on the arc of a great circle.

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(1) See : *The Development of Great Circle Sailing*, by C. W. LITTLEHALES, Washington 1899; and *Studie uber die Schiffahrt im Grossten Kreise*, by August ROTH. *Annalen der Hydrographie* 1904, page 375.

SIGSBEE, then HARRIS, perfected the method, showing that a single network is sufficient and avoiding the rotation, always difficult, of two figures about their common centre.

In 1906, the Azimuth Diagram of G. W. LITTLEHALES was constructed on the same principles.

Richard A. PROCTOR uses the polar stereographic projection. The great circle passing through two points is represented by a circle which can be traced when a third point has been fixed, for example, the antipodes of one of the points of departure or arrival. It is then easy to fix the vertex, the course *angles*, (which are preserved because the projection is *conformal*) and, by a simple enough construction, the distances.

Prof. Dr. TEEGE in the *Annalen der Hydrographie* of 1918, page 146, used the *polar stereographic projection* to construct a very simple azimuth diagram based on the following principle. Let a great circle passing through the points A and B be considered; it passes also through the antipodes C of A, and the angle BAC, which is preserved in projection, is none other than the desired azimuth (or its supplement, according to the convention adopted). This property can also be used when the *transverse stereographic projection* (on a meridian plane) is employed, as has been shown, in the *Annalen der Hydrographie*, by A. WEDEMEYER in 1918, page 215 and H. TEEGE in 1922, page 229. This projection also gives the azimuths very rapidly: placing the point A on the meridian of the plane of projection, an angle ABC is obtained, which is equal to the azimuth increased by a right angle.

Fredreich NEUMANN, in Jahresbericht des Königl. Domgyumasiums in Halberstadt 1906, Prog. M. 291, used the principle which he appears to have been the first to have pointed out, of the small circle passing through the pole (see above (*a*) A. WEDEMEYER) to deduce the track of a great circle. He uses a *polar stereographic projection* and a small circle passing through the projected pole. This small circle will be represented by a straight line from which the great circle may be easily deduced.

d) *ALESSIO PROJECTION.*

In the *Rivista Marittima* of July-August 1908, page 107, Lieutenant A. ALESSIO explains the principle of a diagram which the ITALIAN HYDROGRAPHIC SERVICE has brought out to solve all nautical astronomy problems relative to the triangle of position. He uses a perspective projection of the sphere on a plane tangent to the Equator. This projection differs from the Stereographic projection by the fact that the centre of perspective is at the distance $\dfrac{R}{\dfrac{\pi}{2} - 1}$ from the centre of the sphere instead of being at the distance R. The poles are thus projected at their true distance from the Equator, but the projection is no longer *conformal*. The meridians and parallels are drawn for every 15'; they are arcs of ellipses.

e) *GNOMONIC PROJECTION.*

The *Gnomonic projection* has the advantage of representing the great circles by straight lines, also it has been very often used for Navigation on the Great Circle. In 1858, Hugh GODFREY drew up two gnomonic charts taking the poles as points of contact; these charts extend to 70° from each pole.

In 1879, Lieutenant HILLERET published orthodromic charts of the three oceans on the gnomonic projection on planes tangent to the Equator. These charts are issued by the FRENCH HYDROGRAPHIC OFFICE under the numbers 3680 and 3682.

The diagram of Gustave HERRLE, which bears the name of "Direct Track Scale" is none other than a gnomonic projection on a plane tangent to the Equator. The meridians are parallel straight lines, the parallels hyperbolas. The scale has been reduced as regards the longitudes, which does not hinder the great circles from continuing to be represented by straight lines.

In 1881, Gustave HERRLE drew up orthodromic charts adopting as the tangent point of the plane of projection a point situated toward the centre of the ocean which he desired to represent.

It is according to this system that the charts of the WASHINGTON HYDROGRAPHIC OFFICE N^os^ 1280, 1281, 1282, 1283, 1284, 1286 have been constructed.

Chart N° 5400 is specially designed for use with radiogoniometric bearings on the EAST COAST OF NORTH AMERICA.

Starting from the Coastal radiogoniometric stations, the great circle arcs, along which the bearings are taken, are represented by straight lines. These lines can be drawn by means of arcs of circles divided in degrees for values of the bearings. As the gnomonic projection is not conformal they are not equally separated and the divisions of the circles are not equal. If on the other hand bearings are taken of a station from a point of which only the approximate position is known, it will be necessary to correct the bearing with double the Givry correction and then deduce from it the bearing which should have been obtained from the radiogoniometric station.

It is this value which enables the straight line corresponding to the observation to be drawn on the chart. A second approximation might be necessary if the approximate position were found to be too inaccurate.

In 1923, the GERMAN ADMIRALTY also published, on gnomonic projection, chart N° 8, which represents a part of the North Sea, indicating the directions of the bearings sent out from the radiogoniometric stations of the German, English and Danish coasts and, in 1926, chart N° 7, which gives the same information for the Southern part of the Baltic Sea.

In 1914, the BRITISH ADMIRALTY published, on the gnomonic projection system, Ocean charts N^os^ 11, 18, 132, 42 and 53.

Distances can be measured on the gnomonic projection by means of simple constructions indicated by HILLERET (1874) and by G. PELLEHN (1917). See also *Annalen der Hydrographie*, 1919, page 22. W. IMMLER.

f) *LAMBERT'S PROJECTION.*

The Coast and Geodetic Survey of the United States of America publishes for its coasts and for the North Atlantic, outline charts Nos 3069 (1921), 3060 (1919), 3068 (1919) and 3070 (1918) on Lambert's conformal conical projection. On these charts, the great circles differ so little practically from straight lines that it is generally useless to make any distinction. Besides, this projection being conformal and the meridians being straight lines, the *course angles* are easy to measure and the distances are so little modified in the zone for which the chart is drawn up, that they can be measured as if the scale of the chart were constant.

g) *POLYCONIC PROJECTION.*

In 1921, the Coast and Geodetic Survey of the United States of America published chart No 3080 of the North Pacific on the system of *transverse polyconic projection,* taking as a base a straight line representing a great circle which passes very near Singapore and Panama. The distances below 500 miles are affected by an error of less than 2 1/2 %, or 12.5 miles. The meridians are slightly curved.

h) *LITTROW'S PROJECTION.*

J. J. Littrow, in his book "Chorographie oder Anleitung alle Arten von Land-. See- und Himmelskarten zu verfertigen,—Wien 1833" describes on page 122 a conformal projection which has the property of representing lines of equal azimuth by straight lines, that is to say, the locus of the points from which a known point is seen at a constant azimuth.

In this projection (which has also been studied by Gretschel, Herz, Fiorini, Holzmuller and Tissot) the parallels are ellipses and the meridians homofocal hyperbolas.

Taking as base meridian the meridian of the point of which the bearing is taken, which is fixed by its latitude, the line of equal azimuth is a straight line, which makes an angle equal to the supplement of the azimuth, with the meridian. The diagram, founded on the principle of this projection, gives the solution of all spherical triangles; it also enables the problem of finding the intersection of two lines of equal azimuth to be solved.

For this latter problem, the meridian of one of the points, of which the bearing is taken, and of which the position is known, should be taken as the base meridian, and the straight line, representing the first locus, should be drawn; then draw the line which would represent the second locus, if the meridian of the second point, of which a bearing is taken, was taken as the base, and this line would be transferred to the first system by correcting the longitude of each of these points, without altering the latitude.

It is Littrow's projection which Captain Weir used in his *Azimuth Diagram* published in May 1890 by J. D. Potter. It is this also which is the basis of the "Azimuth und Kurstafel" proposed by Dr. H. Maurer. in

1905, and which, since 1917, forms part of the Chart Sets of the GERMAN ADMIRALTY, under the name of *Azimuth-Messcarte.*

It is on the same principle that the diagram is based which serves, since 1892, on the gnomonic charts of the HYDROGRAPHIC OFFICE OF THE UNITED STATES OF AMERICA, to measure the course angles for navigation on an arc of a great circle.

See on this subject :

Traité des Projections, by GERMAIN, 1866, p. 74.
Rivista Marittima 1895, GELCICH.
Annalen der Hydrographie 1905, p. 125, Dr. Hans MAURER.
» » 1918, p. 209 et 1919, p. 183, A. WEDEMEYER.
» » 1919, p. 14 and 212, Dr. Hans MAURER.
» » 1920, p. 265, Prof. W. IMMLER.
Annales Hydrographiques 1904, p. 170, Lt. de Vaisseau PERRET.
Nautical Magazine, December 1927, p. 485.
Leitfaden der Kartenentwurfsehre, by Prof. Dr. Karl ZOPPRITZ, Leipzig and Berlin 1921, p. 221.
Breuzing-Meldau-Steuermannskunst, supplement, Bremen 1927, p. 89.

i) *VARIOUS DIAGRAMS.*

Numerous diagrams which are not based on a system of cartographic projection, have also been designed to solve these problems. They do not come into the scope of this article.

We have doubtless omitted, although involuntarily, several graphic processes derived from cartographic projections and used in certain countries. We will be glad to repair our omissions if the HYDROGRAPHIC OFFICES will be good enough to make them known, by sending charts or diagrams concerning this subject to the INTERNATIONAL HYDROGRAPHIC BUREAU.