ASTRONOMICAL REFRACTION IN HISTORY.

by

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During my research into refraction at low altitudes, I found in the library of the observatory of Leiden a book on refraction, in the shape of the prize thesis by Dr. C. BRUHNS, professor at Leipsig (Leipsig, 1861), on the subject, *Die astronomische Strahlenbrechung in ihrer historischen Entwickelung* (Astronomical refraction in its historical development). This had been set for competitive discussion, and in compiling his essay Dr. BRUHNS consulted about seventy books. I propose here to summarise very briefly the work in question, giving also some information obtained from other sources.

The first mention of astronomical refraction is by CLEOMEDES (probably about the beginning of our era), in his work, *Circularis inspectio meteorum.* He says he has heard it stated that an eclipse of the moon can be seen when both the sun and the moon are below the horizon. Here is his explanation: "In the same way that a ring in a barrel becomes visible above the edge when water is poured into it, so the sun, on account of refraction, can already be seen even when it is actually still below the horizon."

There are many more details in PTOLEMY, professor at Alexandria (first half of the II century A. D.), in his work on optics, in which he gives tables of refraction for air and water and for air and glass. He then proceeds to deal with astronomical refraction, on the subject of which he states that the higher the star, the smaller the difference between the true and the apparent positions; and that at the culminating point the true and apparent positions coincide, no doubt because vertical rays do not undergo any refraction.

He shows by a sketch that refraction always brings stars together at the culminating point and that this also applies in certain special cases with regard to the pole; in the latter case, however, the opposite may also occur. He evidently refers here to stars which culminate between the summit and the pole.

According to PTOLEMY, the cause of refraction is the passage of the light rays from the ether into the denser air, which he represents as being spherical round the centre of the earth. To determine the value of the refraction, he proposes observing the sun and moon or, better, the fixed stars, at all culminating altitudes possible, and to compute the true culminating altitudes, the differences between the two culminating altitudes thus giving the refraction.

This determination was not possible for him on account of the lack of precision of his instruments; neither was it possible for him to draw up a refraction table by theoretical methods, because he did not know the depth of the layer of terrestrial atmosphere.

He says, further, that there must be a law giving the relation between the angle of incidence and the angle of refraction.

It follows from all this that PTOLEMY had an absolutely correct conception of refraction. Many later experts, among them TYCHO BRAHÉ, failed to arrive at this correct conception.

After PTOLEMY the advance of science marked time for a considerable period. Continual wars distracted attention from science, while in the middle ages the dominating influences which prevailed repressed every effort towards the light of truth, and more than thirteen centuries elapsed in the gloom of superstition.

There was, however, during a brief period, a meagre gleam of light. The Arabs, whose caliph OMAR had already in 641 burnt the precious library of Alexandria, devoted themselves to astronomy a century and a half later, when they had converted Western Asia and Northern Africa to Islam. The caliph ALMANUM himself (about 820) studied this science.

About 1100 there lived among the Arabs the savant ALHAZEN, who left a work on optics in which for the first time refraction is mentioned (1). It is more than probable that ALHAZEN was unaware of PTOLEMY's work which we mentioned above. He states that he found by experiment that the refracted and the incident rays lie in a plane surface, and that the ray of light, when it enters a denser medium, approaches the normal. Nevertheless it does not seem possible to him that a definite ratio can exist between the angle of refraction and the angle of incidence for the whole of the quadrant.

He states that the refraction can be determined by observing stars passing through the culminating point. By means of an armillary sphere (2), the polar distance is measured at the time of the upper and lower transit, and the refraction is determined from the difference of the polar distances found.

PTOLEMY expresses himself more categorically in stating that refraction disappears at the zenith.

There are also indications which tend to prove that ALHAZEN was well acquainted with the behaviour of refraction: for example, he observes that under the influence of refraction the stars do not exactly follow their diurnal arc and, further, that the median line (vertical) of the heavenly bodies must appear smaller near the horizon owing to the influence of the refraction.

Later, mention is made of refraction by Roger BACON (1214-1294), professor at Oxford, but only very superficially.

More than two hundred years then pass by before mention is made of refraction in any book whatever.

The wealthy Nuremburg patrician Bernhard WALTHER, collaborator with and successor to REGIOMONTANUS, was one of the first to study practical astronomy afresh after all these centuries. For the first time (in 1484) he used a clockwork wheel mechanism for his observations, and introduced the method of determining the position of a heavenly body by measuring its distance with respect to two fixed stars. Refraction was discovered anew by him.

WILLEM, landgrave of Hesse, and his mathematician ROTHMANN, and MAESTLIN, KEPPLER's teacher, confirmed by their observations the existence of the refraction found by WALTHER.

As observations could, from that time on, be made with greater accuracy than previously, what PTOLEMY and ALHAZEN had found theoretically was determined empirically. No doubt the values found by WALTHER and his immediate successors must have contained grave errors.

It was TYCHO BRAHÉ (1546-1601) who, at the observatory (Uranienburg) built for him in 1576 by King FREDERICK II of Denmark on Huene Island in the Sound, obtained more accurate values for the refraction.

He determined the polar altitude (latitude) by observing circumpolar stars and also by observing the meridian altitude of the sun at the winter solstice ($\pm$ 11°).

In doing so, he found a difference of 4', and first sought for the cause in the instrument used. But when he found the same difference with other instruments, the happy idea occurred to him of attributing it to refraction.

From that moment he commenced, with an armillary sphere of ten feet radius which he had built himself, and with a quadrant, to measure the height of the sun from sunrise to sunset. He also calculated these heights. The height observed, minus

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(1) *Noted by Vitello (Basle, 1572).*

(2) *An armillary sphere consists of a system of wooden and copper circles by means of which the astronomical latitude and longitude of a heavenly body can be determined. The instrument must have been invented in the earliest past, for Hipparchus (II century B. C.) and Ptolemy had already made use of it. Tycho Brahé also made some observations with the armillary sphere.*

the height calculated, gave him the refraction. His results are given in the following table.

TYCHONIS TABULA REFRACTIONUM SOLARIUM

Altit. ⊙ *Gradus*	*Refractio* ′	″	*Altit.* ⊙ *Gradus*	*Refractio* ′	″
0	34	0	23	3	10
1	26	0	24	2	50
2	20	0	25	2	30
3	17	0	26	2	15
4	15	30	27	2	0
5	14	30	38	1	45
6	13	30	29	1	35
7	12	45	30	1	25
8	11	15	31	1	15
9	10	30	32	1	5
10	10	0	33	0	55
11	9	30	34	0	45
12	9	0	35	0	35
13	8	30	36	0	30
14	8	0	37	0	25
15	7	30	38	0	20
16	7	0	39	0	15
17	6	30	40	0	10
18	5	45	41	0	9
19	5	0	42	0	8
20	4	30	43	0	7
21	4	0	44	0	6
22	3	30	45	0	5

Even this early table shows a certain agreement, with regard to low altitudes, with the values which are accepted nowadays.

In spite of this, TYCHO BRAHÉ does not seem to have had a clear conception of refraction. He states, for example, that the sun's parallax contained in this value must be subtracted from the amount of the refraction. Assuming a value of 3′ at the moment of culmination of the sun, the refraction must change sign even for an altitude of about 25° — an error into which PTOLEMY himself did not fall. By faulty reasoning, TYCHO BRAHÉ concludes from this that refraction relative to the stars and the planets must differ from that relative to the sun.

TYCHO BRAHÉ also made use of two quadrants in his measurements. With one of these quadrants (vertical and slewing) he measured altitudes, particularly those of the sun near the solstices; with the other, which was arranged horizontally and included a movable alidade, azimuths. With latitude, declination and azimuth as arguments, he computed the altitude of the sun. This method was also followed more recently by PIAZZI (Palermo, early XIX century) and gave at the time some good results.

But to TYCHO BRAHÉ is due the honour of having been the first, by effectively measuring the refraction, to obtain as high a degree of precision as the instruments of the times permitted.

KEPPLER (1571-1630) devoted a considerable amount of work to the determination of refraction, but was not, as it happened, as fortunate as he deserved to be on account of his zeal. He did not succeed in improving what TYCHO BRAHÉ had found, and the other astronomers of the times did not risk challenging the authority of TYCHO, so that nearly a whole century elapsed before the idea occurred to anyone to improve on TYCHO's tables.

In 1617 there appeared the book, *Refractiones coelestes, sive solis elliptici,* by the celebrated optician, Fr. SCHREINER, S. J. (1580-1650).

The great merit of this book lies in the fact that it gives everything that had been found up to date, and, in addition, numerous observations on the subject of the elliptical shape of the sun at low altitudes; but neither the practical nor the theoretical aspect of refraction made any appreciable advance as a result of it.

LANDSBERGER (1541-1632), a Dutch astronomer at Middelburg, attained celebrity through his solar and lunar tables, but his tables of refraction agree fairly well with those of TYCHO BRAHÉ and are, in consequence, also inaccurate.

RICCOLI (1598-1671), professor at Bologna, wrote a new almagest in which is an appendix devoted to refraction. In it he analyses the works of ALHAZEN, TYCHO and KEPPLER, and gives the law of refraction enunciated by DESCARTES in 1637, but without applying it to astronomical refraction.

At the end, he gives tables of refraction for air and water, air and glass, water and glass, and vice versa (all from VITELLO); for air and water by KEPPLER and by himself, etc.; finally, refraction tables by TYCHO, KEPPLER and LANDSBERGER, besides his own, the latter computed from his observations at Bologna. The only thing he gives more than TYCHO did is a series of three different tables for summer, the equinoxes and winter, followed by remarks on the influence of temperature. For horizontal refraction in winter he finds a good two minutes more than in summer, which, for normal temperatures of 6° C. (42.8° F.) in January and 24° C. (75.2° F.) in July (at Bologna) agrees closely with the refraction according to our own refraction table, viz. 174" — 54" (= 2').

For horizontal refraction in summer he gives the sun 32'25", the moon 33'0" and the stars 29'50", and he thinks, with TYCHO BRAHÉ, that there are different refractions corresponding to different heavenly bodies.

HEVEL (1611-1687), the celebrated Danziger astronomer, was also a faithful partisan of TYCHO.

In his *Prodromus astronomiae* of 1640 he gives refraction tables for the sun and the stars with respective refractions of 30' and 26'. He gives much information on earlier writings, explains how it happens that refraction raises the stars, because it is strongest near the horizon and disappears at the zenith. He also knows that refraction lengthens the day, makes the sun near the horizon nearly elliptical, and is not always of the same magnitude at the same altitude.

He speaks of the work of Professor LINNEMANN at Königsberg, and of the British mathematician GRAVES, according to whom the terrestrial atmosphere must be divided into three layers of different densities. Further changes are caused in this density distribution by rising vapours, as a result of which the path of the light is not always curved in the same way, but corresponds sometimes to a parabola and sometimes to a hyperbola.

He shows also that the curvature of a comet's tail is not a consequence of refraction.

We have already spoken several times of the law of refraction. KEPPLER sought it in vain, but it was found in his lifetime by a young Dutch mathematician WILLEBRORD SNELLIUS (1590-1626).

The work on optics in which SNELLIUS included his results was never published — he died before; but VOSSIUS and HUYGENS were able to examine the work in question and announced that SNELLIUS had discovered that:

"The sine of the angle of refraction is equal to the sine of the angle of incidence multiplied by a constant."

DESCARTES (1596-1650) in this *Dioptrica* gives the same law under the form: $\frac{\sin i}{\sin r} = a$ constant. He reaches this result in a different way from SNELLIUS.

It is not known whether DESCARTES discovered this law himself, or whether he knew of it in Holland and presented it differently, for in the whole of the work he does not mention the source of his information, nor the name of its discoverer.

VOSSIUS, in his *De lucis nature et proprietate* (Amsterdam, 1662), has no doubt that DESCARTES heard talk of this law during his sojourn in Holland; HORTENSIUS, a friend of SNELLIUS, had announced it in public. HUYGENS thinks that DESCARTES was able to examine SNELLIUS' manuscript. What is certain is that DESCARTES lived for a long time in Holland and was in friendly relations with the first families of the country, including among other people the father of HUYGENS.

As soon as the law of refraction was known, it was possible to build up theories on astronomical refraction. Two things, as it happened, were of great assistance: the discoveries of the thermometer by Corn. BREBBEL (1638) and of the barometer by TORRICELLI (1643).

The thermometer was considerably improved by the substitution, as liquid, of alcohol and later of mercury for the original cupric solution, but particularly when at the beginning of the XVIII century the method was introduced of graduating between boiling point and freezing point (FAHRENHEIT, RÉAUMUR and CELSIUS).

The discovery that the weight of a given quantity of gas was equal to the product of the volume and the pressure was also of great importance in the theory of refraction. This law, now universally known, was discovered independently by BOYLE in 1662 and by MARIOTTE in 1679.

The first to establish his tables on more modern bases was CASSINI (1625-1712), who originally lived at Bologna but was afterwards summoned by LOUIS XIV to Paris where he became the first Director of the observatory completed in 1670. He developed an exact theory of the movement of the satellites of Jupiter and made many discoveries. He had already occupied himself at Bologna with making observations for his solar tables, and had noted that it was necessary for that purpose to have accurate refraction tables. He found the law of refraction already discovered thirty years earlier by SNELLIUS, determined the horizontal refraction as 30'20" and the refraction for an altitude of 10° as 5'28"; from this he deduced the coefficient of refraction and the depth of the atmospheric layer, and from the results obtained drew up his tables of refraction. These tables give values which agree closely with our refraction table, for example 0'59" (58".3) for an altitude of 45°, but not, however, at low altitudes. For 5°, 4°, 3°, 2°, 1° and 0° of altitude they give respectively 10'32" (9'52"), 12'48" (11'46"), 16'6" (14'23"), 21'4" (18'19"), 27'56" (24'39") and 32'20" (35'14").

Many celebrated physicists and astronomers have tackled the task of determining atmospheric refraction more accurately.

NEWTON (1643-1727) showed the way that was to lead to the goal by theoretical means. He showed that refraction is a consequence of the force of gravity, on which depend the differing densities of the atmospheric layers.

Using formulae based on this theory, HALLEY (1656-1742) worked out tables which also agree very closely with present-day ones: for altitudes of 0°, 10° and 45° they give respectively 33'45" (35'14"), 4'52" (5'19".3) and 0'54" (0'58".3).

A table prepared theoretically by Daniel BERNOULLI (1738) gives the refraction at 0°, 10° and 45° of altitude respectively as 34'53" (35'14"), 5'28" (5'19".3) and 1'3" (0'58".3).

LACAILLE in 1755 published an empirically computed table and produced the first tables for the influence of the height of barometer and thermometer.

SIMPSON deduced formulae on NEWTON'S principle, which were published in 1743 and somewhat modified by BRADLEY (1693-1762). The tables computed by BRADLEY with these corrected formulae were used for a long time after the tables of CASSINI (from the second half of the XVIII century). For altitudes of 0°, 10° and 45° they gave respectively 33'0", 5'15" and 0'57".

These tables were valid for a temperature of 50° F. and the barometer at 29.6 English inches; for other heights of thermometer and barometer, correction tables were given.

EULER (1754) and LAMBERT (1759) produced theoretical solutions, as also did LAGRANGE (1772) and ORIANI (Milan, 1788).

However, it was known towards the end of the XVIII century that none of the theories hitherto established could be absolutely accurate: for example, the depth of the layer of the earth's atmosphere is much less than as deduced from the duration of twilight and from experience acquired in mountaineering and in ascents in the balloon invented by MONTGOLFIER in 1783.

It was to the scientists of the XIX century that the possibility was given of establishing more and more accurate theories, thanks to the continued improvement in instruments and the great progress of physics and mathematics. KRAMP (Strasbourg, 1799), LAPLACE (1805), BESSEL (1819, in *Fundamentis astronomiae*), YOUNG (1819, 1824), IVORY (1823, 1835, 1838), SCHMIDT (1828), BIOT (1839, 1841, 1854) and LUBBOCK (1840, 1855) in succession developed new theories or compared those which already existed.

The tabular values determined from the different formulae agree relatively well for heights above 2°. It is only for very low altitudes that big differences arise ; these must probably be attributed to local circumstances such as solar radiation in the case of a comparatively low-lying observatory.

With regard to refraction in different parts of the earth, observations made in different parts of Europe, at the Cape and in the East Indies have given nearly the same results.

BESSEL's tables, which for altitudes between 90° and 5° depend on the formula discovered by their author and for low altitudes were drawn up empirically, were drawn for the first edition of BROUWER's tables (1862) from the *Berliner Jahrbuch* for 1844 (barometer 29.6 English inches, thermometer 48°.75 F.).

BROUWER computed the familiar table for mean refraction by BESSEL's method ; it had already appeared in the first edition of his collection of tables and was re-introduced in the second edition, then in the press, of HAVERKAMP's tables.

In the third edition of BROUWER's tables (1896) the original BESSEL's tables were omitted.