UNITS OF LENGTH (*)

BY

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1) The seventh International Conference on Weights and Measures was held in Paris, from 28th September to 6th October 1927. Twenty-five States were represented thereat. This was, at the same time, the celebration of the fiftieth anniversary of the foundation of the International Bureau of Weights and Measures, which was inaugurated near Paris at the Pavillon de Breteuil, on grounds given by the French Government and declared by it to be neutral territory.

The Conference adopted decisions with regard to Measures of temperature, electrical units and on the fundamental standard of length. This last directly concerns geodetic measurements and consequently hydrographic surveys, which, when accurate, are always connected with operations of this type.

It is known that the States which adhere to the International Bureau of Weights and Measures use the metre as the unit of length. This length, as intended by the creators, was to be the forty-millionth part of the Earth's meridian. In reality it is represented by the distance which separates two lines drawn on a standard made of iridized platinum, which is deposited in the vaults of the Pavillon de Breteuil. (**) This standard was made in accordance with the most precise rules of metrology; the greatest precautions are taken to ensure its preservaation and numerous copies, made with the same care, are periodically compared with the original. Each adherent Nation has been supplied with one or several copies of the international prototype and their lengths have also been compared with that of the standard.

By means of this unit of length and of this international agreement all measures of length made either by physicists or by geodesists of the adherent nations are not only expressed in the same unit but are also absolutely comparable. This advantage is capital, and it is of but minor importance that more recent measurements of the form of the Earth have shown that the terrestrial meridian is a few thousand metres longer than provided for by the original definition.

(*) See: Notes from the U. S. BUREAU OF STANDARDS — *Journal of the Franklin Institute*, December , 1927.

(**) See: "*Le Cinquantenaire de la Convention Internationale du Mètre*", by E. PICARD. *Annuaire pour l'An* 1928, " published by the BUREAU DES LONGITUDES, Gauthier-Villars, Paris, "*La Création du Bureau International des Poids et Mesures et son Œuvre*," par les soins de Ch. Ed. GUILLAUME, Gauthier-Villars, Paris, 1927.

Although minute precautions are taken for the preservation and comparison of the Metric Standard, it would obviously be of the greatest advantage if its definition were based on a physical phenomenon capable of giving very precise measurement and the permanence of which could be absolutely assured. The idea of employing in this connection the wave-length of light which was put forward by S. D. GILL in 1889 and developed by Gould, Michelson, Benoit, Fabry and Pérot, was submitted by the United States with a proposal to express the length of the international metre in wavelengths under well-defined conditions of the red ray of Cadmium vapour. The seventh Conference on Weights and Measures took this proposal into consideration and decided to adopt *temporarily* the value obtained by Messrs. BENOIT, FABRY and PÉROT in 1906, according to which :—

1 metre = 1,553,164.13 wave-lengths,

in dry air at a temperature of 15° centigrade, at a pressure of 760 millim. of mercury and under normal conditions of gravity. (*)

This adoption was merely temporary in order to allow for the possible adoption, at a later date, of some other ray, such as for instance, one of those from the spectrum of Krypton, if future research shows that this is desirable.

2) It is to be hoped that, as a result of this decision, an understanding will be reached for determining the length of the yard, in relation to the same wave-length. It will then be easy to compare measurements expressed in metres and in yards, an operation which actually presents somewhat serious difficulties.

Although progress has been made since Clarke studied this question and arrived at the ratio :—

$$\frac{1 \text{ metre}}{1 \text{ yard}} = \frac{39.370432}{36}$$

(Comparison of the standard of Length of England, France, etc., Ordnance Survey, 1866).

Divergences still exist which will now be indicated briefly ; (**) they cause difficulties in the practice of geodesy and explain why various conversion tables for measures of length occasionally give different results. (***)

The standard for measurements of length in Great-Britain is the "*British Imperial Standard yard,*" (****) which is a piece of bronze with linear terminal

(*) Since this determination was made, the necessity of making several slight corrections to the result has been realized and thus the following relation is obtained :—

1 metre = 1,553,163.7 wave-lengths.

(See : Création du BUREAU INTERNATIONAL DES POIDS ET MESURES, page 224).

(**) See : "*A fundamental basis for measurements of length*", by H. W. BEARCE, Senior physicist to the Bureau of Standards, Washington, 1926.

See : B. S. Sci. paper N° 17 or B. S. Mis., publ. N° 64, by L. A. Fischer, Washington.

(***) See : "*Geographical Journal*", December 1927 - pages 600 and 601.

(****) The United Kingdom had legalised the Metric System in July 1864 by Act of Parliament. This Act was repealed for some years. Finally, in 1897, a new Act was passed by which the employment of the Metric System became optional.

marks constructed in 1845 and adopted as the fundamental standard in 1855. It is preserved in the Tower of London, and the length is defined as the distance between two linear terminal marks measured at a temperature of 62° *F*. In 1878 it was compared with the standard metre, when the following ratio was adopted :—

$$\frac{1 \text{ metre}}{1 \text{ yard}} = \frac{39.37079}{36}$$

deduced from comparison made by ARAGO and KATER,

Then an Order in Council dated 19th May, 1898, was issued after comparative measurements had been made at Breteuil by BENOIT and CHANEY, which fixed the ratio :—

$$\frac{1 \text{ metre}}{1 \text{ yard}} = \frac{39.370113}{36}$$

This ratio is still legal, though more recent measurements made by Messrs. JOHNSON and MANDET have given the following ratio :—

$$\frac{1 \text{ metre}}{1 \text{ yard}} = \frac{39.370137}{36}$$

Difficulties are encountered in deciding whether the slight diminution thus ascertained in the value of the yard is due to observational errors or whether it indicates that the Imperial Standard has contracted as certain standards bronze have done.

The "*Indian Standard*" is a 10 feet bar, which was found to be equal to 9,99995658 British feet. Hence the following ratio is obtained :—

$$\frac{1 \text{ metre}}{1 \text{ Indian yard}} = \frac{39.370284}{36}$$

The yard employed in the United States was, at first, *de facto* that of Great-Britain. It was realized that the Throughton Scale, constructed in 1814 and provisionally adopted as the standard, was insufficiently accurate for the purpose.

However, an Act dated 28th July, 1866, legalized the use of the Metric System in the United States, and defined the yard in relation to the metre by the proportion :—

$$\frac{1 \text{ metre}}{1 \text{ U.S. yard}} = \frac{39.37}{36}$$

The United States having joined the International Bureau of Weights and Measures received copies N° 21 and 27 of the International prototype metre.

The Mendenhall Order of 5th April, 1893, declares copy N° 27 to be the national prototype of the metre and maintains the above quoted relation between the yard and the metre, which was fixed by the Act of 1866. (*) It may be appreciated how complicated the calculations are rendered by the use of these different ratios when desiring to convert one unit into the other. Nevertheless, they must be taken into consideration in precise measurements. If, for example, the length of the Earth's radius is expressed in yards, a difference of about 20 metres will be found according to whether the yards used are British or American.

A proposal to unify and simplify these ratios was made at the Congress of Associations for Weights and Measures, held in New-York on April 20th 1926. The 16 States represented were: Austria, Belgium, Cheko-Slovakia, Chile, France, Germany, Great Britain, Italy, Japan, Norway, Poland, Russia, Sweden, Switzerland, the United States. A wish was expressed that all the units of length should be made comparable with the international metre by the ratio :—

1 inch = 25.4 millimetres.

Thus, for the various units of length, the following definitions would be obtained and they have the advantage of not carrying the decimals beyond one tenth of a millimetre.

1 foot = 12 × 25.4 = 304.8 millimetres.
1 yard = 36 × 25.4 = 914.4 ditto
1 fathom = 72 × 25.4 = 1,828.8 ditto
1 statute mile = 1760 yards = 1,609.344 metres.

3) An uncertainty of a different order, but which likewise is not without certain drawbacks, exists with reference to the length of the Sea-Mile. This measure represents, in principle, the length of a minute of arc of the meridian, or a minute of latitude (**). But owing to the elliptical shape of the meridian, its value varies with the latitude and depends upon the elements adopted for the terrestrial ellipsoid.

When expressing a distance in Sea Miles, the unit used should not depend upon the latitude ; generally however, a sufficiently close approximation has been sought so that slight variations in the length of the minute of latitude will have but little importance. It is not quite the same thing, however, when the Mile is used to indicate the speeds of ships. These are often expressed, in speed trials, in hundredths of a Mile. Consequently, if the speed of a ship which is making about 30 knots has to be measured to within 0.01 of a Mile, as is the case in some countries for obtaining subsidies, there should not be more than 0.6 metres of uncertainty as to the length of the Mile.

On the other hand, as charts often have a scale of Sea Miles, the knowledge of the length which was adopted in drawing this scale is of value, and it may

(*) See: "*Bulletin* 26", U. S. Coast and Geodetic Survey, Washington, 1893.

(**) It is interesting to note that, originally, the fathom was one thousandth and the cable's length one tenth of the Sea Mile, thus forming a decimal system of units of length.

be possible, sometimes, when the dimensions of the frame are not inscribed on the chart, to deduce therefrom the approximate shrinkage of the paper.

The INTERNATIONAL HYDROGRAPHIC BUREAU has collated some information as to the lengths used for the Sea-Mile. Some of the results are given here :

BELGIUM : The value adopted is 1854 metres.

DENMARK : The value adopted is 1851.91 m. but in practice the value 1852 metres is employed.

FRANCE : 1852 m. This length is established by the Ministerial Decree dated 1st September 1905.

GERMANY : The value adopted is 1852 metres.

GREAT BRITAIN (*) : Length of the minute of latitude at the mean latitude of the chart, calculated with compression $\frac{1}{294}$; this is the length shown on the Admiralty Chart under the legend : Scale of latitude and distance, The mean Mile of 1853.18 m. (6080 feet) is also often used.

GREECE : 1852 m.

HOLLAND : Has not been fixed officially.

ITALY : 1851.85 m.

NORWAY : 1852 m.

PORTUGAL : 1851.8 m.

SPAIN : 1851.8 m.

SWEDEN : 1852 m.

UNITED STATES (**) : 1853.248 (6080.20 feet).

On the other hand, the lengths of the minute as obtained on the basis of the various definitions which are in frequent use, are :—

1) Length of the minute on a sphere whose meridian is 40.000.000 m. long..... 1851.85135 m.

2) Length of a minute *of the equator* of the international ellipsoid (Madrid 1924)(Sometimes called the Geographical Mile). 1855.39786 m.

3) Length of the minute on a sphere of the *same surface area* as the international ellipsoid ..... 1853.31502 m.

4) Length of the minute on a sphere of the same *volume* as the International ellipsoid ..... 1853.31314 m.

5) Length of the minute on a sphere whose radius is the mean $\frac{2a \times b}{3}$ of the axes of the international ellipsoid ..... 1853.31548 m.

6) Length of the minute on a circle of the same *length* as the meridian of the international ellipsoid ..... 1852.27561 m.

7) Length of the minute at latitude 45° on the meridian of the international ellipsoid ..... 1852.25585 m

(*) See : "*Admiralty Manual of Navigation*", volume II, page 7, London 1922.

(**) Tables published by the OFFICE OF STANDARD WEIGHTS AND MEASURES, September 1898. This length is defined as being the minute of arc of the Great Circle of a sphere, whose surface area is equal to that of Clarke's ellipsoid (1866).

The Directing Committee would be glad to receive the necessary information to complete the list of lengths used for the Sea Mile by the different States, as well as the definitions and the laws which have determined the value in each country.

It considers that this length, with which the Seaman is so familiar, should be based on an international definition, which would depend neither on the latitude nor on the dimensions of the ellipsoid of reference. The Committee suggests that agreement could be reached, defining this unit in relation to the International Metric Standard and it is of opinion that the value 1852 metres is that which approximates most closely to the figures generally adopted.

P. S. *It is of interest to extract from the Report of Proceedings of the Seventh General Conference for Weights and Measures, which met in Paris from September 27 to October 6, 1927, the following paragraphs, which appeared in the* "Bulletin of Scientific Relations", *published by the Institute for Intellectual Cooperation :-*

" Report of the progress of the Metric System. — In the U. S. S. R., the metric system, " which was declared compulsory by a law promulgated in 1918 has, in spite of the present " difficulties, been introduced compulsorily in all Government services so that, from the 1st of " January, 1927, this reform may be considered to have become an accomplished fact.

" The American campaign in favour of the metric System is being actively carried on. A " draft law is now being submitted to the House of Representatives by Mr. F. A. BRITTEN and " to the Senate by Mr. F. H. GILLETT. This law provides for the compulsory use of the Metric " System for trade purposes in the year 1935.

" The Argentine has completed its metrical legislation by a decree dated 20th January, 1926, " which sets up a control service.

" The Metric System became compulsory in French Morocco by a "Dahir" of the 16 of " Moharrem, 1302 (29th August, 1923), which was promulgated on the 7th September.

" Persia adopted the Metric System at the beginning of 1924 but retained the old Persian " names for the new units.

" The Metric System became compulsory in Afghanistan from New Year's Day, 1305 " (21st March, 1926). The change should be completed within three years, *i. e.*, by the 21st " March, 1929.

" In Greece, the law dated 31 August-13 September, 1920, has been put into force from " the 1st March, 1926, for linear measure, and from the 1st March, 1927, for measures of weight " and capacity.

" In Japan the period of conversion has begun. The period of five years, which was " originally fixed for the full application of the law of the 11th April, 1921, has been increased " to 10 years.

" The Metric System was definitely introduced in Siam by an act., N° 2466, of the 17th " December, 1923. This act lays down a first period of 12 months for a partial application and " a second minimum period of 5 months, within which compulsory use will become absolute, " by decree, either all at once, or successively, in the various provinces.

" In Turkey, the metrical law was passed quite recently.

" The Conference unanimously adopted, except for two abstentions (Canada-Great Britain " and the United States), a motion put forward by the representatives of the Argentine Republic " and of Mexico, recommending the delegates to urge their respective Governments to take all " possible steps in order that the Governments of the United States and Great Britain should " make the use of the Metric System compulsory, and thus obtain world-wide standardization " of systems of measurements."
