THE DETERMINATION OF THE MAGNETIC MOMENT OF LIQUID COMPASSES

by

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Although the study of nautical instruments only indirectly concerns hydrography, it is thought that a discussion in the *Hydrographic Review* of the methods of verification applicable to these instruments is not altogether out of place, observing that some Hydrographic Offices possess, or have possessed, under their jurisdiction a section which especially undertakes the supply, receipt and inspection of nautical instruments issued to men-of-war and merchant ships. Besides, the hydrographer, in the course of his duties, is himself called upon to make use of these nautical instruments and to study the conditions of their use with far greater care than the navigator, so that it is often thanks to the studies and labours of officers attached to Hydrographic Departments that nautical instruments and the methods of using them have been perfected and modernised.

In these days, the magnetic moments of the systems of magnetic needles, which form the modern liquid compass cards, have been considerably increased as compared with those of older cards (their value sometimes reaches 5,000 gauss, whereas it was of the order of 200 gauss for the Thomson Compass). Consequently, when a card is put in position in the bowl of the liquid compass, it becomes necessary to make sure if its magnetic moment actually possesses the required value and, moreover, the knowledge of this value will afford valuable information to guide the compass adjuster to place in position the correctors of the quadrantal deviation and to proportion the Flinders bar.

Up till now, this magnetic moment of compass cards has seldom been specified on the supply notes which accompany the compass or spare cards, and on this subject the catalogues of instrument makers give only general and incomplete indications. And yet it is quite natural that before putting such appliances into use, they should be carefully examined and verified in such a way that a kind of measurement or evaluation of the magnetic moment of the cards is carried out by the inspection departments receiving the compasses

Some of the methods which, to our knowledge, have been used for this purpose, are briefly described in the following ; and as, up to the present, little has been written on the subject of the determination of the magnetic moment of a liquid compass card enclosed in its bowl, we should be grateful to readers of the *Hydrographic Review* who are interested in this question, if they would kindly forward to the International Hydrographic Bureau additional information of which they have knowledge, and which they consider useful to make known and to publish.

Firstly we will summarise a certain number of articles which appeared in 1913 and 1922 in the *Annalen der Hydrographie und Maritimen Meteorologie*, then we will describe in detail, with examples pertaining thereto, the method of measurement devised by Vice-Admiral PERRIN and perfected for use in 1923-24 by the French Hydrographic Office for the examination and the receipt of liquid compass cards.

Before proceeding, it is well to recall the simple, the elementary conception of the rôle played by a magnetised bar of magnetic moment $\mathfrak{m}$ on a magnetical pole $+ 1$ at a distance r from the centre of the bar. Compass adjusters should always bear this conception in mind. It constitutes, in some sort, the surest guide to direct the hand of the operator in the manipulation of the compensating magnets of the semi-circular deviations B and C, for it indicates at once the direction and the intensity of the action of the correctors. It is known that this action, which represents the intensity of the field created by the magnet, is expressed by $\frac{\mathfrak{m}}{r_3}$ in the equatorial plane of the magnet, and by $\frac{2\mathfrak{m}}{r^3}$ along the polar axis (See Fig. 1) and also that, for the pole $+ 1$, it is exercised in a *North and South* direction or *Red and Blue*, after the usual manner of painting magnets.

Therefore, at an equal distance, the components created by a magnet on a pole $+ 1$, are either $\frac{\mathfrak{m}}{r^3}$ or double, according to whether the magnet is made to act transversely or on end.

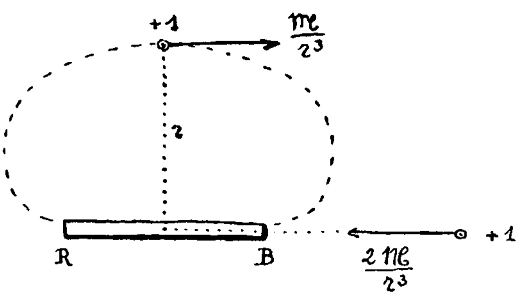


Fig. 1

From this expression of the action exercised by a magnetised bar on a pole $+ 1$, the method of measuring its magnetic moment is derived. In fact, if in a known magnetic field of horizontal intensity H (as for instance the magnetic field of the Earth) a fixed magnet of magnetic moment $\mathfrak{m}$ is made to act, at a certain distance r, in the horizontal plane of a slightly magnetised needle, (called deviation compass), the needle will equilibrate itself under the action of the two components H and $\frac{m}{r^3}$ in a direction which will make with the direction of the field H (magnetic North) an angle of deviation or deflection a such as :

$$a = \sin a = \tan a = \frac{m}{r^3} : H \text{ (*)}$$

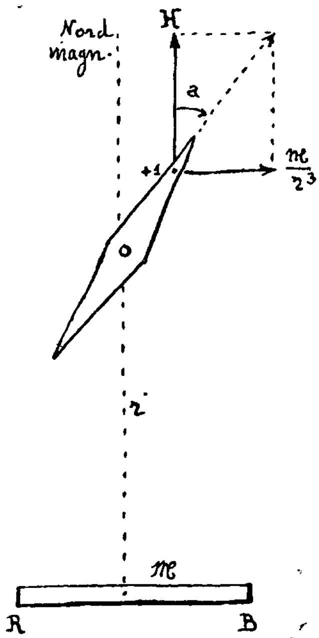


Fig. 2

This equality, which remains true so long as a does not exceed 20° of arc, constitutes a formula whereby the magnetic moments of magnetised bars or systems of compass card needles can be estimated by geometrical measurements, when the intensity H of the field in the place of operation is known.

In this way the measure of the magnetic moment is reduced to a measurement of length (measure which should be carried out with a certain degree of accuracy because the *cube* of a is included in the formula), and to estimating the angle of deflection a, which can be made within $\frac{1}{10}$ of a degree, if a small goniometric compass is used, the points of which turn around a limb divided in degrees of arc (or any other kind of division).

The application of the above method to liquid compasses is very simple, if they are fitted with a contrivance to lock the card. After the card has

(*) This deflection a will be double if the magnet instead of acting transversely, acts, at the same distance, "on end", in the conditions described above.

been locked in the bowl, the latter can be oriented on the trial table so that the *N.* and *S.* line of the card, and consequently the magnets pertaining to it, act transversely on the deviation compass according to the East and West direction magnetic. In this way the magnetic moment of the mounted card, without removing it from the bowl, is measured just as well as if the card itself or a spare card not immersed in the liquid were used.

This method is more difficult when the compass card has no locking contrivance or when it is not easy to invert the bowl so that the card rests on the glass and thus becomes immobile.

In 1913, R. TOPP, in the *Annalen der Hydrographie und Maritimen Meteorologie* (page 167) described a method by which the magnets of the compass card can be oriented in stable equilibrium so that they act on the deviation needle in an East and West direction. For this pui·pose an auxiliary deflector, placed on the compass glass, is used, which causes the magnetised needles of the compass card to be deflected through exactly 90°.

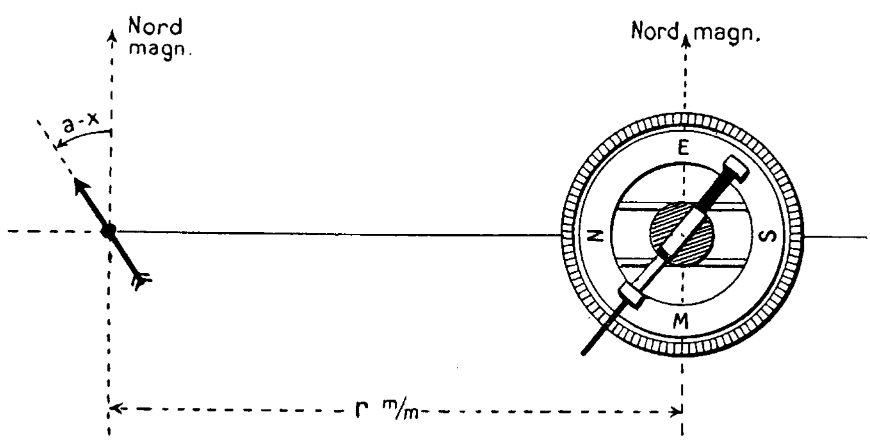


Fig. 3

The contrivance is shown in figure (3). The small deviation needle and the liquid compass are mounted in the same plane, so that the line joining the points of the two pivots lie East and West magnetic from each other. The deviation needle is made to rest in a horizontal position and is placed at such a distance that the deflection does not exceed 20°.

The deflector is then placed on the cover of the liquid compass with its pointer forming a fixed angle of 45° with the North-South line, and the distance between the poles of the deflector, that is to say its intensity of action on the compass card, is adjusted so that the card is deflected exactly 90°, *i. e.* in such a manner that the East point of the card is made to coincide with the lubber's point. In this position, the magnets of the compass card cause the small deviation needle to be deflected an angle $(a - x)$, x being the deflection due to the influence of the deflector itself. This angle x remains very small, if the distance between the deflector and the small deviation needle is sufficient. It is determined experimentally, for different distances, by making the deflector only, inclined at an angle of 45°, act on the needle (Fig. 4). The following deviations have been obtained with Thomson's deflector the power of which is just insufficient to deflect the modern liquid compass through 90°) :—

$r = 300$ m/m	$x = 5°$
400 m/m	2°4
500 m/m	1°5
600 m/m	0°7

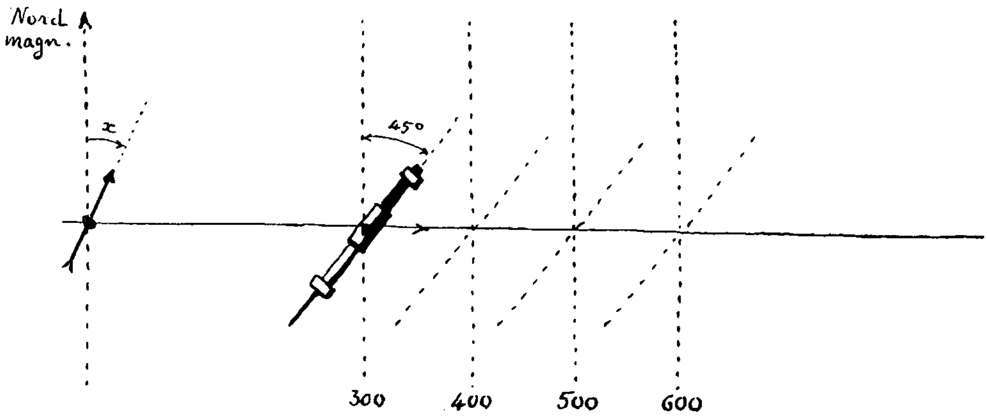


Fig. 4

Once the angle x has been determined, the readings of the angle $(a - x)$ corresponding to different distances are added to it, and the sum gives the angle a by means of which the magnetic moment is calculated from the known formula:

$$\mathfrak{m} = \frac{1}{2} H r^3 \tan a.$$

In practice it is advantageous to repeat the operation, placing the deflector on the compass with its pointer directed to the *NW* magnetic, so that the West point is on the lubber's line, and the mean values thus obtained are used for calculating the angle a.

According to the author this method cannot claim absolute accuracy, owing to the influence of the deflector on the deviation compass, and the reciprocal influence of the deflector and the compass card placed at an angle of 45° one from the other. However, it is good enough when only a first approximation is required.

In the *Annalen der Hydrographie und Maritimen Meteorologie* of 1922, Heft III, page 101, H. Coldewey recommends the following method for obtaining more accurate results. The magnets of the compass card, preferably immobilised in an *E.* and *W.* direction (magnetic) are made to act on the deviation compass by placing them on the trial table at a properly marked distance, either to the East or to the West of the deviation compass, then the compass card is revolved through 180°, and the reverse deviation is obtained. Half the difference of the readings on the divided limb of the deviation compass will give the angle a which is introduced into the formula

$$\mathfrak{m} = \frac{1}{2} H r^3 \tan a.$$

If the experiment is carried out thus, and the magnets of the compass card and the deviation compass are at the same height above the table and the correct distance apart, it is justifiable to consider the result as correct.

In this same article the author describes a method which is applicable to all kinds of compasses, by which the orientation and the immobility of the compass card with its group of magnetised needles can be easily made in the East and West direction. This method consists of nullifying the terrestrial magnetic field at the centre of the compass by means of a suitable deflector, which is placed above the bowl. By slow movements (Fig. 5 & 6) it is easy to succeed in bringing the ensemble into the required position for which the magnet of the deflector being pointed towards the South, the distance apart of its poles (*i. e.* its intensity of action) is regulated so that the compass card remains in equilibrium in no matter what azimuth and thus

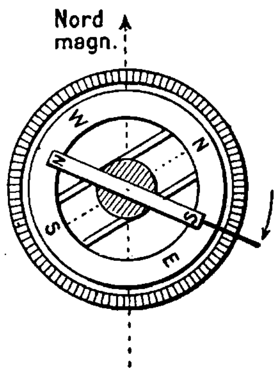


Fig. 5

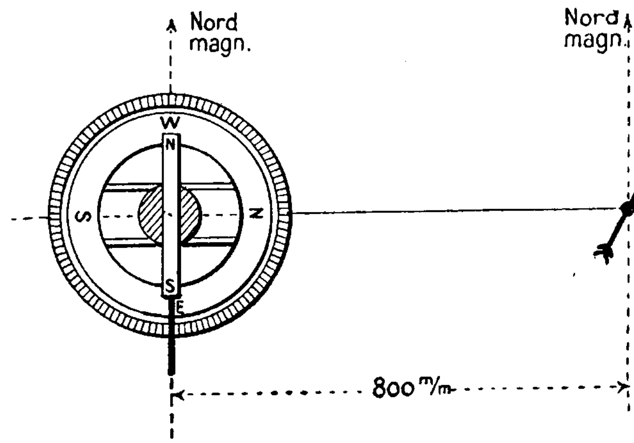


Fig. 6

the compass card can be made to take up a position either *EW* or *WE* by means of a small magnet (Fig. 6). The deviation compass is then placed at a certain distance — either *E* or *W* magnetic — from the centre of the compass and the corresponding angle of deviation is read, then the card is turned 180° by means of the small magnet, so that a second position of equilibrium is obtained, and a second deviation is read.

At the experimental laboratory of Geestemünde the value $a = 4°58'$ was found when using Bamberg's small liquid compass placed at a distance of 800 m/m from the deviation compass. It should be noted, however, that in using this method the deflector influences the intensity of the field at the centre of the deviation compass, and this fact should be most scrupulously taken into account, especially when experimenting with large liquid compasses, of which the distance between the needles and the cover of the bowl reaches as much as 10 centimeters, this necessitating the use of special powerful deflectors which even at distances of 800 m/m from the bowl exercise an appreciable action on it. At Geestemünde a suitable small deflector has been adopted, the magnet of which lies close to the cover of the compass so that its influence is negligible and the determination of the magnetic moment of the liquid compass presents no difficulty. The largest liquid compasses deflect the needle of the deviation compass 12° at a distance of 800 m/m, and a table has been constructed, calculated for angles from 4° to 13° which greatly facilitates the operation.

The value $H = 1793$ gauss has been adopted at Geestemünde, and when using a Thomson's deflector it was found that the greatest error in the determination of the magnetic moment for a Bamberg compass of magnetic moment of 3900 gauss, was of the order of 30 gauss.

In the *Annalen der Hydrographie und Maritimen Meteorologie* (Heft VI. - 1922) Dr. H. MAURER observes that the force of the deflector itself influences the position of the deviation compass needle, as well as the horizontal field of the Earth, at the point occupied by the needle, and he describes a very simple contrivance by means of which the resulting corrections need not be considered.

For this purpose (Fig. 7 seen from the South) the deflector is replaced by an ordinary bar magnet sufficiently powerful to deflect the compass card 90° when placed vertically close to the compass. The magnet *M* rests against the hinged cover of a box *B*; it is moved about and inclined so as to deflect the compass card contained in the bowl *K*, 90°.

It is the lower pole U which acts with preponderance on K, the actions of O and U do not cause any horizontal result on the deviation needle A, especially if the magnet M is maintained in a vertical plane passing through A, and the deflecting magnet will not interfere with the horizontal field at A. In this way the magnetic moment of the compass is not changed by

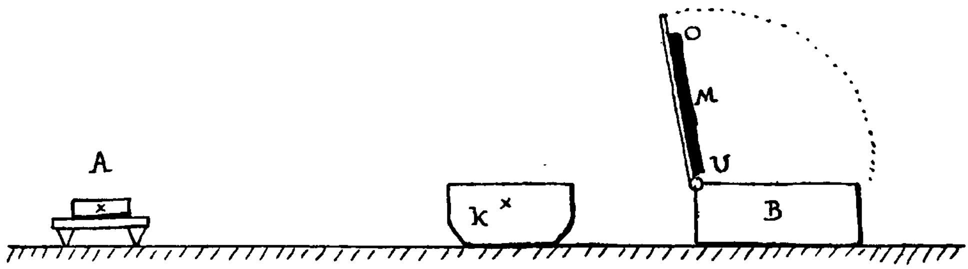


Fig. 7

induction on the part of the deflecting magnet. This latter should be just sufficient to place the liquid compass card in the East-West direction, and to do this a suitable distance from the box B to the compass K should be chosen. A deflector magnet of the 3rd class (of magnetic moment 540) will generally be sufficient for this operation.

A deflection apparatus specially constructed by the firm of W. Ludolph A. G. at Bremerhaven for determining the magnetic moment of liquid compasses has been described by H. Coldewey in the *Annalen der Hydrographie und Maritimen Meteorologie* - 1924, pp. 60 to 62.

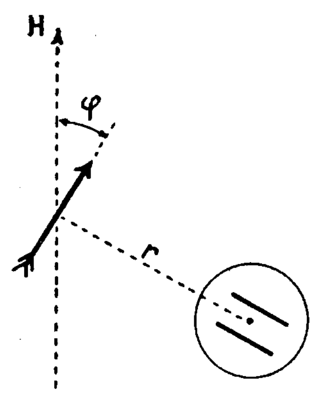


Fig. 8

The methods described above, developed in Germany, lead to the determination of the magnetic moment by the method of deflections. As a rule (Fig. 8) the deflector angle for which the North and South line of the compass card is perpendicular to the deviation needle is used.

The magnetic moment of the compass card is then proportional to the sine of the angle of deflection.

$$\mathfrak{m} = \frac{1}{2} : H\, r^3 \sin \varphi \qquad (3)$$

This formula and the method of operating connected with it gives only an approximate result. The exact solution of the problem requires a much more complicated formula.

In the *Annalen der Hydrographie und Maritimen Meteorologie* (1922 - Heft IX), Captain J. J. Larsen, Director of Navigation at Christiania (Oslo) describes the method used in Norway for the examination of liquid compasses. In ordinary practice, it is not necessary to dismount the compass bowl and extract the card; it suffices to place the compass in a factitious field of 0.5 gauss (which is sufficient to infer that the magnetic moment of the compass is adequate in relation to quality).

But although this cursory examination was done both at a temperature of 18° centigrade as well as at a temperature of 5° centigrade, so as to have a clear idea of the influence of a variation of temperature on the fluidity of the liquid of immersion it only constitutes an expedient which gives very little information concerning the magnitude of the magnetic moment itself, and Captain LARSEN indicates different methods of obtaining the measurement of the magnetic moment without opening the compass bowl.

After mentioning that the British engineer, Mr. B. FIELD, has constructed an apparatus with which he could immediately measure the magnetic moment by means of a torsion balance, the compass card being turned in an East-West direction, with the help of a system of astatic needles connected with the balance, the author points out that for several years he has applied, in Norway, a method by means of which he ascertains the magnetic moment, not by a deviation but, on the contrary, by means of the durations of oscillations.

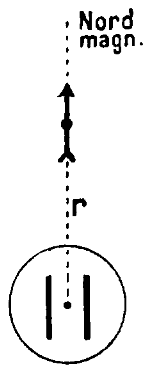


Fig. 9

The theory of the method can be deduced from Figure (9) where the compass card and the oscillation needle are placed on the same North-South line. The intensity of the field created by the compass card at the centre of the oscillation needle is equal to $\frac{2\mathfrak{m}}{r^3}$. Here the earth's magnetic field is augmented by this quantity and the total horizontal intensity is $H + 2\frac{\mathfrak{m}}{r^3}$.
If the compass is taken away, the horizontal intensity becomes again H. The duration of the oscillation of the needle of the oscillating compass is observed and it is noted that:—

t_0 = length of the double oscillation in the field H

t_1 .... $H + \frac{2\mathfrak{m}}{r^3}$

Then:

$$\frac{H + \frac{2\mathfrak{m}}{r^3}}{H} = \frac{t_0^2}{t_1^2}$$

Therefore:

$$\mathfrak{m} = \frac{1}{2} H r^3 \left(\frac{t_0^2}{t_1^2} - 1\right) \qquad (4)$$

Another simple method, based upon this abridged formula, makes it possible to determine the magnetic moment with about the same precision as that obtained by the abbreviated method of deviations.

The difficult problem of determining the magnetic moment of the system of needles of a compass card by means of the durations of oscillation, would necessitate observations being taken to the north and to the south of the needle and at two distances, and these two determinations should be made both with the north and south ends of the needles of the card which is to be examined.

To increase the exactitude of the result, $\mathfrak{m}$ may be ascertained at the same distance to the North and to the South of the needle, and the magnetic moment may be considered as an average of the two values $\mathfrak{m}_1$ and $\mathfrak{m}_2$ which are thus obtained.

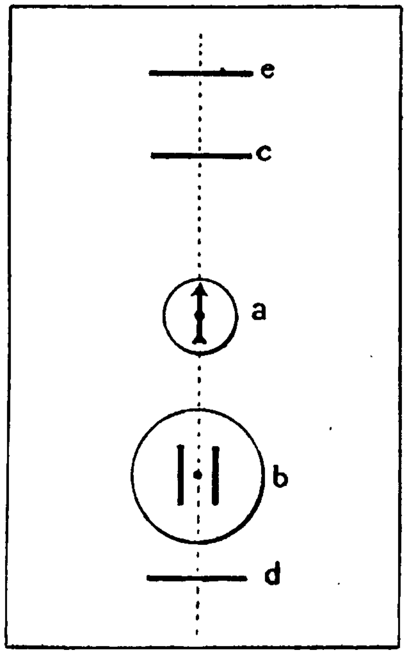


Fig. 10

Figure (10) shows the general arrangement recommended for this method. In the centre *a* of the trial table is the case containing the oscillation compass. The needle is placed approximately 25 centimetres above the level of the table; *b, c, d* and *e* are metal clasps which hold the support of the compass bowl in such a way that the distance between the pivot of the compass card and the pivot of the needle of oscillation is exactly 300 m/m for *b* and *c*, and 500 m/m for *d* and *e*.

The support has a mechanism for raising and lowering it, by means of which the system of needles of the compass card can be placed in the horizontal plane of the oscillation needle. A distance of 300 m/m is used for the compasses with short needles and a distance of 500 m/m for compasses with long needles and with a large magnetic moment.

The operating table is fixed to the floor, in such a way that the line *a, b, c, d* and *e* lies in a north-south direction magnetic.

For the formula used in the calculation, the duration of 10 oscillations is utilised, which is deduced from several series of measurements and which is taken with a chronograph giving 1/10th of a second.

The Norwegian station gives the following examples concerning the magnitude of the magnetic moment of compasses :—

For cards with a diameter of	200 m/m	not below	2,600 gauss.
" "	175 m/m	" "	2,000 "
" "	150 m/m	" "	1,500 "
" "	125 m/m	" "	1,100 "
" "	100 m/m	" "	800 "

Having reviewed the methods employed in Germany and in Norway for determining magnetic moments of compasses, a method which has been pursued by the French Hydrographic Office with the same end in view will now be described. The method of deviations and the method of oscillations were used simultaneously. However, the first method was reserved simply for comparative measurements, when great precision is not essential. The method of oscillations, on the other hand, was applied when it was necessary to determine fundamental data and the constants of appliances (magnets or compass cards) which are destined to be used as standards of comparison.

The material and accessories which have been used and which are, moreover, rather rudimentary, are given below in detail. Any one who is interested in these kinds of experiments may, at a small cost, create similar contrivances which, as will be seen from the numeric examples quoted, give results which may be considered quite satisfactory.

The successive operations consist in :—

1) Measuring the terrestrial field in which the work will be done :
2) Measuring the magnetic moment of a magnetised bar which will be used afterwards as a standard for comparison :
3) Measuring the magnetic moment of the liquid compass card by the oscillations of the deviation compass.

The duration t of the double oscillation (of an infinitesimal amplitude) of a magnet of moment of inertia I and of magnetic moment $\mathfrak{m}$ is expressed by

$$t = 2\pi \sqrt{\frac{I}{\mathfrak{m} H}} \qquad (5)$$

On the other hand, the moment of inertia of a cylindrical horizontal bar magnet of diameter D c/m and length L c/m and weight M grammes in relation to a vertical axis passing through its centre of gravity is expressed by :

$$I = M \left(\frac{L^2}{12} + \frac{D^2}{16} \right) \qquad (6)$$

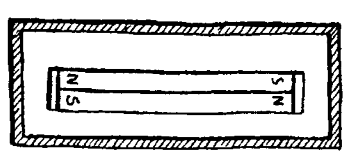


Fig. 11

A good bar magnet is selected for standard which after use is kept in a wooden box in contact with another bar magnet, taking care to close their magnetic circuit by means of pieces of iron. The magnetic moment of this bar in relation to the surrounding field is determined as follows. The bar is made to oscillate at the centre of the trial table, and the periods of oscillation are measured. In course of time, for example every two or three months, the measurement of the field and of the magnetic moment should be taken in order to make sure that these elements have not considerably changed, and

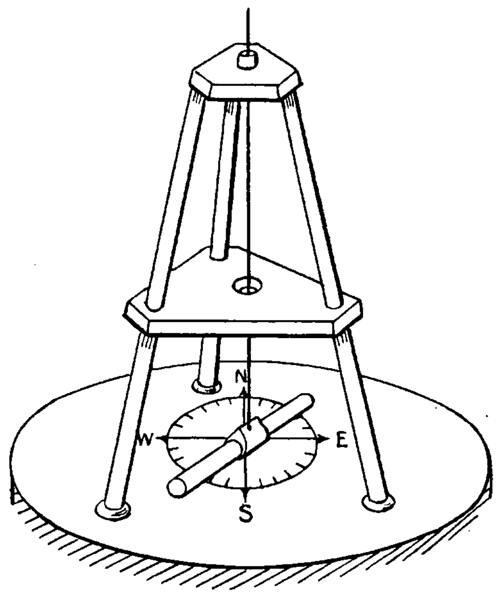


Fig. 12

that the value taken as a term of comparison can still be used. To oscillate the bar, (Fig. 12) a rigid tripod is constructed with broom handles and wooden boards. Each of the feet rests on a piece of thick felt to deaden the vibrations. The bar is suspended in a parchment stirrup attached to a fine hair, of which the length is adjusted so that the horizontal bar lies very close to and above a cardboard circle divided *N.* & *S.* & *E.* & *W.* and graduated in degrees, over which it is centred.

At rest the magnet lies along the $\overline{S\ N}$ line in the magnetic meridian.

In order to oscillate the bar, the pole of a small auxiliary magnet is brought near to one of the poles of the bar, so as to impart an impulse to it, whereby it is deflected 20° to 30° from its position of equilibrium. Then the auxiliary magnet is taken away. The oscillations continue for some time, gradually dying off. The amplitude of each oscillation at each end of a stroke can be read off on the graduated circle. The duration of several series of 10 oscillations each can also be counted with a chronograph, without having to set the bar in motion again. The mean value of the amplitude α is noted for each series, in order to refer the durations of measured oscillation to the oscillation of very small amplitude, by means of the table of reduction annexed.

Table of Réduction

α	fact. (1 + β)	diff.	Log. (1 + β)	diff.
0°	1.0000		0.00000	
1	00		01	2
2	01		03	5
3	02		08	5
4	03	2	13	9
5	05	2	22	8
6	07	3	30	13
7	10	2	43	10
8	12	4	53	16
9	16	3	69	14
10°	1.0019	4	0,00083	17
11	23	4	100	19
12	27	5	119	21
13	32	5	140	22
14	37	6	162	24
15	1,0043	6	0,00186	26
16	49	6	212	28
17	55	7	240	29
18	62	7	269	30
19	69	8	299	33
20°	1,0077	8	0,00332	34
21	85	8	366	36
22	93	9	402	37
23	102	9	439	39
24	111	9	478	41
25	1.0120	10	0,00519	43

α	fact. (1 + β)	diff.	Log. (1 + β)	diff.
25°	1.0120	10	0.00519	43
26	130	11	562	44
27	141	10	606	46
28	151	12	652	48
29	163	11	700	50
30°	1.0174	12	0.00750	51
31	186	13	801	53
32	199	12	854	54
33	211	14	908	57
24	225		965	58
			1023	60
35	238	13		
		14		
36°	1.0252	15	0.01083	62
37	267	15	1145	63
38	282	15	1208	65
39	297	16	1273	67
40°	1.0313	17	0.01340	69
41	330	17	1409	71
42	347	17	1480	72
43	364	18	1552	74
44	382	18	1626	76
45°	1.0400	18	0.01702	78
46	418	20	1780	80
47	438	19	1860	81
48	457	20	1941	84
49	477	21	2025	85
50	1.0498	21	0.02110	87

α	fact. (1 + β)	diff.	Log. (1 + β)	diff.
50°	1.0498	21	0.02110	87
51°	519	21	2197	89
52	540	23	2286	91
53	563	22	2377	93
54	585	23	2470	95
55°	1.0608	24	0.02565	96
56	632	24	2661	99
57	656	25	2760	100
58	681	25	2860	103
59	706	26	2963	104
60°	1.0732	26	0.03067	107
61	758	27	3174	108
62	785	28	3282	111
63	813	28	3393	112
64	841	28	3505	115
65°	1.0869	29	0.03620	116
66	898	30	3736	119
67	928	31	3855	121
68	959	31	3976	123
69	990	31	4099	125
70°	1.1021	33	0.04224	127
71	1054	33	4351	129
72	1087	33	4480	132
73	1120	35	4612	134
74	1155	35	4746	135
75°	1.1190		0.04881	

From this first manipulation, the value $\mathfrak{m}H$ can be calculated from the formula :—

$$\mathfrak{m}H = \frac{4\,\pi^2\,I}{t^2} \qquad (7)$$

With a bar magnet of length 20 c/m, diameter 1 c/m, and weight 143 gr.

$log.\ 4\,\pi^2\,I = 5.27539$ is obtained.

The following calculation is then made. Example :

1. Oscillations of a free bar magnet :

Amplitude α	duration of 10 double oscillations					reduced duration		
14°	181^s.0	— 2.2577	0.0016	2.2561	—	180^s,4	mean	180^s .1
11°	180^s.5	2565	0010	2555		180 .1	t =	18^s.01 — 1.25551
8°	180^s.2	2558	0005	2553		180 .0		I^2 — 2.51102
19°	181^s.1	2579	0030	2549		179 .9		$4\pi I$ — 5.27539
15°	180^s.9	2574	0019	2555		180 .1	log $\mathfrak{m}$ H =	2.76427

Next the bar magnet is laid at the centre of the trial table and oriented in the magnetic meridian, previously determined and known, with its red pole towards the north.

The effect of the bar in this position is to create a horizontal field directed towards the South which diminishes the intensity of the horizontal field H of the earth.

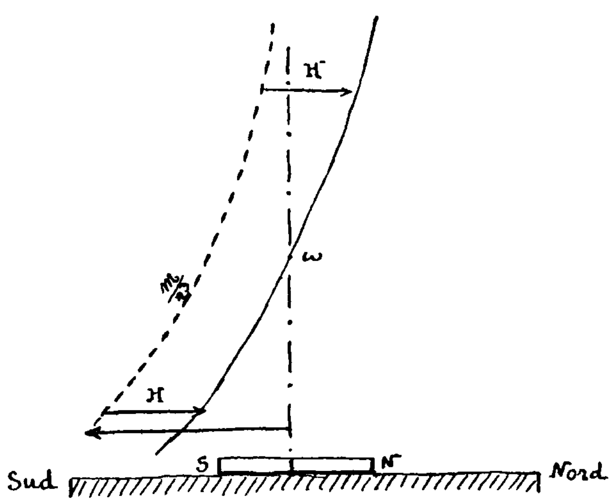


Fig. 13

If the value of the total field thus produced on the vertical passing through the centre of the trial table is analysed, it can be seen by referring to the graphical production of the vectors representing the fields in Fig. 13, that when at a slight height above the table the field is neutralised, and that above this point it is again directed towards the North. The field regains the value H if the bar magnet is withdrawn.

This resulting and changing field can be examined by trial at different heights by means of a small oscillating compass enclosed in a wooden box with glass lid, which contains a well-sharpened pivot on which rests a light oscillating needle. The box which encloses it, protects it from the effects of air currents.

A bristol board, on which the graduation in degrees of a compass card *N. E. S. W.* has been drawn, is placed in the box below the oscillating needle, so as to be able to measure as before the amplitudes of the angles of swing of the needle from one side to the other of the *N.* & *S.* line magnetic (Fig. 14).

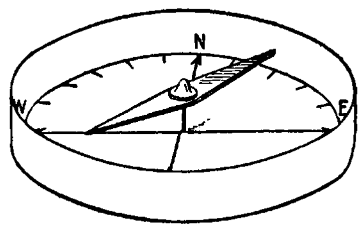
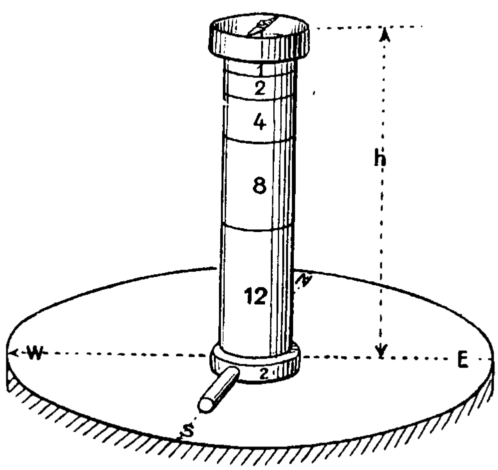


Fig. 14

In order to explore the field along the vertical of the centre of the trial table, it is necessary to know very exactly and precisely the height of the plane of the oscillating compass needle above the plane of the trial table or rather above the median plane of the bar magnet.

For this purpose carefully drilled wooden cylinders of nearly the same diameter as the oscillating compass are constructed. The height of each is an exact number of centimetres :— 1 c/m, 2 c/m, 4 c/m, 8 c/m, 12 c/m, 16 c/m, and



h = 2 + 12 + 8 + 4 + 2 + 1 + 3 = 32 cm

Fig. 15

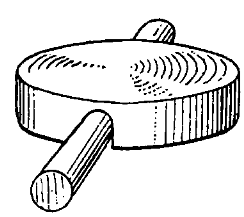


Fig. 16

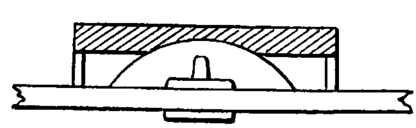


Fig. 16 *bis*

20 $^c/_m$. By superimposing these different cylindrical parts so as to form a column (Fig. 15) any required height can be obtained. A cylindrical round wooden cup is placed like a cap over the bar magnet and forms the base of the column.

With such a contrivance the oscillating compass can be made to oscillate at various fixed heights above the bar magnet, and observations of the durations of a series of 10 oscillations can be taken at each position.

Actually the frequency of the oscillations of the oscillating compass needle is much greater than that of the bar magnet, but nevertheless the duration of 10 oscillations or of 5 double oscillations can be counted before their diminution is too considerable.

A small auxiliary magnet is always used to impart to the oscillating compass needle the impulsion which causes it to oscillate from one side to the other of the magnetic meridian.

The oscillations are so much the more slow as the neutral point is approached, on one side of this point the needle points towards the South, and on the other side to the North.

In order judiciously to assess the measurements, it is well at once to take into account the approximate height of the neutral point ω, then to distribute the oscillating stations 1 - 2 - 3 - 4 ; 5 - 6 - 7 - 8, etc. as nearly symetrically as possible on either side of this point : above and below (Fig. 17). The experiment should be terminated by withdrawing the bar magnet and making the needle oscillate freely under the influence of field H only.

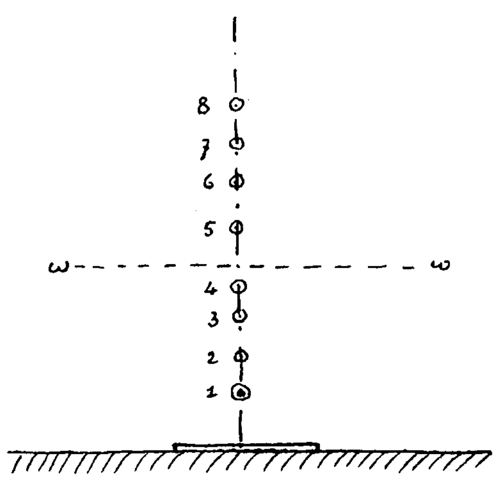


Fig. 17

If the period of oscillation of the free needle under the influence of field H only is denoted by t_0 and the period of oscillation of the same needle under the influence of the component fields existing at the stations : 1, 2, 3 — at heights h_1, h_2, h_3, etc., are denoted by t_1, t_2, t_3, etc. then the respective value of the periods and of the fields will be :—

$t_0 \ldots\ldots H$

$t_1 \ldots\ldots H - \dfrac{\mathfrak{m}}{h_1^3}$

$t_2 \ldots\ldots H - \dfrac{\mathfrak{m}}{h_2^3}$

Hence :—

$$Ht_0^2 = \left(H - \frac{\mathfrak{m}}{h_1^3}\right) t_1^2 = \left(H - \frac{\mathfrak{m}}{h_2^3}\right) t_2^2 = \ldots\text{etc.}$$

i. e.

$$H\,(t_1^2 - t_0^2) = \frac{\mathfrak{m}}{h_1^3}\, t_1^2 \quad \text{or} \quad \frac{H}{\mathfrak{m}} = \frac{t_1^2}{t_1^2 - t_0^2} \times \frac{1}{h_1^3} \qquad (8)$$

and by putting :

$$\begin{cases} x = \tau^2 = \left(\dfrac{t^2}{t^2 - t_0^2}\right)^{\frac{2}{3}} & (9) \\ y = h^2 \end{cases}$$

the formula :

$$\frac{H}{\mathfrak{m}} = x^{\frac{3}{2}} \times \frac{I}{y^{\frac{3}{2}}} \quad \text{or} \quad \left(\frac{y}{x}\right)^{\frac{3}{2}} = \frac{\mathfrak{m}}{H} \qquad (10)$$

is obtained.

Thus the ratio $\dfrac{\mathfrak{m}}{H}$ is known from this formula. As the product $\mathfrak{m}H$ has been previoulsy determined, the requisite values of $\mathfrak{m}$ and H can easily be obtained by adding and subtracting the logarithms of these values.

The calculations are arranged in the following manner. Example :

2. Oscillations of a needle above a bar magnet (calculated 3rd June 1923)

(*see opposite*)

n°	h	α moy	10 oscill.	log	log[1+β]	diff.	durée	
1	13cm.45	45°	32s.1	1.5065	0.0170	1.4895	30s.87	30.77
		35°	31.4	4969	0102	4867	30.67	
		43°	31.9	5038	0155	4883	30.78	
		51°	32.3	5101	0220	4881	30.77	
2	17.45	41°	49.2	1.6920	0.0141	1.6779	47.63	t_2 47.51
		33°	48.3	6839	0091	6748	47.29	
		36°	48.8	6884	0108	6776	47.60	
		41°	49.1	6911	0141	6770	47.53	
3	21.55	40°	87.6	1.9425	0.0134	1.9291	84.94	t_3 85.01
		36°	86.6	9375	0108	9267	84.47	
		41°	88.0	9445	0141	9304	85.20	
		42°	88.4	9465	0148	9317	85.44	
pointe vers le Sud								
pointe vers le Nord								
4	29cm.45	35°	111s.2	2.0461	0.0102	2.0359	108s.61	t_4 108.03
		32°	109.6	0398	0085	0313	107.48	
		36°	111.2	0461	0108	0353	108.46	
		34°	110.0	0414	0097	0317	107.56	
5	33.45	38°	87.6	1.9425	0.0121	1.9304	85.20	t_5 85.30
		37°	87.6	9425	0115	9310	85.32	
		38°	87.8	9435	0121	9314	85.34	
		40°	88.0	9445	0134	9311	85.33	
6	36.90	40°	79.2	1.8987	0.0134	1.8853	76.79	t_6 77.11
		42°	80.0	9031	0148	8863	76.97	
		42°	80.0	9031	0148	8863	76.97	
		42°	80.4	9053	0148	8905	77.72	
boussole libre	0	33°	63s.6	1.8035	0.0091	1.7944	62s.29	t_0 62.38
	indiff. ω	37°	64.0	8062	0115	7947	62.34	
	max.	36°	64.1	8069	0108	7961	62.53	

	t^2	$t^2 \pm t_0^2$	log	log	$\left[\frac{t^2}{t^2 \pm t_0^2}\right]$	2[ ]	⅔[ ]	x		y	
t_0	3891.26										
t_1	946.79	- 4838.05	2.97625	3.68467	- $\bar{1}$.29158	- $\bar{2}$.58316	- $\bar{1}$.52772	- 0.33707	-1.60014	- 180.90	- 949.80
t_2	- 2257.20	6148.46	3.35357	3.78877	- $\bar{1}$.56480	- $\bar{1}$.12960	- $\bar{1}$.70987	- 0.51271		304.50	
t_3	- 7226.70	11117.96	3.85894	4.04603	- $\bar{1}$.81291	- $\bar{1}$.62582	- $\bar{1}$.87527	- 0.75036		- 464.40	
t_4	- 11670.48	- 7779.22	4.06709	3.89093	0.17616	0.35232	0.11744	- 1.[illegible]50	+5.00680	- 867.30	+ 3347.81
t_5	7276.09	3384.83	3.86190	3.52954	0.33236	0.66472	0.22157	- 1.66560		- 1118.90	
t_6	- 5945.95	- 2054.69	3.77422	3.31275	0.46147	0.92294	0.30765	- 2.03070		- 1361.61	

3.40666 .... - 0.53233

2398.01 - 3.37985

$\frac{1}{6}\Sigma(x) = 1.10116$ $\log \frac{1}{6}\Sigma(x) = 0.04538$

$\frac{1}{6}\Sigma(y) = 716.27$ $\log a = 2.84752$

$ax = 781.45$ $\log ax = 2.89290$

$b = -65.18$

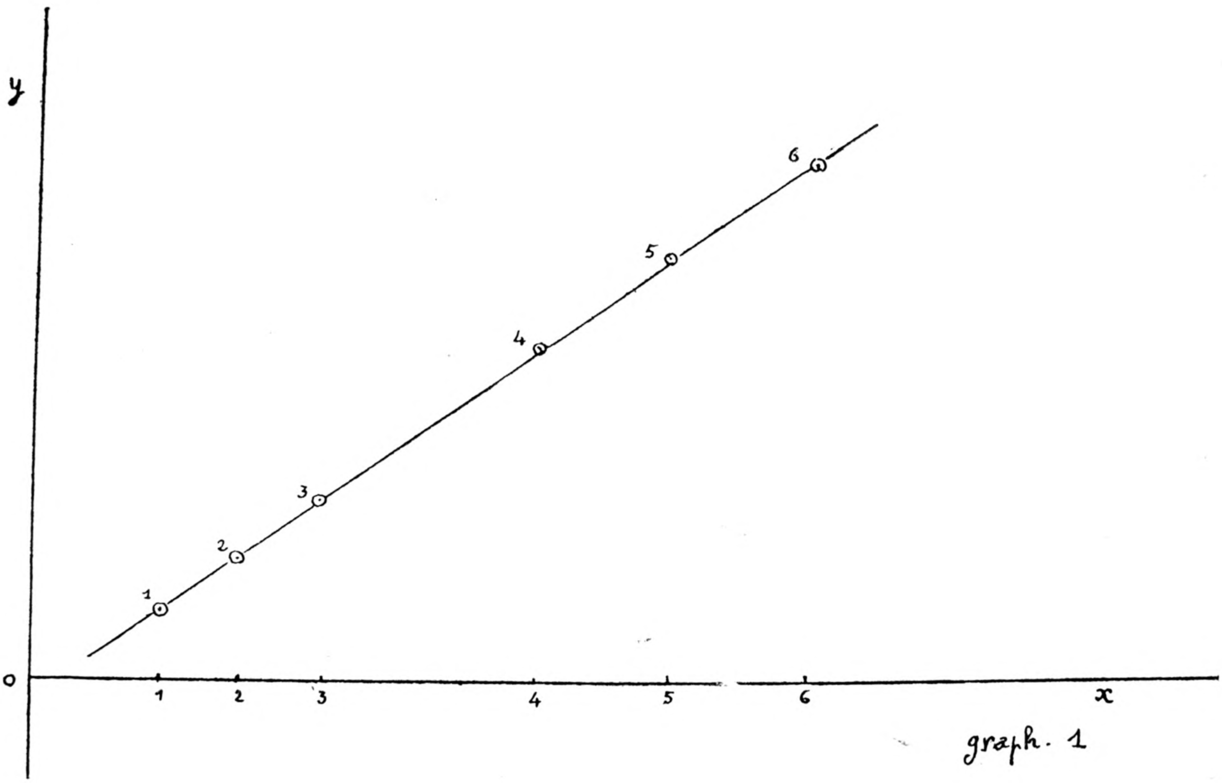
$\lambda = \sqrt{-b} = 8^{cm}.07$

	- 3.37985
	- 0.53233
× 3	2.84752 = log a
2	- 8.54256 = '
$\frac{M}{H}$	- 4.27128
MH	- 2.76427
M^2	- 7.03555 - 3.51778
H^2	- $\bar{2}$.49299 - $\bar{1}$.24650

M = 3294 gauss

H = 0.1764

The quality or regularity of the observations can be verified by drawing a graph of the points representing the 6 stations, taking as co-ordinates the values of x and y from formula (9). According to the preceding equation (10) these points must necessarily fall on a straight line.



graph. 1

The equation of this line is $y = ax + b$ (11) in which $\sqrt{b}$ represents the half of the polar length of the magnet.

Thus the characteristic elements of the field at the place of operation, and also those of the standard bar magnet, which henceforth can be used for rapid measurements by the method of deflections, are known.

If the bar magnet is placed on the trial table at the same distance from the deviation compass, the reading of the deviation, say α°, which the bar magnet oriented to the East-West imparts to it, may be measured (Fig. 18). Having determined this angle the standard bar magnet is replaced by, for example, some trial magnets or some spare unmounted compass cards, which are always placed East and West and at the same marked distance. The deviations α' α'' α''' obtained with each combination are rapidly noted : their magnetic moments are to that of the standard bar magnet in the ratios of the respective deviations :—

$$\frac{\alpha'}{\alpha^\circ} \qquad \frac{\alpha''}{\alpha^\circ} \qquad \frac{\alpha'''}{\alpha^\circ}$$

To operate still more rapidly, the calculation of these proportions is dispensed with, and the following method is used :

Assume that the magnetic moment of the standard bar magnet is 3290 gauss ; it is moved towards the deviation compass as in Fig. 18 until the

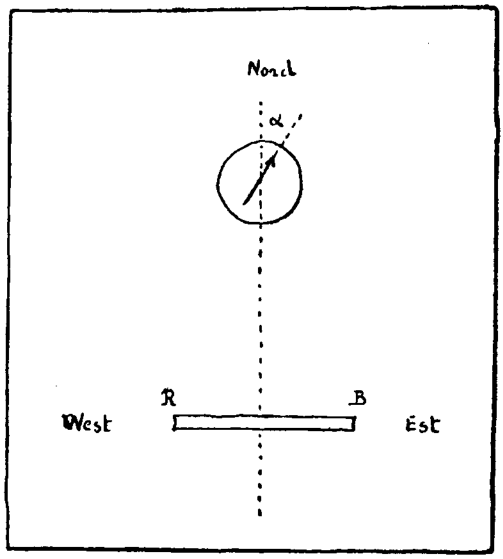


Fig. 18

reading of the angle α° on the divided limb is exactly 32°9. Then mark the exact distance of the magnet from the deviation compass, and place another magnet or spare card in the position occupied by the standard magnet and read off direct its magnetic moment on the graduated limb of the deviation compass without making the least calculation.

As a matter of fact the deviation compass which was used was not graduated in degrees, it was divided in "millièmes" (6,400 to the circumference) which still further facilitated the direct reading of the magnetic moment, when the swings of the needle were small.

Another qualitative method, which is very easy to demonstrate, consists in using the neutral point of a small pocket compass to estimate the ratio $\frac{\mathfrak{m}}{H}$.

These pocket compasses are made with needles less than 1 ½ c/m in total length, and are therefore perfectly suited to explore the field created around a magnet or a compass card.

The magnet being placed at the centre of the trial table, the pocket compass is moved away to the East, until its needle becomes neutral. This occurs at a distance d (Fig. 19) (it should be noted, however, that the value of d is not absolutely accurate). At that moment

$$\frac{\mathfrak{m}}{d^3} = H \quad \text{or} \quad \frac{\mathfrak{m}}{H} = d^3 \qquad (12)$$

A table can be compiled of values of d which correspond, at a certain place, to magnetic moments of 200, 400, 600, 1000, 2000, 3000, 5000 gauss, so that the magnetic power of a magnet or compass card can be at once determined by simply approaching it with a pocket compass. But this result is only qualitative.

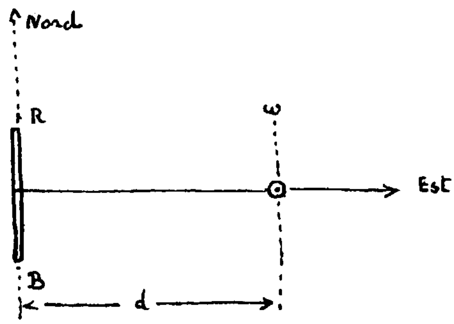


Fig. 19

From the foregoing may be seen all the advantages to be gained from the determination of the magnetic moment of a standard magnet and of the value of the magnetic field.

The original problem still remains to be solved, *i.e.* the determination of the magnetic moment of a mounted liquid compass.

To do this, the bowl of the compass should be placed at the centre of the trial table. The card will orient itself in the magnetic meridian. The value of the horizontal field of the Earth is already known, either by a former calculation, or it is measured at the time, as previously described, from the oscillation of a bar magnet at the place itself.

Measures of oscillations of a small oscillating compass at varying heights above the bowl can be repeated in exactly the same manner as previously carried out above the bar magnet. The same materials and the same accessories are used; but for this purpose the round wooden plate which forms the base of the column of cylinders (Fig. 16) is hollowed out to receive the pivot of the alidade and to centre itself on it. The same method of operating on either side of the neutral point of the oscillating compass is used. The slight swings of the liquid compass card in relation to the magnetic north are of no importance, their period being much slower than that of the oscillations proper of the oscillating compass.

The only difference is that the vertical distance b' separating the mean plane of the needles of the card under examination from the upper plane of the glass of the bow, is not known and that also, in this case, the heights of the needle of the oscillating compass *above the glass of the liquid Compass bowl* are denoted by the letter h'. If h denotes these same heights counted above the plane of the needles, then

$$h = h' + b' \quad \text{(Fig. 20)}.$$

b' can be calculated by combining the observations 2 by 2.

If the stations are numbered 1 - 2 - 3 - 4 ...... m ...... n, the observation m,

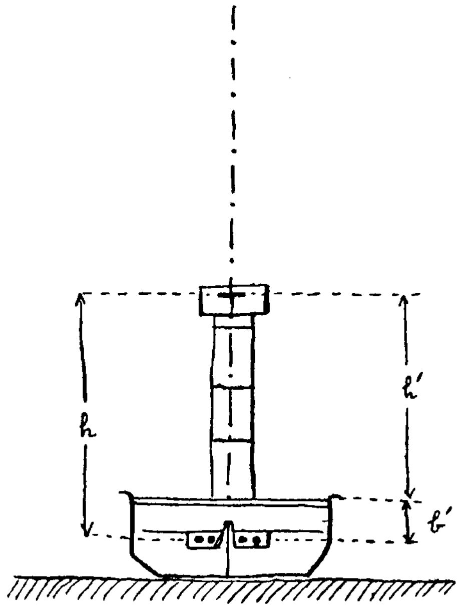


Fig. 20

for example, should be combined with the observation n in such a way that $h'_m - h'_n$ is sufficiently large.

Now,

$$h_m = h'_m + b'$$
$$h_n = h'_n + b'$$

Therefore by subtracting :

$$h_m - h_n = h'_m - h'_n$$

adding :

$$h_m + h_n = h'_m + h'_n + 2b'$$

and multiplying together $h^2_m - h^2_n = (h'_m + h'_n)(h'_m - h'_n) + 2b'(h'_m - h'_n)$

and from the preceding formula (11) for the magnet :

$$h^2_m = a\ x_m = \lambda^2$$
$$h^2_n = a\ x_n + \lambda^2$$

$$h^2_m - h^2_n = a\ (x_m - x_n)$$ so that

$$(h'_m + h'_n)\ (h'_m - h'_n) = a\ (x_m - x_n) - 2b'\ (h'_m - h'_n)$$ or

$$h'_m + h'_n = a\,\frac{x_m - x_n}{h'_m - h'_n} - 2b' \qquad (14)$$

By taking

$$\left\{ \begin{aligned} y'_{mn} &= \frac{1}{2}\,(h'_m + h'_n) = h'_{mn} \qquad (15) \\ x'_{mn} &= \frac{x_m - x_n}{2\,(h'_m - h'_n)} \end{aligned} \right.$$

Compas Doignon Mod 1923 - n° 5906

n°	h'	10 oscill doubles	α moy	log	log (1+β)	diff.	durée	
1	5 cm 2	30 s 2	43°	1.4800	0.0155	1.4645	29 s 14	29 s 55
		30 0	36	4771	0 108	4663	29. 26	
		30 6	33	4857	0 091	4766	29. 96	
		30 6	36	4857	0,108	4749	29. 85	
2	7 cm 2	40 s 0	34°	1.6021	0.0097	1.5924	39 s 12	39 s 08
		39 8	35	5999	0102	5897	38. 88	
		40 0	26	6021	0056	5965	39. 49	
		39. 6	32	5977	0085	5892	38. 83	
3	9 cm 2	52 s 8	29°	1.7226	0.0070	1.7156	51 s 95	52 s 02
		52. 6	33	7210	0091	7119	51. 51	
		54. 2	36	7340	0108	7232	52. 87	
		52. 6	29	7210	0070	7140	51. 76	
4	11 cm 2	76 s 0	36°	1.8808	0.0108	1.8700	74 s 13	73 s 68
		74 7	28	8733	0065	8668	73. 59	
		74 7	28	8733	0065	8668	73. 59	
		74. 3	25	8710	0052	8658	73. 42	
vers le Sud.								
vers le Nord.								
5	22 cm 2	83 s 3	30°	1.9207	0.0075	1.9132	81 s 89	81 s 81
		84. 0	40	9243	0134	9109	81. 45	
		83. 3	30	9207	0075	9132	81. 89	
		82. 7	31	9175	0080	9095	81. 19	
6	24 cm 2	78 s 0	36°	1.8921	0.0108	1.8813	76 s 09	77 s 11
		77. 7	28	8904	0065	8839	76. 54	
		80 0	40	9031	0134	8897	77. 57	
		80. 7	40	9069	0134	8835	78. 25	
7	26 cm 2	75 s 7	27°	1.8791	0.0061	1.8730	74 s 65	73 s 76
		75. 0	38	8751	0121	8630	72. 95	
		75. 3	31	8768	0080	8688	73. 93	
		75. 7	39	8791	0127	8664	73. 52	
8	28 cm 2	72 s 7	38°	1.8615	0.0121	1.8494	70 s 70	70 s 82
		73. 0	37	8633	0114	8519	71. 10	
		74. 0	50	8692	0211	8481	70. 49	
		73. 0	38	8633	0121	8512	70. 99	

boussole libre	0	63 s 3	30°	1.8014	0.0075	1.7939	62 s 22	62 s 36	62 s 50
		63. 7	29	8041	0070	7971	62. 68		
		63. 0	30	7993	0075	7918	61. 92		
		64. 3	37	8082	0115	7967	62. 62		
	indiff.	64 s 3	34°	1.8082	0.0097	1.7985	62 s 88	62 s 59	
		64. 0	35	8062	0102	7960	62. 52		
		65. 3	37	8149	0115	8034	63. 59		
		62. 3	28	7945	0065	7880	61. 38		
	max.	63 s 3	31°	1.8014	0.0080	1.7934	62 s 15	62 s 55	
		63. 7	36	8041	0108	7933	62. 13		
		65. 3	40	8149	0134	8015	63. 32		
		64. 0	34	8062	0097	7965	62. 59		

n°	t^2		$t^2 \pm t_0^2$	log	log	$\left[\frac{t^2}{t^2 \pm t_0^2}\right]$	2[ ]	2/3 [ ]	x	h' + b'	
0	3906. 25										
1	873. 20	+	4779.45	2.941114	3.679378	$\bar{1}$.261736	$\bar{2}$.523472	$\bar{1}$.507824	0.32198	10. 3	106. 09
2	1527. 25		5433. 50	3.183910	3.735080	$\bar{1}$.448830	$\bar{2}$.897660	$\bar{1}$.632553	0. 42910	12. 3	151. 29
3	2706. 08		6612. 33	3.432341	3.820354	$\bar{1}$.611987	$\bar{1}$.223974	$\bar{1}$.741325	0. 55122	14. 3	204. 49
4	5428. 74		9334. 99	3.734699	3.970114	$\bar{1}$.764585	$\bar{1}$.529170	$\bar{1}$.843057	0. 69672	16. 3	265. 69
									1. 99902		727. 56
5	6692. 88	−	2786. 63	3.825613	3.445079	0.380534	0.761068	0.253689	1.7934	27. 3	745. 29
6	5945. 95		2039. 70	3.774221	3.309566	0.464655	0.929310	0.309770	2.0407	29. 3	858. 49
7	5440. 54		1534. 29	3.735642	3.185907	0.549735	1.099470	0.366490	2. 3254	31. 3	979. 69
8	5015. 47		1109. 22	3.700312	3.044017	0.656295	1.312590	0.437530	2.7386	33. 3	1108. 89
									8. 89810		3692. 36
									6. 89908		2964. 80

Calcul de b' $h'_{mn} = \frac{1}{2}(h'_m + h'_n)$ $x'_{mn} = \frac{x_m - x_n}{2(h'_m - h'_n)}$

m & n	$h'_m - h_n$	h'_{mn}	$x_m - x_n$	$2(h'_m - h'_n)$	log	log	diff.	x'_{mn}
1 - 3	4 cm 0	7 cm 2	0.22924	8.0	$\bar{1}$.360290	0.903090	$\bar{2}$.457200	0,02866
2 - 4	4. 0	9. 2	0. 26762	8.0	$\bar{1}$.427519	0.903090	$\bar{2}$.524429	0,03345
3 - 5	13. 0	15. 7	1. 24218	26. 0	0.094185	1.414973	$\bar{2}$.679212	0,04778
4 - 6	13. 0	17. 7	1. 34398	26. 0	0.128393	1.414973	$\bar{2}$.713420	0. 05169
5 - 7	4. 0	24. 2	0. 53200	8. 0	$\bar{1}$.725912	0.903090	$\bar{2}$.822822	0. 06650
6 - 8	4. 0	26. 2	0. 69790	8. 0	$\bar{1}$.843793	0.903090	$\bar{2}$.940703	0. 08724

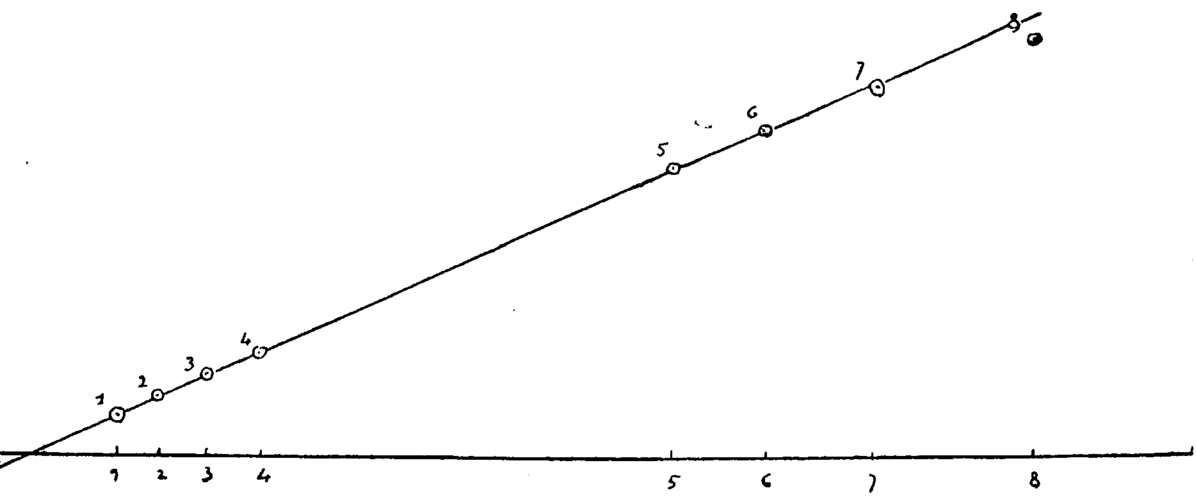
b' = 5 cm 1

3.47200
0.83879
2.63321
1/2 - 1.31661
$\frac{\mathfrak{M}}{H}$ - 3.94982
H - $\bar{1}$.24650
$\mathfrak{M}$ - 3.19632 = 1571 cgs

it can be seen from the equation (14) that the graph of the points of co-ordinates y'_{mn}, x'_{mn} will be a straight line of which the ordinate at the origin with sign changed will give b'.

Having thus obtained this value b' with an approximation near a millimetre, the sums $h' + b' = h$ are done, and the calculation completed as previously described for the bar magnet.

The form of calculation is as shown on the sheet opposite.



graph. 2

The method of oscillations is rather delicate to put in practice; the measurement of the oscillations should be made with care. For the three calculations above, the measurement of the terrestrial field, the observations to find the moment of the standard magnet and the magnetic moment of a liquid compass, lasted from 1 ½ to 2 hours. The calculations of reduction take from 4 to 5 hours. It is true that they could be shortened a little after a first trial calculation, and also by using the table of squares of numbers from 1 to 31,000 by Dr O. J. Broch (64 pages). The measurement of the field and that of the magnetic moment of the bar alone require 1 ½ to 2 hours.