SUBMARINE PHONOTELEMETRY

by

CAPTAIN L. TONTA, DIRECTOR.

II.

ACCURACY OF A RADIO-ACOUSTIC POSITION-LINE

16. In the preceding paragraphs (*) we have found that the mean error (or uncertainty) E_t in the radio-acoustic position line of equal $\frac{\theta_a}{\theta_b}$ due to a mean error t in the measurement of the intervals θ_a and θ_b, which the sound takes to cover the distances $PA = D_a$ and $PB = D_b$ (fig. 10), is given by the formula :

$$E_t = vt\ \frac{\sqrt{D_a^2 + D_b^2}}{q}$$

(v being the velocity of sound).

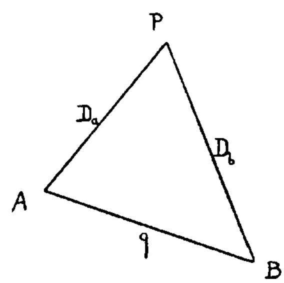

Fig. 10

The displacement of the radio-acoustic position line (**), due to the existence of a current may be determined (and consequently corrected) by means of the formula in Paragraph 11 since the speed and the direction of the current are known. (For the practical method of correcting the effects of current, refer to the observation at the end of the present paragraph).

(*) See "*Hydrographic Review*" Vol. IV, Nov. 1927, pages 139 and following.

(**) Hereafter for simplification the locus of equal $\frac{\theta_a}{\theta_b}$ will be referred to by the name *Radio-acoustic position line*—or, more simply still, the radio-acoustic line.

(For obvious reasons it is well to avoid the necessity of making such corrections by operating in still water (slackwater)).

On the other hand, it is best to allow for an error in the measurement of the current and even if operating in slackwater, the existence of a *very slight* current is not easily estimated and therefore remains unknown. Nevertheless it should be noted that the uncertainty of the radio-acoustic position line from this cause is extremely small, as can be verified by the above formula.

It appears then lawful to assume that E_t represents the total mean error E in the position line with sufficient accuracy; and therefore :

$$(1) \qquad E = vt \frac{\sqrt{D_a^2 + D_b^2}}{q}$$

REMARK : Instead of calculating the lateral displacement of the radio-acoustic position line by the above formula, it is preferable to correct the effect by applying to each observed time θ, a particular correction to reduce it to the value which would have been obtained had there been no current. To determine this correction the following hypothesis may be made:

If the existence of a current is not considered, a time $\theta' = \frac{D}{v}$ would be obtained, D being the distance and v the velocity of sound in still water. But actually the time $\theta = \frac{D}{v + \Delta v}$ is observed, where Δv represents the increment (positive or negative ; which is equal to the projection of the current *along the direction considered,* see paragraph 8), and by which, due to the current, the velocity v is affected.

Differentiating the formula $\theta' = \frac{D}{v}$, in which D is a constant, we have :

$$d\,\theta' = \frac{-D}{v^2}\,dv$$

and for the finite increments we have :

$$\Delta\theta' = \theta - \theta' = \frac{-D}{v^2}\,\Delta v$$

And the correction C which should be applied to the time θ actually observed, to make it equal to the value θ' which corresponds to *no current, i.e.* the quantity $C = \theta' - \theta$ will be found in magnitude and sign from the formula :

$$C = \frac{D}{v^2}\,\Delta v$$

(It is evident that the correction will be positive when the current increases the velocity of sound in the considered direction, and negative in the reverse case).

In calculating the correction, for D and v approximate values may be used. If the value of v is 1500 m. /sec. and if Δv is expressed in metres, then :

$$C = 0{,}000444\, D^{km.}\ \Delta\ v, \text{ sec.}$$

If, for example, $D = 100$ kms.. and $\Delta v = +$ 1 metre the correction will be + 0,0444 seconds.

17. If the base AB is known, a radio-acoustic position line can be determined for each point P of the plane which, for a known value t of the mean error in the measurement of the time, will be affected by a mean error E given by the formula (1) of the preceding paragraph.

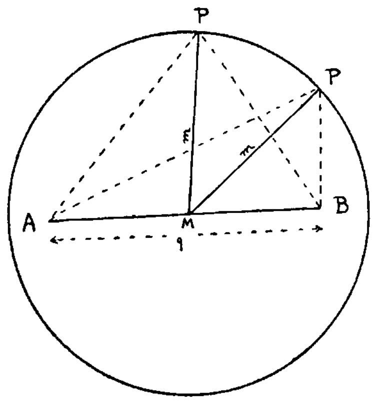

Fig. 11

The succession of points for which the value of E is the same, forms a series of continuous curves, which will be called the curves of equal accuracy of the radio-acoustic position line or, for short, the *accuracy curves.*

It is easy to show that the accuracy curves are concentric circles whose centres are at the central point M of the base AB (fig. 11). In other words, for all points P, situated at equal distances from the centre of the base, radio-acoustic lines are obtained which are affected by the same mean error E. It is known that the locus of the points for which the sum of the squares of the distances to two points A and B is equal to a known constant, lies on the circumference of a circle whose centre is M, the central point of AB.

In the position triangle PAB (fig. 2)

$$D_a^2 + D_b^2 = 2\ (m^2 + \frac{q^2}{4})$$

m being the medial line relative to the base AB.

Therefore

$$E = vt \sqrt{2\ (\frac{m^2}{q^2} + \frac{1}{4})}$$

From this formula it is seen that, for a known base AB= q, and giving to t a constant value (for example: 0.01 sec.) the values of E depend entirely on the length of the medial line m.

If $t = 0.01$ sec., and v has the approximate value of 1500 m/sec.

Then $$E' = 21{,}21 \sqrt{\frac{m^2}{q^2} + \frac{1}{4}}$$

A table showing the values of $\sqrt{\frac{m^2}{q^2} + \frac{1}{4}}$ is given below, calculated for different values of $\frac{m}{q}$:—

$\frac{m}{q}$	$\sqrt{\frac{m^2}{q^2} + \frac{1}{4}}$	$\frac{m}{q}$	$\sqrt{\frac{m^2}{q^2} + \frac{1}{4}}$
0	0.500	3	3.041
0.5	0.866	3.5	3.535
1	1.118	4	4.031
1.5	1.581	4.5	4.527
2	2.062	5	5.025
2.5	2.550		

From the following approximate rule, a rapid and rough appreciation of the uncertainty of the radio-acoustic position line due to an error of a hundredth of a second in the measurement of the time, can be obtained. The mean error is equal to as many times 20 metres as the base is included in the distance from the position to the centre of the base.

With this rule the error may be determined with an approximation of about 10 metres in the most unfavourable case ($\frac{m}{q} = 0$) and with a closer approximation in the more usual case when $\frac{m}{q} > 1$.

Since it is considered that in the most frequent cases of determination, effected by ordinary means, it is advisable to admit that the mean error in the measurement of the times can reach 0.01 sec., it seems permissible to conclude that, for a good determination of a radio-acoustic position line, it is necessary that the position to be determined be not more distant from the middle of the base than 5 times the length of the base. Only with very accurate means and measuring instruments can the limiting value $\frac{m}{q} = 5$ be exceeded.

THE RADIO ACOUSTIC POSITION BY THREE DISTANCES.

18. The radio-acoustic position line PT forms with the (small) side PB of the position triangle PAB (fig. 3) an angle $90° - \alpha$, α being the angle at the apex A, adjacent to the longest side PA (*),

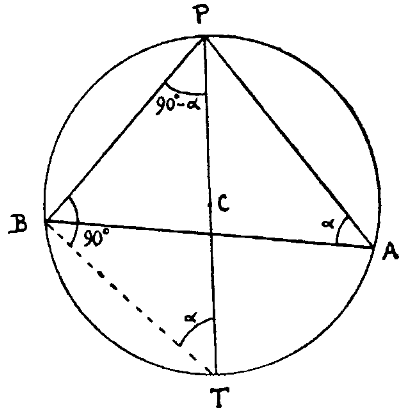

Fig. 12

In other words, the radio-acoustic line *coincides with the diameter PC of the circle circumscribing the position triangle ;* an examination of fig. 12 will demonstrate this.

Take a group of three points 1, 2, 3, (**) (fig. 13) and let it be desired to determine the point P by measurement of the times θ_1 θ_2 θ_3 that the sound takes to traverse the distances $P1 = D_1$, $P2 = D_2$, $P3 = D_3$, respectively.

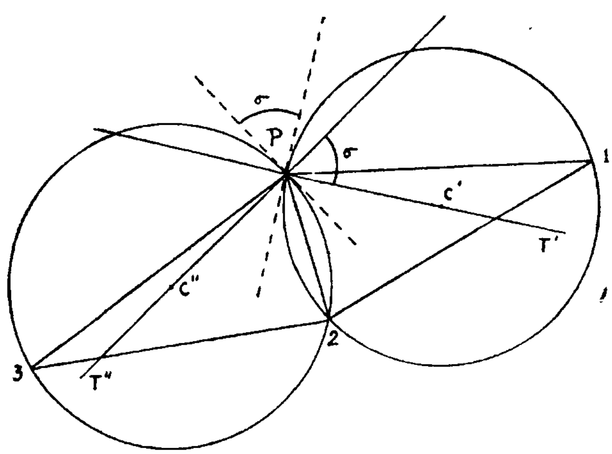

Fig. 13

(*) *See* paragraph 3 of preceding note.

(**) Hereafter it will always be assumed that the points 1, 2, 3 are in such positions that an imaginary line pivoting on P successively meets the three points in the order 1, 2, 3 or 3, 2, 1.

The ratio $\frac{\theta_1}{\theta_2}$ determines the position of the radio-acoustic line *PT'* which coincides with the diameter *PC'* of the circle circumscribed about the triangle *P*12; and the ratio $\frac{\theta_2}{\theta_3}$ determines the position of the radio–acoustic line *PT''* which coincides with the diameter *PC''* of the circle circumscribed about the triangle *P*23.

The two lines *PT'* and *PT''*, and the two circles circumscribed about the triangles *P*12 and *P*23 intersect at the same angle σ (*).

This interesting geometrical analogy between the radio-acoustic position in which we are concerned and the Pothenot (or Snellius) position by two subtended angles, permits the statements : —

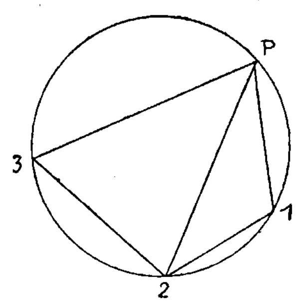

Fig. 14

(*a*) that our radio-acoustic position is indeterminate when the quadrilateral *P* 1 2 3 is circumscribable (fig. 14) because then the two circles coincide with the circle circumscribed about the triangle 1 2 3 and the diameters *PT'* and *PT''* coincide with the diameter *PT* of the latter.

(*b*) that, if the point *P* is situated, relatively to the points 1, 2, 3, in a favourable position to obtain a good intersection of the two Pothenot circles determined by the measurement of the angles 1*P*2, 2*P*3, a good intersection will also be obtained of the radio-acoustic position lines, determined by the ratios $\frac{\theta_1}{\theta_2}$ and $\frac{\theta_2}{\theta_3}$.

19. Let 1, 2, 3, be the points selected for the determination of the radio acoustic position. Taking pairs of the three distances D_1, D_2, D_3, calculated with the same approximate value v' of the velocity of sound, ($D_1 = v'\theta_1$, $D_2 = v'\theta_2$, $D_3 = v'\theta_3$), and working as has been indicated in the preceding note, three determinating points are obtained, and more precisely :

(*) If the angle 21*P* is denoted by φ the angle *P*32 by λ then $\sigma = \varphi + \lambda$.

The determinating point P_{12} by solving the triangle which has 12 for base.
The determinating point P_{23} by solving the triangle which has 23 for base.
The determinating point P_{31} by solving the triangle which has 31 for base.

P_{12}, in other words, is the intersection of the circle I, *locus of equal distance* D_1 to 1 with the circle II, locus of equal distance D_2 to 2 ; etc. (fig. 15).

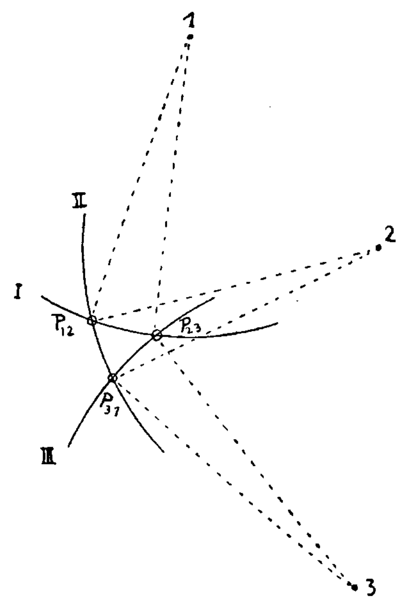

Fig. 15

The three determinating points form a small triangle, the sides of which may coincide with the arcs of the circles $P_{12}P_{23}$, $P_{23}P_{31}$, $P_{31}P_{12}$, which belong respectively to the circles I, II and III.

It should be noted once more that these sides (which for simplicity will be called : side I, side II, side III) represent in the vicinities of the exact point, *the loci of equal distance,* I being the line of equal distance D_1 to 1, II the line of equal distance D_2 to 2, etc., etc.

Let the triangle considered be called the *determinating triangle.* Let the parallel displacements of the sides I, II, III be respectively s', s'', s''', (fig. 16) and, more exactly, let the side I be moved parallel to itself a distance s' *towards* the point 1, the side II parallel to itself a distance s'' *towards* 2, and the side s''' *towards* 3, these three displacements being respectively proportional to the distances D_1, D_2, D_3 (*i.e.*, $s' = KD_1$; $s'' = KD_2$, $s''' = KD_3$ where K denotes an arbitrary proportional coefficient) the triangle P'_{12} P'_{23} P'_{31} is obtained, similar to the determinating triangle ; and further these two triangles are homothetic. The lines which join the homologous apices (*i.e.*, the three homologous vectors $P_{12}P'_{12}$, $P_{23}P'_{23}$, $P_{31}P'_{31}$) meet at the homothetic centre P.

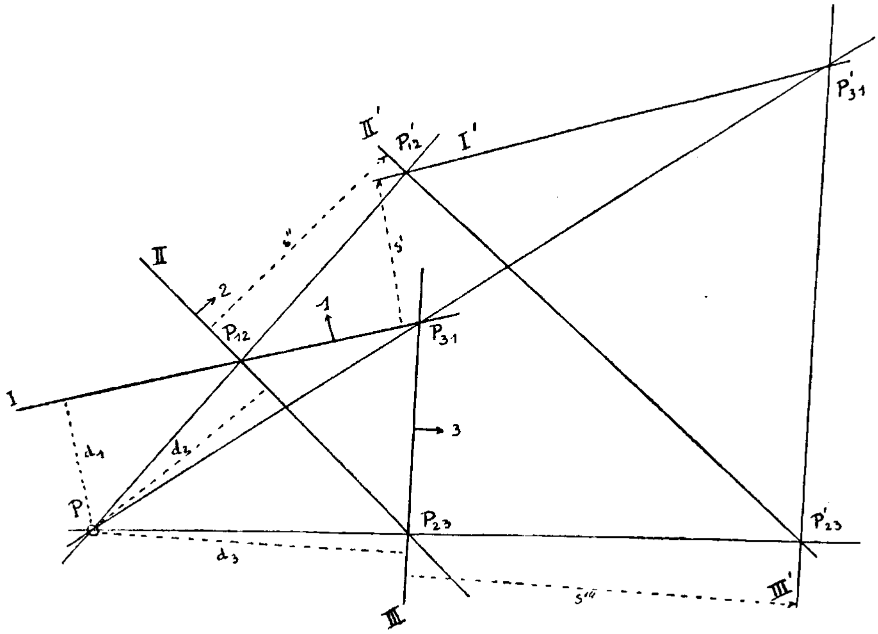

Fig. 16

It is now stated that the homologous vectors coincide with the three radio-acoustic position lines determined from the ratios $\frac{\theta_1}{\theta_2}$, $\frac{\theta_2}{\theta_3}$, $\frac{\theta_3}{\theta_1}$ and therefore the homothetic centre P is the radio-acoustic position required.

The demonstration is extremely simple :

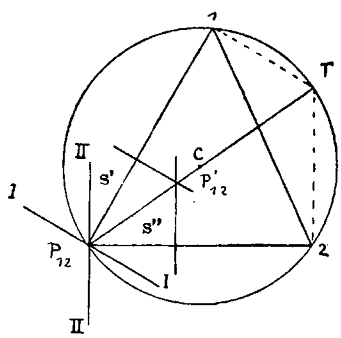

Fig. 17

For example let (fig. 17) the triangle P_{12} 12 be considered, and let a circle be drawn circumscribing it. The side I of the determinating triangle is perpendicular to P_{12} 1 and side II is perpendicular to P_{12}2. Let the sides I and II be given respectively the parallel displacements $s' = KD_1$ and $s'' = KD_2$, then the transposed lines I' and II' intersect at a point P'_{12}, which is on the diameter $P_{12}\,T$ of the circle circumscribed about the triangle P_{12} 12. This is evident from a consideration of figure 17. Therefore the vector $P_{12}\,P'_{12}$ coincides (as indicated in paragraph 18) with the radio-acoustic position line determined from the ratio $\frac{\theta_1}{\theta_2}$.

An analogous consideration may be repeated for the triangles P_{23} 23, P_{31} 31. Thus it has been shown not only how to determine graphically the radio-acoustic position by means of these determinating points, but it has also been shown that the three radio-acoustic position lines, determined by the ratios $\frac{\theta_1}{\theta_2}$, $\frac{\theta_2}{\theta_3}$, $\frac{\theta_3}{\theta_1}$, obtained by taking the three intervals θ_1, θ_2, θ_3, in pairs, *converge at the same point.*

20. Lastly, there is one other property of the determinating triangle which is evident. The distances d_1, d_2, d_3 from P (fig. 16) to the sides I, II, and III of this triangle (*) measure the difference between the *actual* distances of P to the three points 1, 2 and 3 and the value of the same distances calculated with the systematic erroneous value v' of the velocity of sound.

If v is the true value of this velocity, then:

$$d_1 = (v - v')\,\theta_1$$
$$d_2 = (v - v')\,\theta_2$$
$$d_3 = (v - v')\,\theta_3$$

and therefore

$$v - v' = \frac{d_1}{\theta_1} = \frac{d_2}{\theta_2} = \frac{d_3}{\theta_3}$$

This relation affords the means of calculating the correction which must be applied to the approximate value v' to obtain the true value v. In other words, the radio-acoustic determination is not only independent of the exact value of the velocity of sound, but it may be itself used to measure this value.

The quality of this measurement naturally depends on the quality of the determination of the radio-acoustic position, which can be best obtained when the base points have been appropriately selected and when the measurement of the times has been effected with the necessary accuracy.

This result *which besides could be foreseen a priori,* does not appear to be quite negligible.

21. A practical example is given below, which reproduces *in an approximate manner,* the determination of the Rochebonne plateau (off the coast of Charentes, on the west coast of France, Lat. 45° 12' N. Long. 2°23' W.), the points 1, 2 and 3 being situated respectively near the point of Corbeau, at Barges and at Chassiron (fig. 18).

(*) Except, naturally, errors arising from errors made in the measurement of the times θ_1, θ_2, θ_3 or from the unknown existence of a slight current.

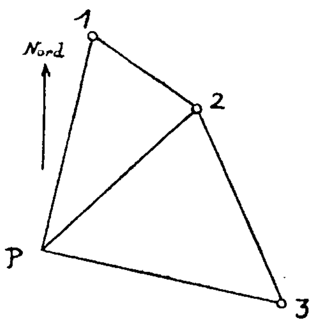

Fig. 18

Example :

The points 1, 2, 3 are known by their rectangular coordinates in metres. The origin of the coordinates is 1 $\begin{cases} x_1 = 0 \\ y_1 = 0 \end{cases}$

We have 2 $\begin{cases} x_2 = + 33485.2 \\ y_2 = - 21755.6 \end{cases}$ 3 $\begin{cases} x_3 = + 65350.3 \\ y_3 = - 71247.4 \end{cases}$

With these details we will determine the lengths of bases 12, 23, 31, which we shall call respectively q_{12}, q_{23}, q_{31}.

$q_{12} = 39932.0$ metres
$q_{23} = 58864.0$ metres
$q_{31} = 96679.2$ metres

Azimuth of 2 from 1 = (12) = 123°00'44".0
» 3 » 2 = (23) = 147°13'28".0
» 3 » 1 = (13) = 137°28'19".2

Details deduced from the observations :

θ_1 = time taken by the sound to traverse the distance $D_1 = 35^s940$
θ_2 = » » » » » $D_2 = 34^s816$
θ_3 = » » » » » $D_3 = 50^s400$

Approximate value of the velocity of sound = 1490 metres-sec.

$D_1 = 35.940 \times 1490 = 53550.6$ metres.
$D_2 = 34.816 \times 1490 = 51875.8$ »
$D_3 = 50.400 \times 1490 = 75096.0$ »

Solving the position triangle $12P_{12}$ the base of which is q_{12} and sides D_1, D_2, the following coordinates for the determinating point P_{12} are obtained :

$$P_{12} \begin{cases} x = - 7959 \\ y = - 52955 \end{cases}$$

and then by solving the position triangle $23P_{23}$, the base of which is q_{23} and sides D_2 and D_3, the rectangular coordinates of P_{23} will be obtained

$$P_{23} \begin{cases} x = - 7600 \\ y = - 53427 \end{cases}$$

Finally the position triangle $13P_{31}$ the base of which is q_{31} and the sides D_3 and D_1 is solved and the rectangular co-ordinates of

$$P_{31} \begin{cases} x = - 7501 \\ y = - 53023 \end{cases}$$

are obtained.

Place on a large-scale graph (fig. 19) the three determinating points

$$P_{12} \qquad P_{23} \qquad P_{31}$$

Draw the homothetic triangle

$$P'_{12} \qquad P'_{23} \qquad P'_{31}$$

giving to the sides I, II, III of the determinating triangle P_{12}, P_{23}, P_{31}, the parallel displacements s', s'', s''' respectively proportional to D_1, D_2, D_3, in the direction of the arrows which indicate respectively the directions of the points.

Plot the homologous vectors

$$P_{12}P'_{12}\,;\quad P_{23}P'_{23}\,;\quad P_{31}P'_{31}$$

which meet in the homothetic centre P which is the position required.

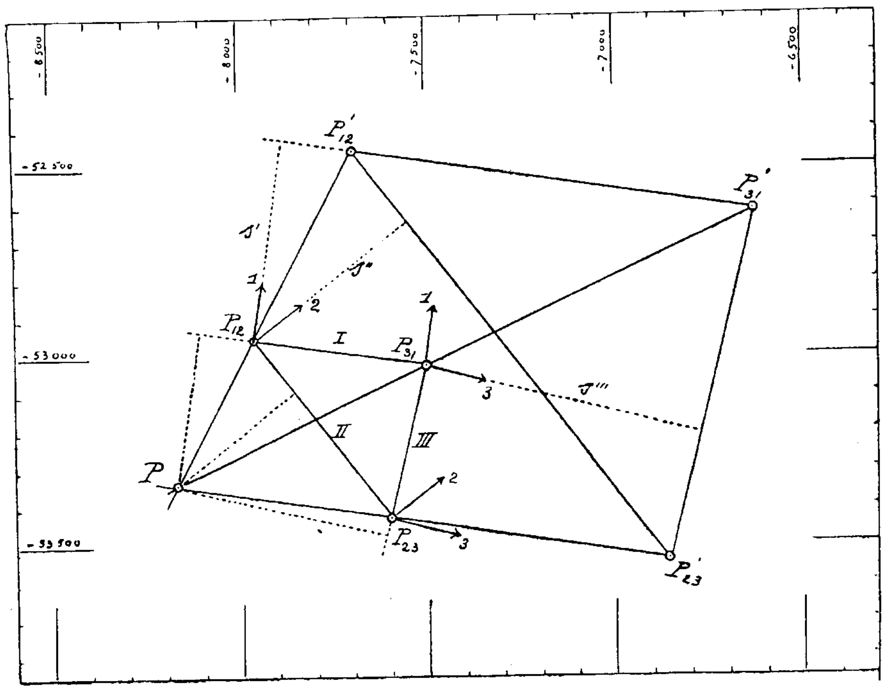

Fig. 19

22. The accuracy of the radio-acoustic position will now be determined. In the following discussion, only the influence of accidental errors committed in the measurement of the times θ_1, θ_2, θ_3 will be considered. Thus the influence of a slight current will be neglected, but it is obvious from examination of the formula which gives the displacement due to the current of the radio-acoustic position line (paragraph 11 of the preceding note) that, in practice this influence will always be very small indeed.

On the other hand it should be noted that the influence of the current manifests itself by a modification of the times θ_1, θ_2, θ_3 which the sound takes to traverse the three distances.

However, it is a question of only very slight modifications to which it seems permissible to attribute the character of accidental errors by including them with the errors truly and properly accidental in the measurement of the times, when these measurements are repeated in different weather conditions.

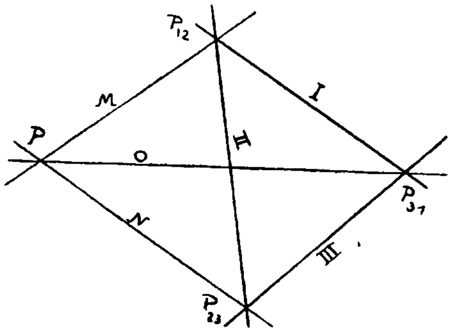

Fig. 20

Assuming the above, let us consider the determinating triangle P_{12}, P_{23}, P_{31} (fig. 20). It must be repeated once again that the sides I, II, III represent the loci of equal distance to the three known points (1, 2, 3). The lines M, N, O drawn through the three apices of the determinating triangle are the radio-acoustic lines obtained by means of ratios $\frac{\theta_1}{\theta_2}$, $\frac{\theta_2}{\theta_3}$, $\frac{\theta_3}{\theta_1}$: they meet at the same point P which is the required position (radio-acoustic position).

To calculate the mean error of the point of intersection P, which error may be taken as the degree of approximation that can be obtained in the radio-acoustic determination, the following hypothesis may be made:

An error t_1 in the measurement of the time θ_1 causes a lateral displacement of the line of equal distance I parallel to itself by a quantity vt_1, likewise an error t_2 in the measurement of time θ_2 will produce a displacement vt_2 of the line II and so on.

The lateral displacement of each of the lines I, II, III parallel to themselves causes a displacement of the point P along the radio-acoustic line from the opposite apex. Associating with each of the three lines I, II, III, one only of the two radio-acoustic lines which are contiguous to it, for example: the radio-acoustic line M to the line I, the radio-acoustic line N to the line II, the radio-acoustic line O to the line III, and supposing that each line, in being laterally displaced, affects only the radio-acoustic line which has been assigned to it, it is easy to see:

1) that a mean error t_1, in the measurement of the time θ_1 which causes the displacement vt_1 of the line I, will in its turn produce a lateral (parallel) displacement (*) of the line M

$$vt_1 \frac{D_2}{q_{12}}$$

2) that, corresponding to this lateral displacement, there is a discrepancy, or mean error, of P along the line N which is expressed by the formula:

(*) *See* paragraph 13.

$$\varepsilon_{I} = vt_1 \frac{D_2}{q_{12}} \frac{1}{sin\ (MN)} \quad (*)$$

and also that

$$\varepsilon_{II} = vt_2 \frac{D_3}{q_{23}} \frac{1}{sin\ (NO)}$$

$$\varepsilon_{III} = vt_3 \frac{D_1}{q_{31}} \frac{1}{sin\ (OM)}$$

Each one of these displacements of the point P is independent of the others, hence, adopting the known principle which consists of measuring the mean total error by the square root of the sum of the squares of the independent errors between them, and assuming $t_1 = t_2 = t_3 = t$, the expression for the mean total error of the point P can be denoted by the formula:

$$\varepsilon_p = vt \sqrt{\left(\frac{D_2}{q_{12}}\right)^2 cosec^2(MN) + \left(\frac{D_3}{q_{23}}\right)^2 cosec^2\,(NO) + \left(\frac{D_1}{q_{31}}\right)^2 cosec^2\,(OM)}$$

If $t = 0.01$ sec. and assuming the mean value of 1500 metres/sec. for v, then $vt = 15$ metres and therefore $\varepsilon_p = 15\sqrt{\Sigma\,\varepsilon^2}$ metres in which $\Sigma\,\varepsilon^2$, for simplicity, represents the quantity written under the radical in the preceding formula.

It may be shown that the angles (MN), (NO), (OM), that is to say the angles formed at the intersection of the pairs of the three position lines, can be expressed as functions of the angles at the base of the triangles $P_{12}\,12$, $P_{23}\,23$, $P_{31}\,31$; but it has not been considered necessary to do so for it would make the formula giving the mean error somewhat complicated.

For the practical use of the formula it should be remembered that the line M is that which passes through the determinating point P_{12}; that the line N passes through P_{23} and that the line O passes through P_{31}.

In the example given in paragraph 21, with

$$\frac{D_2}{q_{12}} = 1.299$$

$$\frac{D_3}{q_{23}} = 1.276$$

$$\frac{D_1}{q_{31}} = 0.554 \quad \text{and} \quad \begin{aligned}(MN) &= 70°58' \text{ approx.}\\ (NO) &= 34°25' \quad "\\ (OM) &= 36°33' \quad "\end{aligned}$$

the value of the mean error of the radio-acoustic position, corresponding to a mean error $t = 0.01$ sec. is:

$$\varepsilon_p = 42 \text{ metres approximately}$$

(*) The symbols (MN), (NO), (OM) are here used to denote the angles formed by the lines M and N, N and O, O and M. The symbols q_{12}, q_{23}, q_{31} denote the lengths of the bases included between the points 1 & 2, 2 & 3, 3 & 1 respectively.

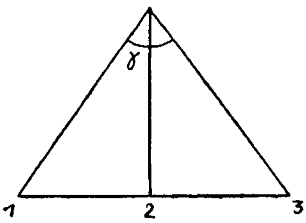

Fig. 21

23. In the particular case in Figure 21, in which the three points 1, 2, 3 are in a straight line, and where the point 2 is half way between 1 and 3 and where, moreover, the point P is on the line perpendicular to the line 13, through the point 2, the three radio-acoustic lines which are obtained by taking in pairs the intervals θ_1, θ_2, θ_3 coincide with the lines $P1$, $P2$, $P3$.

Therefore the mean error of the point P can be expressed in terms of the angle $\gamma = 1P3$, which measures the difference of the azimuths from P to the extremities of the base 13.

By converting the formula of mean error, a simpler result is obtained :

$$\varepsilon_p = \frac{vt}{1-\cos\gamma}\sqrt{6}$$ which if $t = 0.01$ sec. gives :

$$\varepsilon_p = \frac{36.7}{1-\cos\gamma}$$ metres approximately.

Another particular case is where the points 1, 2, 3 form an equilateral triangle, and where P is situated at the centre of the circumscribed circle (Fig. 22).

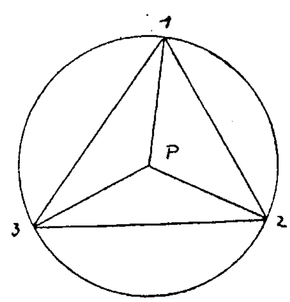

Fig. 22

Then $\varepsilon_p = vt\ cosec\ 60°$, and if $t = 0.01$ sec.
$\varepsilon_p = 23.2$ metres approximately.

24. The formula for mean error of the radio-acoustic position reveals an important property.

As the value of ε_p is a function of the ratio $\frac{D}{q}$ between the distance and the base and of the angle which the radio-acoustic lines form between them, it is evident that, for a known value of the error t made in the measurement of the times, an accurate result will depend entirely on the shape of the quadrilateral *P* 123, in the sense that, for two similar quadrilaterals (fig. 23) ε_p has the same value.

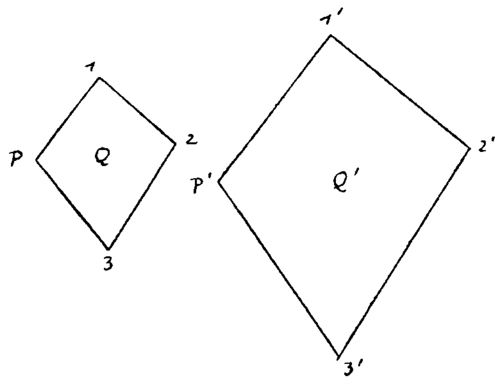

Fig. 23

In other words, in that which concerns the accuracy of a radio-acoustic position, the fact that *P* is more or less distant from points on which rests the determination is of no account, the error depends solely on the shape and not on the dimensions of the quadrilateral *P* 123.

(It should be remembered here that this conclusion has been reached by neglecting the effects of a possible current. It is therefore valid in the case of no current. But if the existence of a current is admitted, it can easily be seen that the error in the position also depends on the distances of *P* to the points 1 2 3).

REMARK : In paragraph 18 of this note, attention was drawn to an interesting analogy between the Pothenot position and our radio-acoustic position.

If then the expression of mean error of the Pothenot position (*) is considered, it is found that for a given value of the mean error made in the measurement of the two angles, its value depends not only on the shape but also on the dimensions of the quadrilateral P 1 2 3. In fact, for similar quadrilaterals the error increases in proportion to the linear dimensions of this quadrilateral.

For example, in the case of Figure 23, in which the sides of the quadrilateral *Q'* are double those of the quadrilateral *Q*, the mean error for the point *P'* determined by two angles (Pothenot) is double that of point *P*.

(*) *See* Dr. W. JORDAN—Handbuch der Vermessungskunde.

LOCUS OF POSITION OBTAINED BY THE ACOUSTIC MEASUREMENT OF THE DIFFERENCES OF THE DISTANCES TO TWO KNOWN POINTS (PHONOHYPERBOLA).

25. If T_a and T_b denote the time taken by sound transmitted from P to traverse the distances $PA = D_a$ and $PB = D_b$, the measurement of the difference.

$$T_a - T_b = \tau$$

that is to say the difference which elapses between the arrival of the sound wave at A and at B, determines a locus of position of the point P. In fact, if v denotes the velocity of sound, the product

$$v\tau$$

measures the *difference of distances* $D_a - D_b = \delta$
between the point P and the extremities of the base AB : the locus of position is a hyperbola the transverse axis of which, MN, (fig. 24) is equal to

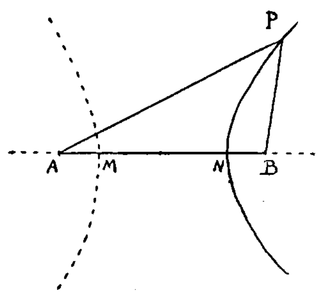

Fig. 24

the difference of the above distances and the foci of which are A and B. The sign of the difference

$$\tau = T_a - T_b$$

determines the branch of the hyperbola on which P is found. The measurement of the interval τ may be made (and it is advisable to do it) independently of the measurements of the intervals T_a and T_b; in other words, to determine τ it is not necessary to observe (or to register) at A or at B either the moment of arrival of the sound wave at the two hydrophones, or the moment of transmission of sound at P. It suffices that the moments of the observed arrival at A or B should be referred to an identical time origin, *i.e.* to a moment *arbitrarily selected* and signalled, for example, by W. T.

26. It is generally admitted that the determination of the position by acoustic measurements (in the air) by differences of distances had been con-

ceived by the French astronomer Charles NORDMANN, at the commencement of the World War. Since then, nearly all the belligerent armies have to a great extent used this method for registering hostile batteries. According to a Note presented at the first International Hydrographic Conference (London 1919) by Captain H. P. DOUGLAS (now Rear-Admiral and head of the Hydrographic Department) and by Captain R. S. H. BOULDING, entitled "Sound-Ranging" and published in the reports of the Conference (*), it seems that this method was extensively used in the British Navy in submarine phonotelemetry for registering positions of mine and torpedo explosions, of monitors, etc.; the excellent results obtained have, since that period, stimulated the use of the same methods for surveys and for navigation (fixing the positions of buoys, light-ships, etc.)

The *Nautical Magazine* of December 1919 contains an official account originating from the Press Bureau of the British Admiralty in which two groups of phonotelemetry stations are described, situated respectively near St Margaret's Bay (Dover) and near Easton Broad (Lowestoft), which are systematically used for locating the position of ships by means of the acoustic measurement of the difference of the distances.

Messrs. WOOD & BROWNE describe in the Memoir entitled : " A radio-acoustic method of locating positions at sea ", an analogous arrangement composed of a group of hydrophones situated near and to the east of the Goodwin Sands. With the exception of that which is described by Captain DOUGLAS in the above-mentioned Note, we are ignorant of the graphical and numerical process which has been used in each of them to obtain the loci and to determine the positions. The process described in the following paragraphs is very simple, even intuitive, and it seems permissible to suppose that it has already been used although, as far as we know, documents to support this assertion are lacking. This method is both very practical and, in our opinion, better suited than any other to a complete and elementary study of this locus of position. In this study, a certain number of inferences will be drawn which, perhaps, are not without importance and which form, unless we are mistaken, the original part of this second division of our work.

27. Let us suppose that the approximate position of the point to be located is known (**); in this way we know the *approximate* distances of the point to the extremities of the base AB. Selecting *one* of these distances, for example, the distance D'_a of A, we are able to determine a point P' of the hyperbola near the point desired by solving the triangle $AP'B$, the sides of which are known :

$$AB = q$$
$$AP' = D'_a$$
$$BP' = D'_b = D'_a + \delta$$

(*) International Hydrographic Conference, London 1919, Report of Proceedings, pp. 168-171

(**) The knowledge of an approximate position is also necessary in using the method described by Admiral DOUGLAS. If two differences of distance are measured to two pairs of stations, and if the details to determine the points by the intersection of two loci of position are thus known, the *approximate* point is given roughly by the intersection of the *asymptotes* to the two hyperbolas, which can be very easily determined.

where δ equals, *in size and sign*, the difference of the distances calculated from the formula :

$$\delta = v\,(T_b - T_a) = v\tau$$

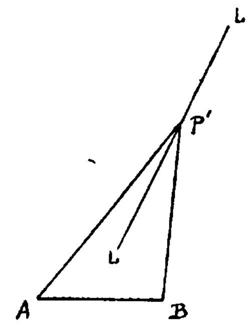

Fig. 25

We will give to the triangle $AP'B$ (fig. 25) the name of the *position triangle* (*). The point P' thus located being near the desired point P, it can be assumed that the tangent on the hyperbola at P' coincides (in the vicinity of P) with the hyperbola. It follows that this tangent may be likened to the locus of position of the unknown point P.

For the sake of brevity, we will in future give the name of *phonohyperbola* to the tangent at P' (and to the curve to which this tangent belongs), and we will call the point P' the *determinating point* of the phonohyperbola. Nothing is easier than to draw the phonohyperbola. In fact, we know by Analytical Geometry that a tangent at a known point of a hyperbola is the bisector of the angle of the radius vectors which join this point to the foci so that, if A and B are the foci of the phonohyperbola, the sides $P'A$ and $P'B$ of the triangle $P'AB$ are the radius vectors of the phonohyperbola. The interior bisector LL of the angle at P' of the position triangle is therefore the required tangent.

DISPLACEMENT OF THE PHONOHYPERBOLA DUE TO A SMALL VARIATION $\Delta\delta$ IN THE DIFFERENCES OF THE DISTANCES.

28. Let

$$\delta = PA - PB$$

be the difference of the distances of the point P to the extremities of the base AB (fig. 26). The bisector LL of the angle APB coincides with the phonohyperbola determined by the difference δ. An approximate point of the hyperbola is obtained corresponding to the difference of distance $\delta + \Delta\,\delta$

(*) This nomenclature is similar to that which has been adopted in determining the radio-acoustic line (*see* preceding note).

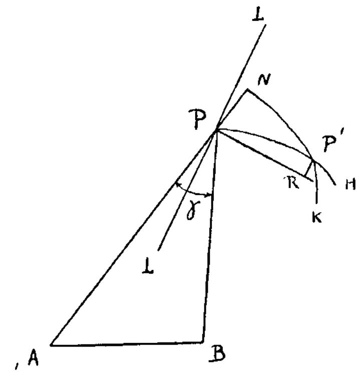

Fig. 26

by the following construction. With B as centre and radius BP describe an arc of a circle PH. Make $PN = \Delta\,\delta$ on the prolongation of the line PA, then, with A as centre and radius AN, describe the arc of a circle NK. It is evident that the point of intersection P' of the two arcs of the circles belongs to the phonohyperbola determined from the difference of distance $\delta + \Delta\,\delta$. This phonohyperbola coincides with the bisector $L'L'$ (not drawn on the figure) of the angle $AP'B$. The mixtilinear triangle PNP' may be considered, by reason of its smallness, to be rectilinear. It is right-angled at N, and we have

$$PP'N = APB = \gamma$$

where γ equals the angle which the base AB subtends at P.

Then

$$PP' = \frac{PN}{\sin\gamma} = \frac{\Delta\delta}{\sin\gamma}$$

and the projection of PP' on the normal to the bisector LL is equal to :

$$PR = PP' \cos\frac{\gamma}{2}$$

therefore

$$PR = \frac{\Delta\delta}{\sin\gamma}\cos\frac{\gamma}{2} = \frac{\Delta\,\delta}{2\sin\frac{\gamma}{2}}$$

If the convergency of the bisectors LL and $L'L'$ is neglected the segment PR measures, in the direction normal to the hyperbola, the *parallel lateral* displacement of the position caused by the small variation $\Delta\,\delta$ due to the difference of distance.

If $\Delta\,\delta$ measures the error made in the determination of δ, the true value of the error, or the displacement e of the position line (phonohyperbola) due to the error $\Delta\,\delta$, can therefore be expressed by means of the formula :

$$(*) \qquad e = \frac{\Delta\,\delta}{2\sin\frac{\gamma}{2}}$$

If, for example, the interval τ has been measured with an error t we would have :

$$\Delta \delta = tv$$

and, therefore, if v equals the approximate value of 1500 metres per second we would have :

$$e = 750 \frac{t}{\sin \frac{\gamma}{2}}, \text{ metres}$$

If $t = 0.01$ sec.

$$e = \frac{7.5}{\sin \frac{\gamma}{2}}, \text{ metres.}$$

With a known base AB the important and simple formula (*) denotes that the locus of the points of the plane, for a known error $\Delta \delta$, corresponding to a displacement e of the position line (phonohyperbola), is the arc of circle drawn through A and B, and subtended by the angle γ. In other words, for all points of the plane at which the base AB subtends a certain angle, there is a definite parallel lateral displacement of the phonohyperbola for a known value of $\Delta \delta$. This relation denotes that the displacement e is smaller—other things being equal—in proportion as the angle γ approaches 180°, and the displacement increases as this angle diminishes.

INFLUENCE OF CURRENT ON THE ACOUSTIC MEASUREMENT OF THE DIFFERENCE OF THE DISTANCES.

29. Let the current be resolved into two parts perpendicular to each other *i.e.* one component *perpendicular to the base AB* and the other *parallel to the base* and let the effect of each be considered separately.

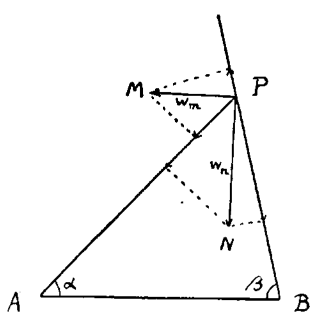

Fig. 27

Firstly let us consider the component

$$W_n = PN \quad \text{(Figure 27)}$$

perpendicular to the base. The projections of W_n on the directions PA and PB represent the increments in the velocity of sound along these lines, due to the presence of W_n.

If α and β are the angles at the base of the position triangle, it is apparent that along the line PA, the projection of W_n is

$$W_n \sin \alpha$$

and along the line PB

$$W_n \sin \beta$$

If T_a and T_b equal the time taken by the sound to traverse the distances $PA = D_a$ and $PB = D_b$ (times which are actually observed) the exact value δ of the difference of the distances $D_a - D_b$ is :

$$\delta = D_a - D_b = T_a \ (v + W_n \sin \alpha) \ - \ T_b \ (v + W_n \sin \beta)$$

The value δ' of the same difference of distances, obtained by neglecting the effects of the current, is :

$$\delta' = v \tau$$

where

$$\tau = T_a - T_b$$

therefore

$$\delta' = T_a v - T_b v$$

and

$$\delta' - \delta = T_b W_n \sin \beta - T_a W_n \sin \alpha$$

In this formula the approximate value $\frac{D_a}{v}$ and $\frac{D_b}{v}$ may be assumed for T_a and T_b and then

$$\delta' - \delta = \frac{W_n}{v} \ (D_a \sin \beta - D_b \sin \alpha)$$

But

$$D_a \sin \beta = D_b \sin \alpha$$

and therefore

$$\delta' - \delta = \text{zero.}$$

It may be assumed, therefore, that the perpendicular component to the base *has no* influence on the acoustic determination of the difference of the distances.

Now let the component $W_m = PM$, *parallel to the base,* be considered. The projections on the lines PA and PB are respectively :

$$W_m \cos \alpha \quad \text{and} \quad - W_m \cos \beta$$

Therefore the velocity of sound along PA will be :

$$v + W_m \cos \alpha$$

and along PB

$$v - W_m \cos \beta$$

Therefore, by giving to δ and δ' the signification attached to them :

$$\delta' - \delta = - \frac{W_m}{v} (D_a \cos \alpha + D_b \cos \beta)$$

But

$$D_a \cos \alpha + D_b \cos \beta = \text{base} = q$$

Therefore

$$\delta' - \delta = - \frac{W_m}{v} q$$

This relation gives the error which is made in the acoustic determination of the difference of the distances, calculated without taking into account the component W_m of the current.

If v is given the approximate value of 1500 metres per second, and if W_m is valued in metres per second, and q in kilometres, then

$$(**)\ \delta' - \delta = \frac{2}{3} W_m q^{\text{kms}}, \text{ metres (absolute value).}$$

For example, when $W_m = 1$ metre per second and when $q = 15$ kilometres, we have (as absolute value)

$$\delta' - \delta = 10 \text{ metres}$$

In the case illustrated in the figure, in which the component W_m acts in the direction from B towards A, the correction which must be applied to the calculated value $\delta' = v(T_a - T_b)$ (where the interval $T_a - T_b = \tau$ must be considered both as to magnitude and as to sign, consequently δ' may be positive or negative) to obtain the correct distance $\delta = D_a - D_b$, is positive.

If the component W_m acts in the direction from A to B, the correction to be applied to the same value δ' (still considered as to magnitude and sign), is negative.

CONCLUSIONS : When a current exists, the value of the difference of distance, calculated without taking the current into account (*i. e.* assuming slack water) will be affected by an error which is *independent* of the component perpendicular to the base but which depends only on the *component parallel* to the base and on the length of the base, and it is proportional to each of these two quantities.

In practice, owing to the difficulty of measuring the current with accuracy, it is best to operate when the water is motionless, *i.e.* at slack water. However, as the existence of a slight current must not be excluded, we are able, by means of the relation previously ascertained and also by the formula in the preceding paragraph and the formula (**) of the present paragraph, to calculate the error or the lateral displacement of the position line (phonohyperbola) due to this cause. For example, if the existence of a current parallel to the base of velocity $W_m = 0.25$ metres per second ($\frac{1}{2}$ knot) is admitted, there would be a displacement equal to

$$\frac{0.25}{23} \frac{q^{\text{kms}}}{\sin \frac{\gamma}{2}} = 0.0833 \frac{q^{\text{kms}}}{\sin \frac{\gamma}{2}}, \text{ metres.}$$

If $q = 15^{kms}$ and $\gamma = 15°$, then the displacement is 9.57 metres.

30. We have learned to determine the displacements due to an error in the measure of time and in the estimation of the current and, therefore, by giving an average value to these two errors, we are by way of estimating the total average displacement of the position line (phonohyperbola) due to their co-existence and, consequently, to estimate the *quality* of the locus of position determined by an acoustic measure of the difference of the distances.

There remains to be considered the displacement which is due to an error Δv on the value v of the velocity of sound, which is used to determine the difference of distance, δ.

The error made in the difference of distance δ will be :

$$\Delta \delta = \tau \Delta v$$

which is nil when τ = zero, that is to say when P is equidistant from the extremities of the base AB. It would then be necessary to try to operate in these conditions which, as a matter of fact, are quite exceptional.

When τ does not equal zero, *i.e.* when P is not equidistant from the extremities of the base, the position line (phonohyperbola) will be affected by an error of a systematic character, the value of which is :

$$\frac{\tau \Delta v}{\sin \frac{\gamma}{2}}$$

and which it will be generally very difficult to estimate because (independent of the existence of an unknown current) so many causes may intervene to modify the value of v to an appreciable extent.

Everyone knows that it is nc easy matter to measure exactly the temperature and salinity of water, and it is consequently easy to understand that, in the determination of v, errors can be made which are not compatible with the accuracy required to determine positions intended for hydrographic purposes.

We may therefore conclude that, in order to obtain a good phonohyperbola in which confidence may be placed, it is necessary to select bases in such a way that their extremities are perceptibly equidistant from the point to be determined.

The greater the difference of distance δ, the less will be the degree of confidence that may be attributed to the phonohyperbola which is thus determined, and it may be affirmed that, in consequence of an error in the velocity of sound, there is a systematic error in δ which it is almost impossible to estimate and the effects of which may be considerable.

DETERMINATION OF THE POSITION BY TWO DIFFERENCES OF DISTANCE TAKEN IN CONNECTION WITH THREE GIVEN POINTS (Point common to two phonohyperbolas having a common focus).

31. Given three points 1, 2, 3, let us suppose that it is necessary to determine the position of P by the intersection of the two hyperbolas, corresponding to the differences of the distances:

$$D_2 - D_1, \quad D_3 - D_1.$$

Let T_1, T_2, T_3 be the times taken by the sound to traverse the respective distances D_1, D_2, D_3, which are unknown, but of which we know the differences:
If

$$\tau_{21} = T_2 - T_1$$
$$\tau_{31} = T_3 - T_1$$

then

$$D_2 - D_1 = \tau_{21}\, v$$

and

$$D_3 - D_1 = \tau_{31}\, v$$

Let us suppose that the approximate value of *one* of these three distances is known—for example, the value

$$D'_1 \text{ of } D_1$$

and let

and

$$(***) \quad \begin{cases} D'_2 = D'_1 + \tau_{21}\, v \\ D'_3 = D'_1 + \tau_{31}\, v \end{cases}$$

(In this sum, the terms $\tau_{21}v$ and $\tau_{31}v$ ought naturally to be considered in magnitude and sign; and they should be added algebraically to D'_1).

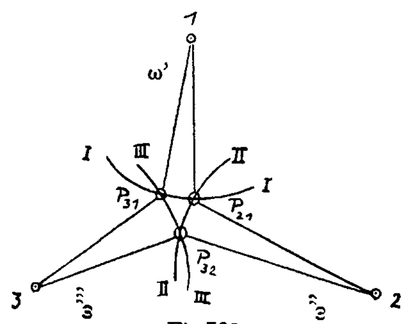

Fig. 28

From 1 as centre and radius D'_1 describe the circle I; with 2 as centre and radius D'_2 describe the circle II; and with 3 as centre and radius D'_3 describe the circle III.

The points

P_{21} (intersection of I and II)
P_{31} (intersection of III and I)
P_{32} (intersection of III and II)

are the determinating points of the three phonohyperbolas determined respectively by the intervals:

$$\tau_{21} = T_2 - T_1$$
$$\tau_{31} = T_3 - T_1$$
$$\tau_{32} = \tau_{12} - \tau_{31} = T_3 - T_2.$$

They are obtained by solving:

(1) the triangle the base of which is 2 1 and the sides D'_1 and D'_2;
(2) the triangle the base of which is 3 1 and the sides D'_3 and D'_1;
(3) the triangle the base of which is 3 2 and the sides D'_3 and D'_2;

In other words, P_{21} is a point of the phonohyperbola τ_{21}; P_{31} is a point of the phonohyperbola τ_{31}; and P_{32} of the phonohyperbola τ_{32}; and each of the three points is an *approximate* position of the point which it is desired to determine. These points form a small curvilinear triangle which can be compared to a rectilinear triangle on account of its dimensions.

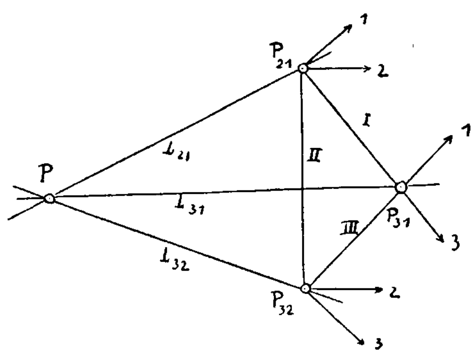

Fig. 29

Let this triangle which (by usual analogy with the radio-acoustic problem previously demonstrated) we will call the *determinating triangle*. be reproduced on a large scale. Fig. 29. The sides I, II, III, represent respectively *the loci of equal distance* D'_1 from 1, D'_2 from 2, D'_3 from 3. Through each of the apices let arrows be drawn, P_{21} 1, P_{21} 2, P_{31} 3, P_{31} 1, etc., respectively perpendicular to the sides which meet at this point and *in the direction* of the points 1, 2, 3.

The bisectors L_{21}, L_{31}, L_{32} of the angles (internal or external) of the triangle, which also bisect the angles formed by each of the three pairs of arrows drawn through each of the apices, coincide with the three phonohyperbolas determined respectively by the intervals τ_{21}, τ_{31}, τ_{32}.

(It is true that in proceeding thus we neglect the convergencies ω', ω'', ω'''. But as the sides of the determinating triangle are so small relative to the distances D'_1, D'_2, D'_3, they are in fact negligible). Moreover, these bisectors are also the bisectors of the angles (internal or external) of the determinating triangle ; consequently, they converge, according to an elementary geometrical theorem, at the same point P, which is the point required.

It is interesting to note the analogy between this construction and that described in paragraph 19 for the determination of the radio-acoustic position. The two constructions lead to calculations of the same type and are resolved by the solution of three position triangles for each of which the three sides are known.

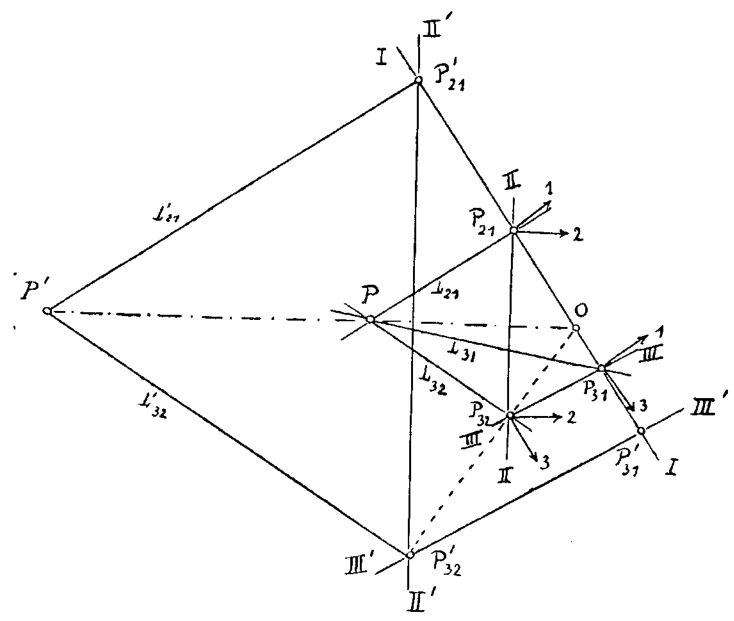

Fig. 30

In figure 30, let P_{21}, P_{31}, P_{32}, be the determinating points calculated by the former method.

Let us assume that the differences of distance

$$D_2 - D_1 = \tau_{21}\, v$$

and

$$D_3 - D_1 = \tau_{31}\, v$$

have been determined by taking an erroneous value $v + \Delta v$, in which Δv denotes the error for v.

In this case the distances

$$D'_2 \text{ and } D'_3$$

calculated by the formulae (***) of the preceding paragraph and used for determining those three determinating points, differ respectively from the quantities derived from the actual velocity of sound, [we will distinguish them by the symbols (D'_2) and (D'_3)] from the quantities :

$$\tau_{21} \Delta v, \qquad \tau_{31} \Delta v$$

Let us suppose that we know Δv, *i. e.* we assume we know (D'_2) and (D'_3). The three determinating points which result from the three combinations of pairs of the three distances D'_1, (D'_2), (D'_3) in place of the distances D'_1, D'_2 D'_3 actually used in the calculation, are obtained from a very simple graphical operation. In fact the position of side I of the triangle P_{21}, P_{31}, P_{32} can be left unchanged, and the two other sides II and III of this triangle moved parallel to themselves in the following manner :

The side II, a distance s'' equal to

$$\tau_{21} \Delta v$$

towards the point 2 (*i.e. in the direction of the arrow* 2), if $\tau_{21} \Delta v$ is a *negative quantity* and *in an opposite direction if positive.*

Similarly the side III, a distance s''' equal to $\tau_{31} \Delta v$, towards the point 3 if $\tau_{31} \Delta v$ is positive and in an opposite direction if negative. (In the figure the error $\tau_{21} \Delta v$ is assumed to be negative and $\tau_{31} \Delta v$ positive.) Hence it is evident that the displaced lines II' and III' coincide with the loci of equal distance (D'_2) and (D'_3) and therefore form with the side I corresponding to the distance D'_1, which remains unchanged, the new and true determinating triangle. The new apices :

$$P'_{21}, \ P'_{31}, \ P'_{32}$$

are in other words the determinating points corresponding to the true value of the velocity of sound.

The two triangles

$$P_{21} \ P_{31} \ P_{32} \text{ and}$$

$$P'_{21} \ P'_{31} \ P'_{32}$$

are homothetic and their centre of homothesy falls at O, the meeting point of the homologous vectors P'_{32}, P_{32} with the side I (common to the two triangles.)

Let the bisectors (phonohyperbolas) L'_{21}, L'_{31}, L'_{32} be drawn through the apex of the new determinating triangle, and let P' be the point of their common intersection.

P' will be the true position desired.

The two quadrilaterals :

$$P \ \ P_{21} \ \ P_{31} \ \ P_{32} \text{ and}$$

$$P' \ \ P'_{21} \ \ P'_{31} \ \ P'_{32}$$

are homothetic and their centre of homothesy coincides with O, and consequently it is common with that of the two determinating triangles. We assume therefore that the true point P' falls on the holomogous vector PO. In other words the line PO which joins the erroneous point P and the centre of homothesy O of the two determinating triangles *is a locus of position of the true point* P'.

It will be noted (and it is a practical inference from the preceding demonstration) that the centre of homothesy O, which has been determined by constructing the triangle P'_{21} P'_{31} P'_{32} on the hypothesis that the displacements $\tau_{21} \Delta v$ and $\tau_{31} \Delta v$ are known, can be determined even if these are unknown, as is actually the case. In fact it is easy to see that if the new determinating triangle P'_{21} P'_{31} P'_{32} is constructed by giving to the sides II and III displacements s'' and s''' *proportional*, instead of equal, to τ_{21} Δv and $\tau_{31} \Delta v$, *i.e.*, proportional to the known values

$$\tau_{21} \text{ and } \tau_{31}$$

the triangle

$$P'_{21} \quad P'_{31} \quad P'_{32}$$

is obtained, homethetic to the triangle

$$P_{21} \quad P_{31} \quad P_{32}$$

and that the centre of homothesy coincides with that of the two triangles considered first, *i.e.* with the point O.

Concerning the directions of these proportional displacements, it can be easily seen that they should comply with the following rule :

They should be in the direction of the arrows (which indicate the direction of the points 2 or 3) when the interval considered (τ_{21} for the line II ; τ_{31} for the line III) is positive, and in the opposite direction when the interval is negative (*). We thus know all the details necessary to draw the locus of position PO *which has the very important property of being exempt from the systematic error by which the value of the velocity of sound may be affected.* These results do not seem to be without interest and after further study, could perhaps be used for important applications in practice.

Furthermore, the possibility of obtaining *a position exempt from systematic error in the velocity of sound* from the acoustic measure of an appropriate number of differences of distances (independent of each other) has been demonstrated.

In conclusion we will remark only that, before extending the application of these new loci of position, it will be necessary above all to discuss the errors by which they may be affected in consequence of the displacement of the phonohyperbolas due to accidental errors in measurement. In other words, it will be necessary to fix a criterion by which the *favourable cases* can be distinguished from the *unfavourable*, as has been done for the radio-acoustic line and position.

(*) The contrary rule could also be used, but it is of course necessary that the same rule be applied to the two groups of displacements.

32. It appears to us that the determination of positions by the *acoustic measure of the differences of distances* ought to be limited to cases for which it is impossible to *make a radio-acoustic measure of distances*; and consequently that the method of the difference of distances (phonohyperbolas) has found and can find its most logical application in the science of war rather than in hydrographic operations in which the radio-acoustic measure of distances will always be possible. As an example—a very advantageous use may be made of phonohyperbolas in ballistic experiments which have some relation to the science of war (*i.e.* to determine the point of impact of a projectile in the sea). In a maritime zone in the vicinity of the coast, along which have been placed a suitable number of hydrophones in favourable positions, if it is assumed that the projectile is fitted with a very sensitive fuse which causes an explosion at the moment the projectile falls in the water, the acoustic measure of the differences of distances which separate the point of impact from various pairs of hydrophones can be made. Moreover, taking into account the great range of modern guns, this is the most practical method of determining the range, and consequently it may be presumed that this method has already been largely used.

We state at once that this argument has nothing to do with hydrography and is not within our competence, and that we only mention it for the *sole object* of rendering a just homage to the pioneers of acoustic submarine methods.

The "*Vie Maritime et Fluviale*" of 10th February 1909 describes, in its broad outlines, an installation for ballistic experiments proposed by Mr. P. Rediadis of the Greek Navy, and based on the following principle.

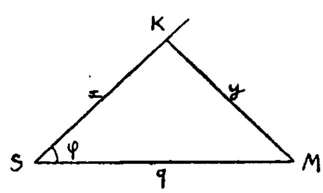

Fig. 31

Let us suppose a gun at S (Figure 31) situated near the sea and that the projectile falls at K, and that two hydrophones are in position, one in the immediate vicinity of S and the other at M. Let us suppose that the difference

$$T_x - T_y$$

of the times which the sound takes to traverse the distances $KS = x$, and $KM = y$ is measured. Let us suppose that the angle φ which is formed by the line SK (which coincides to a very appreciable extent with the direction of the line of fire) and the line SM, has been measured. By very simple considerations which it is needless to reproduce here, the formula

$$x = \frac{1}{2} \frac{q^2 - v^2 (T_x - T_y)^2}{q \cos \varphi - v (T_x - T_y)}$$

is obtained, in which $q = SM$ and v is the velocity of the sound in water.

As may be seen, the method reduces itself to fixing a position by two lines of direction which are: (*a*) the line of direction SK defined by the direction of the line of fire; (*b*) the phonohyperbola determined by the acoustic measure of the difference of distance $x - y$.

Consequently the method suggested by Mr. REDIADIS, and put forward in 1909 contained the germ of the principle of the determination of a position by the acoustic measure of the difference of distances.

ACOUSTIC METHODS IN NAVIGATING.

33. The importance of acoustic methods for navigating, and especially for fixing the position of the ship during fog, is evident. But it seems to us none the less evident that their use will become really practical only when they have been considerably simplified, by limiting them to special cases for which the observations and calculations are simple and rapid (or better still *requiring no calculation* but only an instrumental reading or observations of coincidence). The methods which we have described find in fact their logical application, taken as a whole, to hydrographic surveys for which a necessary and fundamental condition is certainly not the greatest economy of time and personnel but a search for the greatest possible accuracy in the results. We fear that the attempts which have been made up till now to use these acoustic methods for fixing positions for hydrography (or for purposes of war) and which reduce themselves, in principle, to simplifications of the methods of calculation (which do not seem to be always quite justified) to the substitution of graphical construction in place of numerical calculations, *etc.* in others words, to economise on figures and a few minutes in the duration of the calculation, are not likely to meet with success. On the other hand, the two following methods seem more practical — of which the first at least has already received the sanction of sufficiently long experience.

A) RADIO-ACOUSTIC METHOD.

It is described for the first time (subject to contradiction) by J. JOLY in 1918 (Proceedings of the Royal Society, N° A664, August, 1, 1918, page 547, *A method of avoiding collisions at sea*). Professor JOLY writes as follows:

" If signals be simultaneously emitted by wireless and by submarine bell " (or Fessenden oscillator), the former being transmitted with practically infinite " velocity, the latter arrive with a lag which is the time the submarine " sound requires to traverse the intervening medium. The rate of propagation " of sound in water being closely 4800 feet per second, the lag is 0.62 " second for one-half sea-mile.

" In practice the signals may be so ordered as to dispense with the stop-

" watch or chronograph. This is accomplished by sending out the wireless " ticks in groups of, say, 20 " dots " spaced to intervals of 0.6 second. The " stroke of the bell precedes the first of these dots by one of these intervals. " Thus, when the sailor is half a mile from the source he hears the first wire- " less dot along with the bell stroke. If he is 1 mile distant the bell stroke " comes in with the second dot, and so on. He has, in fact, only to count " up the dots till he hears the bell, and the number of the dot coincident " with the bell is the number of half sea-miles intervening between his ship " and the source of the signals. It is possible to estimate the quarter mile " by noting a want of coincidence between bell stroke and dot. This method " of estimating distance is in actual operation in assisting mariners to navigate " the approach to New York Harbour, the signals being emitted from the " Fire Island Light Ship. It is of special value in coastal navigation ".

This method is actually applied on board some important German, Danish and Swedish lightships, in the North Sea and the Baltic. The types of signals differ from those described by Mr. JOLY, but the principle is exactly the same. The lightship transmits radio-telegraphic signals in groups of 16 dashes, each of one second duration. The dashes are separated by an interval of 0.253^s. A submarine sound signal precedes the first dash of each group of 1.253^s. As sound takes an average of 1.253^s to cover one mile, it suffices to ascertain by appropriate means which one of the 16 dashes coincides with the arrival of the sound on board the ship. Thus the distance in miles separating the ship from the light vessel is known within one mile. By this method the navigator determines the radio-acoustic position line, or the circle of equal distance to the light vessel.

B) METHOD BY THE DIFFERENCE OF DISTANCE EQUAL TO ZERO.

This method was invented and probably is already used in England; it enables the ship to take up a position on a fixed line of bearing for entering or passing through a channel. *Although this system is applied in the air, it can also be applied in water for taking up a position at sea on a given line of bearing.* We cite textually a paragraph from the "Electrician" of 10th June, 1917, p. 653, which gives a summary description of the method.

" The system, which is of British invention, is designed to enable vessels " to negotiate narrow channels during foggy weather and thus minimise risk " and loss in time when light vessels are not visible. The underlying principle " of the apparatus is based on the fact that if two sounds are emitted simul- " taneously from two separate points an observer, equidistant from each, will " hear both signals at the same time, whereas, if he hears one before the " other he will know that he has deviated from the centre of the channel. " It is convenient to arrange for one signal to give a single blast and the " other two shorter blasts, with a short interval of time between them. The " centre of the channel would then be indicated when the longer blast was " heard between the two short blasts, thus forming practically one long blast. " The sound apparatus consists of diaphones worked by electrically-driven

" compressors, while the synchronising of the two signals is done by a master " timing device set in action by means of a single push button or switch, " which automatically makes the two signals sound simultaneously for a " straight course, or with a predetermined interval between them in the case " of a curved course. The apparatus for timing the signals functions by " means of electro-magnets, which are adjusted during erection to give and " maintain exact synchronisation ".

The deviation of a vessel from the line of bearing, due to an error in estimating the perfect coincidence of the sound signals may be ascertained by means of formula (*) in paragraph 28.

$$e = \frac{\Delta \delta}{2 \sin \frac{\gamma}{2}}$$

In the case in point

$$\Delta \delta = vt$$

in which $v =$ the velocity of sound (1500 m. per sec. approx.) and $t =$ the error of coincidence ; moreover, the line of bearing being perpendicular to the base,

$$\sin \frac{\gamma}{2} = \frac{1}{2} \frac{q}{D}$$

in which q is the base which separates the two sound signal transmitting stations, and D is the distance of the ship from these stations.

Consequently

$$e = \frac{D}{q} vt$$

If $t = 0.01^s$

$$e = 15 \frac{D}{q} \text{ metres.}$$

MEASURING INSTRUMENTS FOR THE ACOUSTIC DETERMINATION OF POSITIONS.

34. There is a complete description of a series of instruments for radio-acoustic determination in Publication N° 107 of the Coast and Geodetic Survey of the UNITED STATES OF AMERICA which we have mentioned in paragraph 2 of this treatise (*).

From an allusion recently made in an American article it seems that these appliances might be improved, and that experiments are at present

(*) *See* – *Hydrographic Review*", Vol. IV, N° 2, Dec. 1927.

being made with this end in view. Some important particulars concerning the method of operating and the instruments necessary for radio-acoustic determinations are also found in the note by Wood & Browne cited in the same paragraph (Proceedings of the Royal Society 1923). Einthoven's well-known photographic recording galvanometer, which constitutes one of the most perfect oscillographic models in existence at the present time, is used for registering radio-acoustic and radio-telegraphic impulses. Allusion is made to this instrument in the article by Admiral Douglas, published in the reports of the Hydrographic Conference in London, and which has been cited in paragraph 26. Tinsley's Phonic Chronometer has also been used in England for measuring intervals of time. The phonic motor, which is the essential organ of this instrument, merits attention and for that reason we notify it to Hydrographic Services.

Hence the principal instruments necessary for acoustic and radio-acoustic measurements exist, and are within the reach of everyone, since they are found in commerce. The thing is to bring them together in the most suitable manner and we believe that it is not necessary to make long and difficult experiments to accomplish it.

Before concluding these brief considerations, it is well to draw attention to a recent model of a portable photographic recording oscillograph, invented by Mr. Legg, engineer to the Research Laboratories of the Westinghouse C°. of Pittsburg and which is sold commercially by that firm under the name of *Osiso*. This oscillograph is based on the same principle as Einthoven's galvanometer; being, however, extremely accurate, it is an instrument which lends itself to field operations and its use does not necessitate previous lengthy preparations. On the other hand, Einthoven's galvanometer is, as is well known, a true laboratory instrument and is not easily transported. Another advantage of the *Osiso*, which is not negligible, is that its price is comparatively small, and for that matter we consider that it possesses all the requisite qualities for our measurements.

35. There is nothing new to be said on the subject of hydrophones. All navies have used them for some years, and great variety of types is to be found in commerce. Perhaps different navies keep particulars of hydrophonic material secret, but it may be assumed that the secret is apparent or, to express it better, that it concerns improvements which are not divulged owing to a natural habit of discretion.

Concerning the quantity of explosive which must be used for these determinations of acoustic position, we should say that, according to Wood & Browne (see Note cited), a gun-cotton or nitrotoluene cartridge of 9 ounces (270 grammes) is sufficient to effect the determination at a distance of 40 miles. The range of an explosive in water appears to be very great, but it naturally depends on the quantity of explosive used. It has been ascertained that explosions produced at a distance of 400 sea miles could be recorded through very deep water.

36. In concluding this contribution to the study of submarine phonote-

lemetry, we would once more draw the attention of Hydrographers to the very great importance of this subject. We do not believe it an exaggeration to say that the next and most necessary improvement of hydrographic charts will, in great part, depend on improvements in the application of phonotelemetry. Many too many dangers and aids far from the coast and which are of great importance to navigation, are in positions which are at present badly determined or determined by methods of insufficient accuracy. The result is the great number of doubtful positions (*P. D.*) which still exist and which constitute a veritable task for hydrography. The present and the near future of Hydrography are allied in great part to acoustic methods, *i.e.* to acoustic sounding and to phonotelemetry. Acoustic sounding, which represents a real revolution in hydrographic methods, and which constitutes the greatest and most valuable conquest of Hydrography, is already in daily use. It is through it that the true knowledge of the real form of the suboceanic terrestrial relief, so little known at the present day (although this seems paradoxical in this century of progress) will be gained. We are confident that a like brilliant future awaits phonotelemetry. It appears of good omen that the recognition of its great importance is implied by two facts which have a special significance for the I. H. B. and which might well be placed in evidence :

(1) The first scientific memorandum, published in the first document of our International Institution, treats precisely of Phonotelemetry. I allude to the interesting Note (already cited in the course of this treatise) of the British Hydrographer Admiral DOUGLAS (Proc. Int. Hydr. Conference, London 1919, Sound Ranging, page 168 *et seq.*)

(2) The regretted Admiral PARRY, on assuming the presidency at the first sitting of the Conference in London, made use of the following words :

" Of all the many inventions which have been brought into use as practical possibilities during the War, I, personally, consider that the use of directional wireless telegraphy and of hydrophones for position-finding at sea are the most important to us in the simplification and acceleration of hydrographic work, and the eventual universal adoption of these inventions to assist in carrying out hydrographic work is, I think, a certainty. We are already extensively using hydrophones for sound-ranging under water to assist in the exact determination of positions in the North Sea, required in relation to mine-fields and in the replacement of navigational aids, and the possibilities in this connection are enormous ; the accuracy resulting from the use of this method is even now marvellous, yet the invention is only in its infancy".

We believe that to-day the period of infancy is entirely ended because we are sure that many studies and experiments have been carried out by the various National Hydrographic Services. Apart from some exceptions, however, this activity has perhaps lacked the necessary cooperation to yield the most practical results. Work has been carried out in watertight compartments, perhaps due to the fact that acoustic methods can be largely applied (and have been applied in the not very distant past) to the conduct of war. This phenomenon is found to be true in part, and in origin also, in matters concerning the study and use of acoustic sounding, which in our days form part of the common hydrographic possessions.

Those parts which form the basis and the common substance of all phonotelemetrical experiments cannot remain unknown and, in fact, are already partially known to those who carry out patient researches (hydrophone, microphone, stethoscope, chronograph, oscillograph, *etc.*). They form the dispersed elements of a body already constituted but which still remains in obscurity without any well-founded reason.

One of the first duties which the I. H. B. undertakes to accomplish is to collect these particulars and to divulge them.

With this object in view, it invokes that cooperation which was launched under the auspices of the British hydrographic surveyors and which was so generously begun by them during the First Hydrographic Conference. (*)

(*) *See* Proc. Int. Hydr. Conf., 3rd Plenary Session, page 53 :

12. Captain DOUGLAS (GREAT BRITAIN) : referred to sub-aqueous sound-ranging, and suggested that he should write a short account of the methods employed, or else make an oral statement to the Delegates on this subject.

13. The PRESIDENT said he had little doubt that the Delegates would be interested in a paper describing sound-ranging and the use of hydrophones. He then described the advantages which this system of obtaining position presents, and stated that all the surveying vessels of the British Navy would be provided, in the future, with two sets of sound-ranging instruments. He was ready to show any of the Delegates who desired to see it one of the stations which is fitted with the necessary instruments and was prepared to arrange for a visit to Dover for this purpose.

14. The VICE-PRESIDENT thanked the PRESIDENT for permitting the paper proposed by Captain DOUGLAS to be circulated, and he then called attention to a system of taking soundings by sound which is under trial in France. He described the system and the instruments employed and stated that, in a ship moving at 10 knots, soundings up to 60 metres have been taken with a degree of accuracy of about 3 per cent. A short discussion was then held as to the advantages and disadvantages of this method of sounding

,.......The PRESIDENT, however, stated that the Conference must hasten its work, and regretted that he did not think there would be time for discussion of these subjects. He suggested, however, that Delegates who desired any special information might add requests for this to the proposals which each Delegation was sending in with reference to Section X (Establishment of an International Hydrographic Bureau).