INSTRUMENTS FOR OBSERVING EQUAL-ALTITUDES IN ASTRONOMY

(Gauss's method generalised)

by

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INTRODUCTION.

GAUSS'S equal-altitude method, which enables the latitude and longitude of a place to be determined simultaneously and by observations of the same nature, was hardly used during the XIX century. But during the past fifty odd years astronomers and geodesists, anxious to render measurements of geographical positions easier and, incidentally, more accurate, have designed instruments more appropriate than the sextant for use with this method.

The search for increasing accuracy raises complex problems. Instruments of various types correspond to the different solutions found for them and, after a period of more or less rapid evolution, they have acquired a more or less final form. Nevertheless an important improvement still remains to be made, namely the elimination of the personal error of the observer by some arrangement equivalent to the moving-wire micrometer in meridian circles.

All these attempts, made with a view to utilizing GAUSS'S method to the best advantage, are of great interest, and we think it will be profitable to make a comparative study here of the various instruments based on its use.

After a summary of the devices used at the end of the last century we shall deal at greater length with those now in use, dwelling not so much on detailed descriptions as on an examination of their merits and demerits and of the accuracy of the results obtainable from each.

Having had occasion to make practical use of various equal altitude instruments (principally the prismatic astrolabe) for many years, and having always been interested in the application and development of the method, we have felt constrained to make a few personal observations which we have inserted in the various sections of this memorandum.

The latter comprises four parts.

In the first, a short one, we recall the principle of the method and describe the *almucantar* and *nadirinstrument*, which never came into general use among geodesists.

The second part is devoted to contemporary instruments, the chief of which is incontestably the *equilateral-prism astrolabe ;* during thirty years of practical use, it has undergone modifications of detail which have greatly contributed to extending its adoption.

The *circumzenithal,* designed at the same period, has been much modified by its inventors, and has only been in common use for some ten years.

In the same part, also, we examine the *pentagonal-prism astrolabe* and the *bent telescope ;* the latter, having only been described in short articles, will be dealt with in some detail.

Although the existence of the personal equation has been known for a long time in observations of equal altitudes, instruments enabling it to be correctly measured are of quite recent construction and are capable of certain improvements.

Various *impersonal* instruments are being designed, and will make it possible in the near future to determine the time by this method, without appreciable systematic error.

A fourth part, of a practical nature, is reserved for the computations of a preparative and reductional nature necessitated by instruments based on GAUSS's method. In particular we deal there with the use of the tables and diagrams published in large numbers for use with the instruments in being, and more especially with the prism astrolabe. With a view to reducing computation to a minimum we shall show how to make it according to the degree of accuracy desired by the observers and to the conditions under which they are working.

In conclusion we give an appendix containing as complete a bibliography as possible of the principal books and articles which we have consulted in connection with the present memorandum.

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PART I.

A. *EQUAL-ALTITUDE METHOD — GENERAL.*

(1) Conceived by GAUSS in 1808 as a remedy for the imperfections of the sextant, the method of equal altitudes consisted originally in noting the times at which any three stars reached the same altitude, this altitude not having to be known exactly.

At the times of transit it is sufficient to combine, as data for the problem, the positions of the stars, the rate of the chronometer and, if necessary, the readings of the barometer and thermometer, to obtain, by direct trigonometrical calculation, the latitude of the place, the chronometer correction and the true altitude of observation.

Thanks to this procedure the sextant enters only as an instrument of comparison and not of absolute measurement; consequently the results are affected by no systematic errors; they are only affected by possible errors in the elements used as bases.

On several occasions GAUSS applied his method to the determination of the latitude of Göttingen; during the observations made in the course of a single evening the alidade of the sextant remained fixed on the limb. The results he obtained are worth recalling. With observations of α Andromedae, α Ursae Minoris and α Lyrae, on the 25th, 27th and 28th August, 1808, he obtained the following values for the seconds of the latitude of Göttingen : 56.7, 51.5, and 54.4. These isolated values of latitude were in excellent agreement for that period, proving the great value of the method used.

In the *Additions à la Connaissance des Temps pour* 1812 DELAMBRE gave a detailed analysis of GAUSS's method, but he confined himself almost entirely to criticism and simplification of the formulae set up by his illustrious contemporary, without, it appears, realising the importance of this new method in positional astronomy.

After GAUSS, KNORRE at Nikolaev in 1832 and ANGER at Danzig in 1835 expanded and completed the method of equal altitudes by substituting for the direct solution, given by three stars, methods of indirect solution which have the advantage of being applicable to any number of stars. This resulted in increasing the accuracy of the method.

In 1890 Admiral PERRIN, by an ingenious extension of the method of the *estimated position*, used in the Navy and in surveying, showed that the use of a graph in the reduction could appreciably simplify the solution of the problem.

(2) In spite of the efforts of its warmest partisans, the equal-altitude method was not used for determining geographical positions requiring the highest degree of precision. This was largely due to the difficulties associated with stellar observations when using the sextant and artificial horizon. It is, in fact, awkward to keep the two images of a star in the field and to make them pass near enough to one another to appreciate the instant at which they are at the same level. Further, the magnification used is low, $\times$ 10 at the most, for it is necessary for convenience of observation to have a telescope with a fairly large field.

In spite of improvements in the sextant, it became quite evident that this instrument was not entirely suitable for application to GAUSS's method. Nor was the theodolite any more so, for the reference to the vertical, being obtained by means of a spirit level, is consequently not capable of great accuracy.

This is why certain experts have tried to design instruments exclusively for application to the general method of equal altitudes.

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B. *EARLY EQUAL-ALTITUDE INSTRUMENTS.*

ALMUCANTAR BY S. C. CHANDLER Jr. (1883).

(3) The American astronomer S. C. CHANDLER appears to have been the first in point of time to make a serious attempt to design an instrument specially for observing equal altitudes.

He first made the *Chronodeik*, for observing corresponding altitudes of the sun ; this instrument had a vertical telescope pointing downwards and a movable mirror mounted in front of the object glass. (*)

A little later he designed the *Almucantar ;* in a lengthy memorandum (†) he gives a description of it and the mathematical theory on which it is founded, besides a discussion of the results obtained.

A rectangular trough containing mercury turns round a vertical pillar. The mercury supports a float of the same shape, in equilibrium, which is made to follow the trough when the latter turns. The float carries a telescope movable round a horizontal axis, which can be clamped at a determined altitude of observation.

When the instrument is made to revolve round a vertical axis, some point of the optical axis of the telescope describes a small horizontal or *almacantarate* circle. The particular circle adopted by CHANDLER is that which passes through the pole ; this choice leads to simplification of the reductions.

The telescope of the Almucantar had an aperture of 10 cm. and a focal length of 1 metre. The graticule consisted of a series of horizontal threads, for use when observing, this being also facilitated by the use of a bent eyepiece of high magnification.

(*) *The Observatory*, 1881, Vol. IV, p. 14.

(†) *Harvard College Annals*, Vol. XVII.

From his observations, in which Prof. ROGERS often collaborated, CHANDLER deduced some extremely interesting results.

In 1884 and 1885 he observed a certain number of stars with a view to a new determination of the latitude of Harvard College, of the value of which grave doubts had arisen. Confirmed by a fresh discussion of the old determinations, the observations with the Almucantar showed that the latitude of the dome of Harvard, as then given by certain Almanacs, was too high by about a second. Further, these observations furnished Right Ascensions for a certain number of fundamental stars of large declination, which differed from those adopted by the *Jahrbuch*.

The differences between CHANDLER and the *Jahrbuch* sometimes reached 0.3 second.

These corrections were confirmed by observations at Greenwich and Cambridge made after the publication of the AUWERS Catalogue.

This shows the great accuracy of the Almucantar, and it can well be understood that CHANDLER waxed enthusiastic over the equal-altitude method, the advantages of which he seems to have been the first to demonstrate. According to him, instruments enabling it to be applied gave better results than transit instruments. With the former, in fact, the latitude, the time, and the positions of the stars, are determined by observations all of the same type (times of transit); whilst with the latter they result from a combination of observations of different types — to the observations of transits must be added readings of levels and of divided circles, as well as of nadiral sights.

Further, when proceeding from one observation to another, with an equal altitude instrument, the angle between the optical axis of the telescope and the vertical does not change, and consequently the flexion of the telescope remains constant. In the same way, thanks to the constancy of the angle of observation, refraction does not enter into the calculations except in so far as it varies, which is often negligibly. On the other hand, flexion as well as refraction play an important part in the use of transit circles, quite apart from changes which reversal may bring about in certain instrumental constants.

When observations are being made in the field, equal-altitude instruments can be set up in a few moments, while the use of transit circles necessitates the construction of supports and shelters.

It has been objected, in connection with the Almucantar, that locking it on a given azimuth causes oscillations of the mercury and the float, making observations extremely delicate. CHANDLER nevertheless showed that two minutes' wait was sufficient for the instrument to attain complete stability, but it does seem that difficulties inherent in this arrangement of instrument have prevented the popularisation of the Almucantar, at least in its original form.

NADIRINSTRUMENT BY Dr. A. BECK (1890).

(4) In the Nadirinstrument of Dr. BECK, a Zurich astronomer, a constant altitude of observation is obtained by using a prism with a rhomboidal base, forming an angle of 60° and fixed under the object-glass of a vertical telescope, the prism being preferable to a mirror for conserving the angle. (*) (Fig. 1).

The telescope and prism can turn together round a vertical axis *V*. The angles of the prism are 60° at *C* and 120° at *D*. The prism reflects into

(*) A first order error in the position of the mirror actually causes an error of the same order in the observation of the altitude of the star, while a small derangement of the prism only causes a second order error. (This only applies if one of the angles of the prism is 60°).

the telescope the rays from a star *E* whose altitude is 30°, by making them undergo two complete reflections, one on the lower horizontal face *CD*, the other on the face *DB*. The entry face *CA* and the exit face *AB* are normal to the rays.

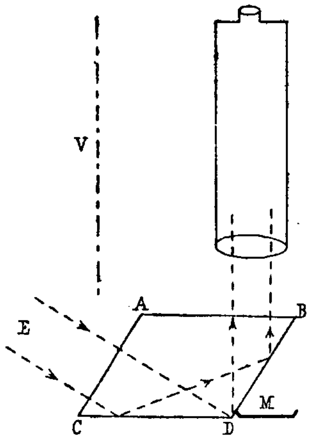

FIG. 1

The observation consists in noting the instants when the image of the star passes behind a series of parallel wires in the focal plane of the object-glass.

A mercury trough *M* serves to verify, by autocollimation, that the optical axis of the telescope is remaining vertical. Thus, the constant angle *A* of the prism is associated with the vertical of the trough in giving a constant instrumental altitude.

In several memoranda in the *Astronomische Nachrichten* BECK has shown in the completest manner the part that may be played by double-reflecting prisms. Among the combinations noted there he mentions the use of a triangular prism in conjunction with a horizontal telescope for observing altitudes of 60°.

According to a comment by Mr. NUŠL, "BECK in 1892 discovered the principle of the astrolabe and the circumzenithal." (*)

With a nadiral telescope of which the object-glass had a focal length of 36 cm. and a diameter of 40 mm., he made numerous observations at Riga in 1890. The mean of eight values of the latitude obtained on different evenings is such that the mean difference of the single determinations does not exceed one second of arc — a truly satisfactory result considering the small dimensions of the instrument.

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PART II.

PRESENT-DAY EQUAL-ALTITUDE INSTRUMENTS.

A. *PRISM ASTROLABE.*

(5) Messrs. CLAUDE and DRIENCOURT, those fervent partisans of the equal-altitude method, the many advantages of which they have frequently demonstrated (†), have been trying since the end of last century to produce an instrument which would lend itself better than the sextant to the application of the generalised equal-altitude method while remaining none the less handy to use.

With no knowledge of the work of Dr. BECK, who had however preceded them along this path, their efforts led them to the construction of the prism astrolabe.

PROPERTIES OF THE EQUILATERAL PRISM :

(6) The instrument is based on the following properties of the equilateral prism :

(*) *Troisième étude sur l'appareil circumzénithal*, Part I, page 6.

(†) *Revue Scientifique* 1905, pp. 972-983 and 1071-1083, and *Description et usage de l'astrolabe à prisme.*

If a ray of light enters by one face in a plane intersecting the prism at right angles, and if it is totally reflected off a second face and leaves by a third, the directions of the incident and emergent rays are symmetrical about the normal to the reflecting face. If the incident ray consists of white light, there will consequently be no dispersion and the emergent ray entering a telescope will give white light at the focus.

If two rays of light normal to the edges of an equilateral prism, having entered the prism by two different faces and been reflected in the interior of it, each from the face of entry of the other, follow the same path on emerging by the third face, they make an angle of 120°.

It follows that any two of the three faces of an equilateral prism can replace, as a measuring instrument, the two-mirror system of a sextant for a fixed angle of 60°.

Description of the prismatic astrolabe.

(7) The instrument comprises in the first place an *astronomical telescope* with a horizontal optical axis.

An *equilateral prism* is placed against the object-glass, its face BC being for practical purposes perpendicular to the optical axis.

It moves with the telescope, as also does a *mercury trough H*, lying slightly below the prism and protected from the wind by a baffled hood. The whole, movable about a *vertical axis*, can be trained on any bearing.

The axis of rotation is carried on a *tripod* with levelling screws; a spherical level enables it to be set vertical with sufficient accuracy.

A *horizontal limb* fixed to the frame is used for readings in azimuth; it is roughly trained by means of a *bearing plate.*

Path of the rays. — If a star is at an altitude of 60° in the vertical plane through the optical axis of the instrument, a ray of light from this star will meet the face AB normally and be reflected completely at I_1 without loss of light; the emergent ray I_1R leaves perpendicularly to the face BC (Fig. 2).

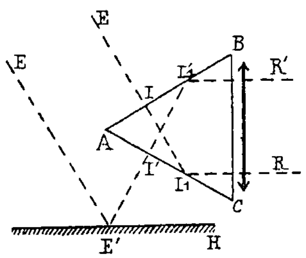


FIG. 2

Another ray EE' parallel to EI, after being reflected at E' on the surface of the mercury, makes an angle of 120° with EI; $E'I'$ passes through the prism and leaves it along I'_1R' parallel to I_1R in accordance with one of the properties mentioned above.

As a result, the once-reflected and the twice-reflected images of the star coincide at a point in the focal plane of the object-glass.

The prismatic astrolabe thus enables one to appreciate the moment when a star reaches an apparent altitude of 60°. For the observer this is the moment at which the two images of the star become merged after crossing the field to meet each other, while remaining on the same vertical.

Instrumental errors.

(8) It is necessary, in the first place, that the prism should be equiangular, with plane faces, and thus with parallel edges, and, in addition, perfectly homogeneous. If the conditions imposed upon the prism were not satisfied or if the glass were not homogeneous, it would be noticed from the lack of clearness of the images.

The makers are succeeding in producing prisms whose angles rarely depart by more than one minute from the theoretical value of 60°. As a general rule the departure does not exceed a few seconds. Such a prism enables one to measure a constant double altitude which, instead of being 120°, differs from this figure by a quantity proportional to the error of the reflecting faces. The proportionality is rigorous and the accuracy within 0.1 sec., so long as the angle of incidence of the rays entering the prism is less than 2°; this condition is always easily obtained in practice by the horizontality of the optical axis of the telescope and its perpendicularity to the rear face of the prism.

It is further necessary for the rays of light to be normal to the edges of the prism. These edges, and especially that which separates the two reflecting faces, must thus be perpendicular to the vertical through the heavenly body. This condition will be attained if the edges are both horizontal and perpendicular to the optical axis of the telescope. This perpendicularity must not be in error by more than 2.5 min. if the accuracy of 0.1 sec. in the observed altitudes is to be maintained.

This has led the makers to place in the focal plane of the telescope a *reticle* formed by two vertical wires marking off a section of the field 5 min. wide within which the observations must be made. The reticle is completed by two horizontal wires for adjusting the prism.

Adjustments.

(9) The adjustment of the spherical level and the perpendicularity of the optical axis of the telescope to the vertical axis of rotation are generally carried out by the maker. It is easy to readjust them when necessary by the usual methods.

Thus to set up the instrument, one starts by getting the axis of rotation vertical by means of the level and levelling screws, (*) then the direction of the adjusting circle is roughly set by the bearing plate; the exact direction, within a few tenths, will be obtained afterwards by observing a bright star whose azimuth has already been worked out.

The perpendicularity of the optical axis of the telescope to the rear face of the prism is obtained by using a self-collimating device which enables images of the reticle to be obtained by reflection from the three faces of the prism. The adjustment is made by pivoting the prism, by means of two suitably placed screws, about two axes at right angles to one another and perpendicular to the optical axis of the telescope, until the wires of the reticle coincide with their reflected images which must coincide if the prism is nearly perfect. This adjustment must be verified at the close of observations.

Finally, horizontality of the edges of the prism is readjusted before each star sight by turning the prism and the telescope round the optical axis until the two images of the star are roughly on the same vertical. In point of fact observers arrange for the images, instead of coinciding, to pass very near one another, and take the instant at which they cross the same horizontal as the instant of coincidence.

Various types of astrolabes.

(10) The various types of astrolabes were examined in the greatest detail by Messrs. Claude and Driencourt, who were thus able to show the makers where certain improvements might be made; but it must be recognised that the first astrolabes made by Vion immediately gained an excellent reception at the hands of geodesists.

(*) This levelling does not require great accuracy, since the horizontality of the edge of the prism is corrected for each star, besides which one is always entitled to work the adjusting screws during the observations so as to make the images coincide at about the same height within the field.

The astrolabes manufactured by Messrs. JOBIN have certain advantages over earlier ones, chiefly as regards the height of the prisms and the shape of the trough containing the mercury. The latter, hollowed out of a copper slab with which the mercury forms an amalgam, produces a bottom which, instead of being a segment of a sphere as in the VION type, is flat to within about a centimetre of the edges, and rises in a uniform slope. The depth of the central part of the trough is of the order of 2 mm. To obtain perfect horizontality of the mercury surface, it is in fact an advantage for the depth to be the same all over the working part of the trough.

These astrolabes must be dismantled into several parts for transport.

On the other hand, the S.O.M. astrolabes, of more recent design, are in one piece, and consequently can be set up very rapidly. Also, the optical axis of the telescope is perpendicular by construction to the vertical axis of rotation. Finally, the adjustment by autocollimation renders a special eyepiece no longer necessary and it can be verified at need during the course of the observations, without risk of displacing the setting of the eyepiece. This last precaution is of the greatest importance, as we shall see in the next paragraph.

There are at present three models of astrolabe :-

The *small* model, in which the object-glass has a focal length of about 25 cm. and a diameter of 25 mm., with a magnification of × 30, is mainly used for reconnaissance in new country.

The *medium* or *geodetic* model, with an object-glass of 40 or 50 cm. focal length, according to the maker, and 40 or 50 mm. diameter, and with magnifications of from × 50 to × 80, is mainly used by geodesists for the accurate determination of geographical positions.

The *large* model, manufactured by JOBIN, with an object-glass of 55 cm. focal length and 60 mm. diameter, and magnifications of × 50, × 100 and × 150, is more particularly intended for observatories, as is the large model made by S.O.M., with an object-glass of 57 cm. focal length and 80 mm. diameter, and a magnification of × 120 (Figs. 3 & 4).

In all these models, the prisms are of crown glass and their dimensions are in proportion to those of the object-glass.

INFLUENCE OF THE EYEPIECE SETTING.

(11) If the eyepiece of an astronomical telescope has a focal length f, the observation takes place in a plane P at a distance d from the eyepiece, such that $\frac{1}{d} = \frac{1}{\Delta} + \frac{1}{f}$ where Δ is the minimum distance of distinct vision.

By setting is meant displacing the eyepiece, i.e. the plane P which is invariably linked to the eyepiece for a given observer, in such a way that it cuts the beam of light from the object-glass at the spot where the rays are most concentrated; it is this position that defines the focal plane of the object-glass (Fig. 5).

In the astrolabe, there are two beams of light each of which covers half the object-glass; the axis of each beam passes at a distance from the centre roughly equal to a quarter of the diameter of the object-glass.

Near the focal plane, the beam may be taken to coincide with the mean (Fig. 6).

If the two beams intersect one another in a plane P_1, the images of the stars coincide in this plane. But in a neighbouring plane P_2, they no longer coincide (Fig. 7). Thus the degree of extension of the eyepiece modifies the instant of coincidence. This is equivalent to changing the instrumental altitude.

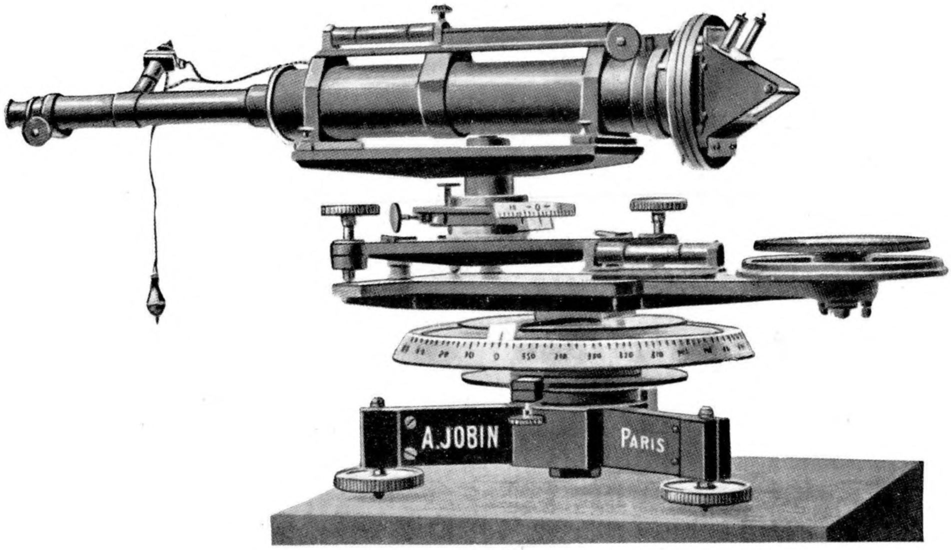


FIG. 3

JOBIN PRISMATIC ASTROLABE.
Large Model.

ASTROLABE A PRISME JOBIN
Grand Modèle

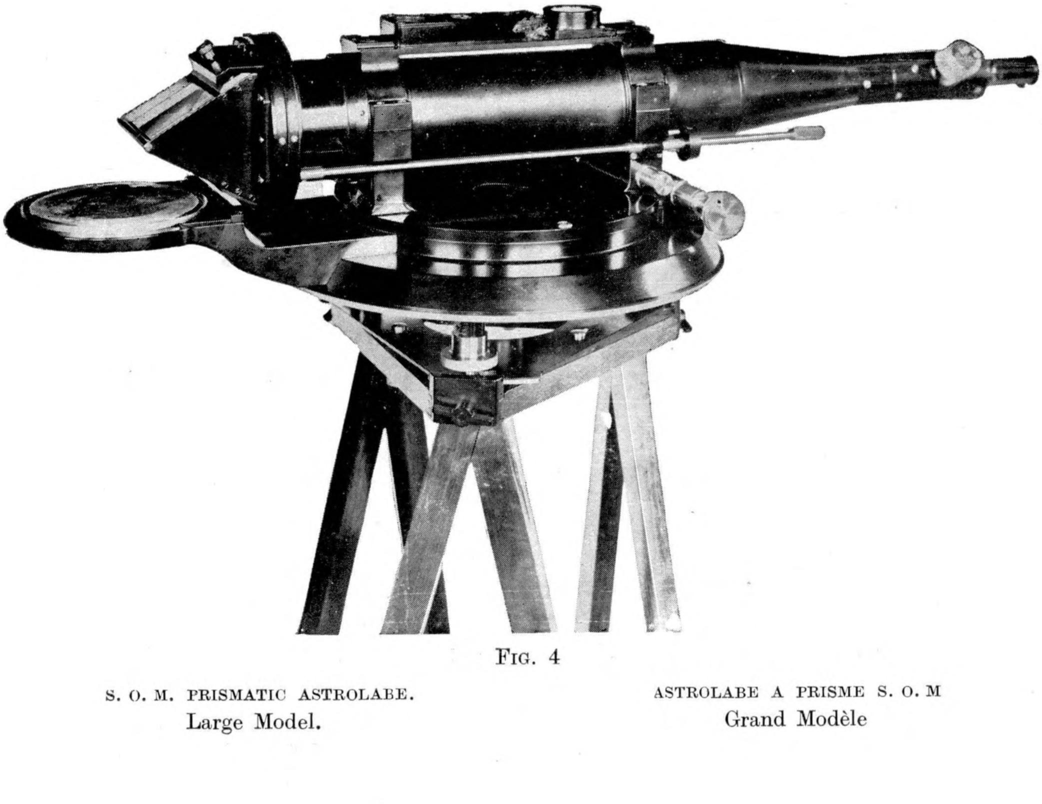


FIG. 4

S. O. M. PRISMATIC ASTROLABE.
Large Model.

ASTROLABE A PRISME S. O. M
Grand Modèle

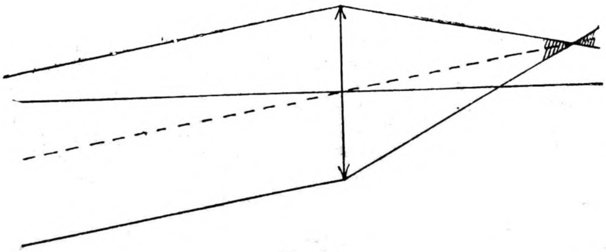


FIG. 5

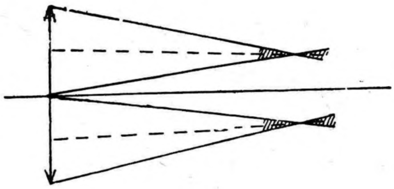


FIG. 6

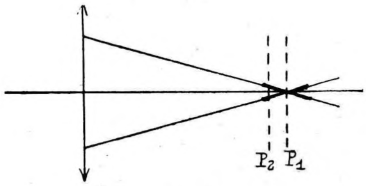


FIG. 7

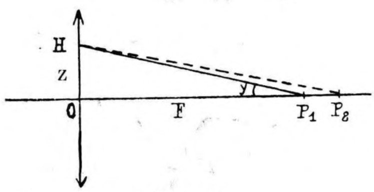


FIG. 8

Influence of the eyepiece setting. — *Influence de la mise au point de l'oculaire*

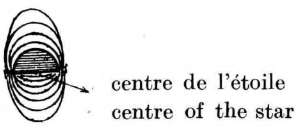


FIG. 9

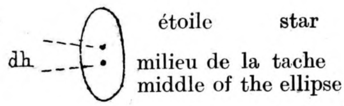


FIG. 10

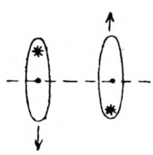


FIG. 11

Equation of magnitude. — *Equation de grandeur*

Let z be the distance between the axis of the beams and the optical centre of the object-glass, and y the angle made by this axis with the optical axis (Fig. 8). At the moment of coincidence, we have for practical purposes $\cot y = \frac{F}{z}$.

If we displace the plane of investigation of the eyepiece from P_1 to P_2 ($P_1P_2 = dF$), by displacing the eyepiece by the same amount, the instrumental altitude varies by dy. Thus:

$$\frac{-dy}{\sin^2 y} = \frac{dF}{z}$$

or again:

$$-(z^2 + F^2)\, dy = z\, dF.$$

We can neglect z^2 before F^2, so we get:

$$dy = -z\frac{dF}{F^2}.$$

If $F = 600$ mm. and $z = 12$ mm., as $\frac{1}{\sin 1''} = 200{,}000$, the above formula gives $dy = 7\, dF$ nearly, dy being expressed in seconds of arc and dF in millimetres.

When the eyepiece is displaced 1/10 mm., the altitude of observation varies by 0.7 sec. It is thus essential not to touch the setting during a series of observations, and the makers have provided a screw on the telescope with which the eyepiece may be clamped at the extension adopted.

Equation of magnitude.

(12) Each image (i.e. the once reflected and the twice reflected) is produced by half the object-glass, the latter being divided in half by the horizontal plane passing through the edge of the prism. It follows that the images are not round, but are elongated vertically. The apparent dimensions of these ellipses vary considerably with the size of the star. Though very appreciable in the case of bright stars, the elongation is hardly noticeable with stars of the 5th or 6th magnitude (Fig. 9).

The point corresponding to the centre of the star falls neither in the middle nor at one end of the ellipse (Fig. 10).

Let dh be the angular distance, as seen from the object-glass, between the point corresponding to the star and the centre of the ellipse (Fig. 11).

If an estimate be made of the coincidence when the centres of the two ellipses arrive at the same level, the error in the height of the observation due to this cause will be dh.

If the same star be observed to eastward and to westward, the errors will be identical in size but on the one hand the coincidence will have been observed too late and on the other hand too soon. This is equivalent to saying that in the two cases the observation will have been made at the altitude $h_0 + dh$, dh being either positive or negative, and h_0 being the instrumental altitude corrected for refraction.

If all the stars were of the same apparent magnitude, $h_0 + dh$ would be constant and no error would result from this fact in the determination of latitude or time. Only the altitude of the observation would be in error, by the amount dh.

But dh varies with the apparent magnitude of the star observed, and therefore the method of equal altitudes cannot be applied in its full rigour. Thus one must contrive to make the images the same size, so that dh may remain constant.

However, it does not seem necessary to insist on any very great accuracy in this unification of apparent sizes, as it seems in practice that the image of the star and the centre of the ellipse are nearly coincident, and that consequently the quantity *dh* is of the order of a small fraction of a second of arc.

This is why, in general, observers vary the lighting of the threads of the reticle by means of a rheostat, giving it its maximum intensity for stars of the first magnitude, and making it as weak as possible for observing the dullest stars.

In this way, on account of the light diffused by the wires of the reticle, the dimensions of the elongated spots corresponding to the bright stars seem reduced, and with a little practice one gets the knack of reducing images of stars of different magnitudes to comparable dimensions.

None the less, the use of *screens* of various thicknesses seems more advantageous for bringing the images to the same apparent magnitude. This is the procedure which we have applied in observations with large model astrolabes; a set of three or four screens is sufficient. The one selected is placed like a cover over the baffled hood; this arrangement has the additional advantage of somewhat increasing the protection of the mercury trough from the wind, and sometimes from oxidation.

Accuracy of results.

(13) The results given by the prism astrolabe have quickly proved themselves to be most satisfactory, and account for the success which this instrument has met with among French and foreign geodesists since its appearance in 1903.

It has been adopted by the Geographical Services of numerous countries — France, Egypt, British India, Spain, U.S.A., Belgium, Brazil, etc.

It owes this popularity to the fact that while providing extremely accurate results its bulk remains small, its setting up and adjustment are easy, and observation with it is very simple and does not require long training.

Among the various equal-altitude instruments, the prismatic astrolabe is by far the most generally used and this long experience is a protection against unforeseen errors which might become apparent in other instruments the use of which is much more restricted.

Thanks to numerous determinations made in the past thirty years, under widely varying conditions, with prismatic astrolabes of different characteristics, we posses reliable data relative to the accuracy obtainable with them.

Latitude. — With the large model the mean error of a single value of latitude taken from a series of about 35 stars, suitably distributed in azimuth, is of the order of 0.15 to 0.20 sec.

With the geodetic model this error is from 0.25 to 0.30 sec.

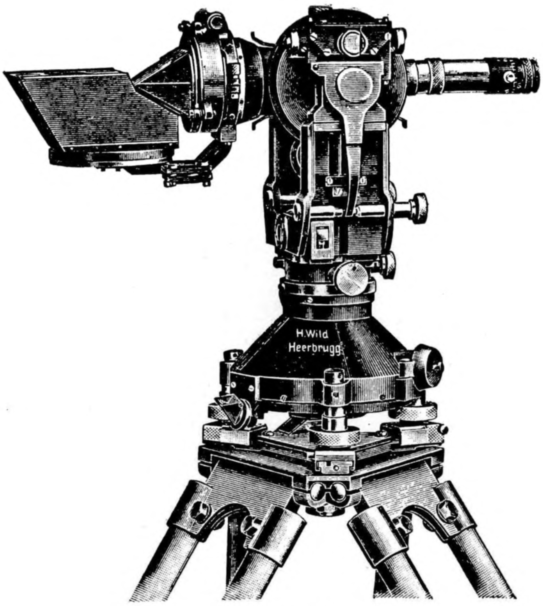
Finally with the small model it reaches 0.4 to 0.5 sec.

In view of such satisfactory results, certain observers have tried to use the astrolabe for studying variation of latitude or deviation of the vertical.

Time. — Working out series of observations with a large model astrolabe gives a mean error of about 0.01 sec. in the determination of time.

The smaller instruments give somewhat larger errors, which may attain 0,05 sec.

It must however be noted that the result of the computation does not represent the absolute time but that quantity modified by a systematic error, the personal equation of the observer. As we shall see presently, it is only recently that attention has been paid to eliminating this error with all the care possible.



WILD THEODOLITE
WITH EXTRA PRISM,
ready for observing.

THÉODOLITE WILD
A PRISME
ADDITIONNEL
en position d'observation

Below: Graph of Observations of 9th March 1933 (Service Hydrographique).

Scale: 2 mm. to 1'' of arc.

En bas: Graphique des Observations du 9 Mars 1933 (Service Hydrographique).

Echelle: 2 mm. pour 1'' d'arc.

FIG. 12

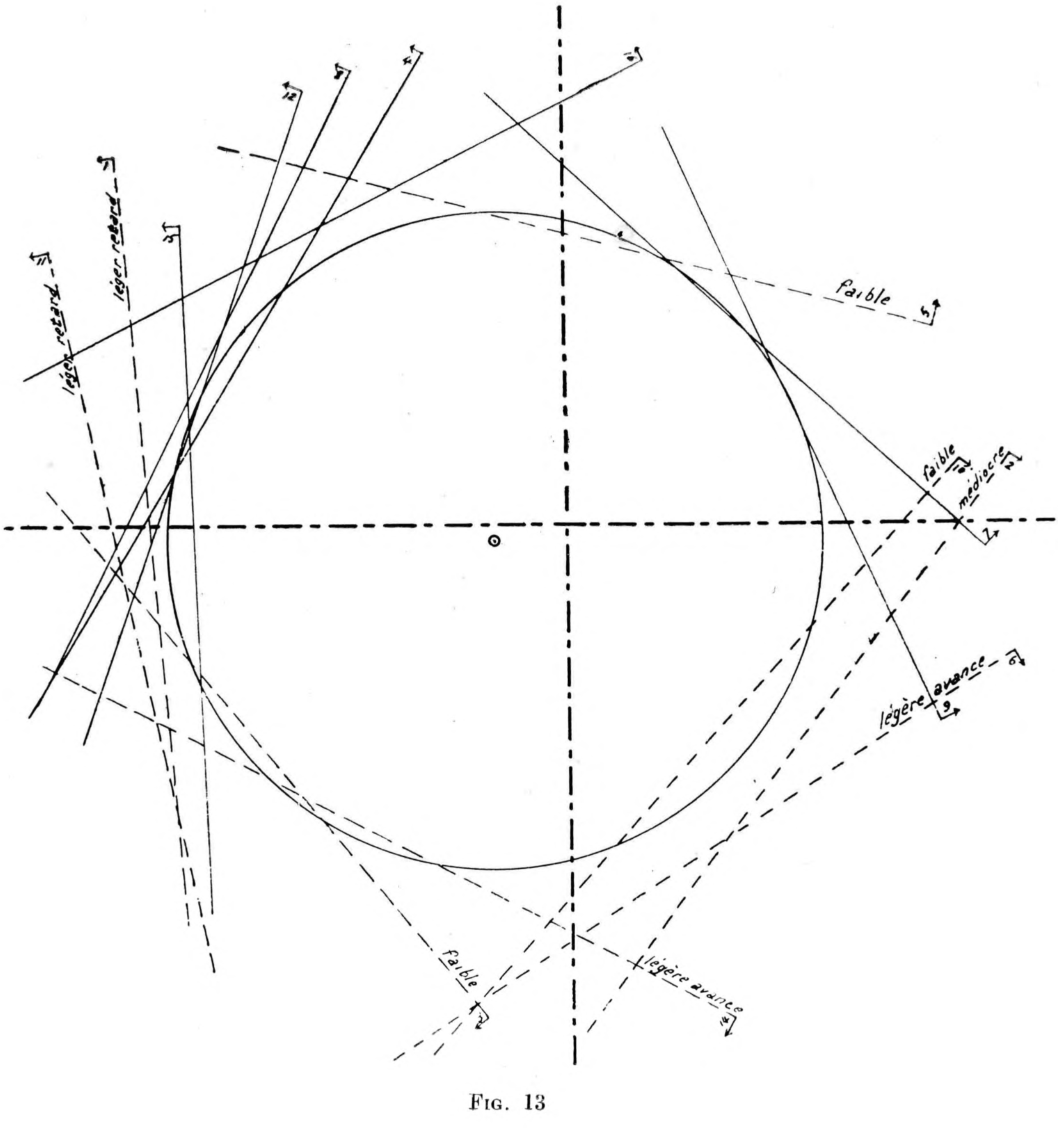


FIG. 13

It may be well, at this juncture, to recall an investigation by Mr. N. Stoyko (*) who, taking advantage of the methods of computation employed by the *Bureau International de l'Heure*, has compared the time-piece corrections furnished by the astrolabes used in Paris and at Zi-Ka-Wei during the months of October and November, 1926, with the time-piece corrections, known as rectified corrections, deduced from the combined corrections from four time-keeping clocks and six observatories; the transit instruments used in these observatories were all fitted with impersonal micrometers.

As a result of his investigation, this astronomer was able to conclude that, "If the observer's personal equation at the astrolabe is constant during the period of observation, the mean error of a determination of time is of the same order as those of transit observations, and these results can be used for getting improved time keeping, allowing for a constant difference with respect to the transit observations".

Number of stars in a series.

(14) If it be desired to determine simultaneously the time and the latitude with the full accuracy of which the astrolabe is capable, it is necessary to observe 30 to 40 stars (†) suitably distributed in azimuth.

But if the latitude is known to within 0.2 sec. from previous observations, it is sufficient for determining the time to observe 8 to 10 stars to the eastward and as many to the westward, in the neighbourhood of the prime vertical.

In the latter case, by using a table of hour angles (see § 98) the reduction of a series of observations takes hardly over four hours. This comment is not without importance, for some observers have occasionally hesitated to use the prism astrolabe for fear of the length of the reduction work.

B. *THEODOLITES WITH AN EXTRA PRISM.*

(15) It will be remembered that the telescope of a prismatic astrolabe must fulfil the following conditions:–

provide sufficient magnification;

be movable about a vertical axis;

be horizontal.

The telescope of a theodolite satisfies these conditions when it is fixed in the horizontal position; a theodolite can thus be used as an astrolabe if provided with a 60° prism and a mercury trough.

This idea is due to Mr. Reeves, Map Curator and Instructor in Surveying and Practical Astronomy at the Royal Geographical Society of London, and dates from 1921. It has been adopted by several theodolite makers, who manufacture accessories enabling these instruments to be transformed into astrolabes.

(*) *Sur la précision dans la détermination de l'heure et sur les moyens de l'améliorer.* — (Comptes-rendus du Congrès des Sociétés savantes 1928, p. 41).

(†) and not a hundred, as certain observers believe (Baker, *Geographical Journal*, Vol. LXXVII).

As a rule the prism and the mercury trough are fixed on a mounting which can be fitted on to the telescope object-glass.

The prism is carried in a little frame which can be turned to the extent of a few degrees, around three axes at right angles to each other, by means of screws with opposing springs.

One of these axes is parallel to the optical axis of the telescope; the corresponding rotation makes it possible to bring the two images of the heavenly body on to the same vertical.

The rotations round the other two axes have the effect of making the rear face of the prism perpendicular to the optical axis of the telescope.

Some types have an immovable self-collimating eyepiece enabling this adjustment to be made by the usual methods. To make it easier, the telescope is sighted at an elevation of 30° and a small mirror is placed on the now horizontal top face of the prism; the reticle is then made to coincide with its image by working the screws of the prism.

In the case of theodolites not fitted with this self-collimating eyepiece, the setting is done by day by sighting on a distant point on land, through the telescope on which the prism has been mounted. The horizontal and vertical limbs are read; the telescope is turned through 180°, the same point sighted on and the limbs re-read; then the telescope is fixed at the mean of the two readings and the screw of the prism is worked so as to bring the image of the sighted point on to the reticle (assumed to be without collimation).

The mercury trough is protected from the wind either by an aluminium baffled hood or by a parallel-faced sheet of glass acting as a cover.

In certain of these theodolites an eyepiece of $\times$ 40 magnification is substituted for the geodetic eyepiece of $\times$ 25 magnification so as to improve the accuracy of the astronomical observations.

(16) These instruments are of interest to triangulation parties who have to work in difficult conditions, particularly in the colonies. Thus the astrolabe case gives place to a very small box containing the extra part and the few necessary accessories (flask of mercury, glass tube, chamois skin, baffled hood, self-collimating eyepiece, $\times$ 40 eyepiece, etc.).

But this saving, slight as it is, in the bulk of the stores, is offset by a certain sacrifice of accuracy.

These instruments in fact furnish less satisfactory results than those obtained with standard astrolabes, because the magnification employed is comparatively low; the speed of the stars in the field being much reduced, the instant of coincidence is not so well determined. Also, the usual illumination of the threads, enabling dim stars to be observed in the centre of the field, is not available. It is thus necessary to observe, either in a lighted field, which greatly limits the size of the stars to be used, or in a dark field; in the latter case there is a risk of achieving coincidence at a considerable distance from the centre of the field, which is fairly big; and so of introducing a considerable error in the appreciation of the phenomenon.

This fault is all the more appreciable in that the micrometer screw which moves the telescope in azimuth has a very fine thread, which is very convenient for geodetic sights but does not always make it possible to follow the movement of stars in azimuth.

To summarize, one cannot count on an accuracy of nearer than one or two seconds of arc in latitude determination, and one or two tenths of seconds of time in time determination (obviously without taking the personal equation into account).

The use of these astrolabes is therefore only to be recommended where the reduction of stores plays a preponderating part.

Indeed, to associate an astrolabe of medium precision with the best theodolites, used in first-order triangulations, is paradoxical; for it is surely indispensable to seek for the maximum of accuracy in the astronomical determinations which serve as point of departure for these operations.

It may be mentioned that the use of these eminently portable astrolabes has led to the establishment of expeditious methods of preparation: auxiliary tables making it possible to obtain, almost without calculation, the settings for stars, well distributed in azimuth, to the number of about fifteen, which can be observed in an interval of an hour and which are sufficient for the determination of the latitude and the correction of the chronometer within the degree of precision obtainable with the instrument.

C. *CIRCUMZENITHAL INSTRUMENT*

MARK I (1901)

Description of the Circumzenithal.

(17) In 1901 Mr. Fr. NUŠL, a professor, and Mr. Jan FRIČ, a manufacturer, submitted to the Academy of Sciences of Prague the theory of an apparatus which they called the *circumzenithal,* intended for high precision determinations of the time and latitude of a place without the necessity for using very sensitive levels.

The circumzenithal was in fact an apparatus enabling the equal altitude method to be used. It comprised (see Fig. 14):–

(*a*) A *telescope* with its axis horizontal and a magnification of $\times 70$;

(*b*) A *prism,* the silvered faces of which were at an angle α; (the makers at first gave α a value of 120 degrees). Its edge was horizontal, and normal to the optical axis of the telescope; it was mounted at a certain distance in front of the object-glass.

(*c*) A *mercury trough* arranged in such a way as to reflect the rays from a heavenly body on to the lower face of the prism.

The system comprising telescope and prism, attached to a graduated limb, mounted on a tripod with levelling screws, could be turned round the mercury trough without touching it, so that the telescope could be set to the azimuth of observation.

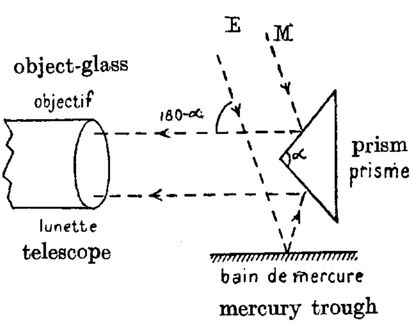


FIG. 14

Path of the rays. — Fig. 14 is a cross-section of the circumzenithal through the vertical plane containing the axis of the telescope, which is the plane of symmetry of the instrument.

A star E, about $180^\circ - \alpha$ in altitude, produces two images in the focal plane of the telescope: one reflected once off the upper face of the prism, and the other reflected twice — off the mercury and off the lower face of the prism. These two images come into coincidence when the star reaches an altitude of $180^\circ - \alpha$, and the observation consists in noting the instant at which this coincidence takes place.

Adjustment. — The above reasoning is rigorous only if the vertical through the heavenly body cuts a normal section through the prism, which implies for the edge the two conditions of being horizontal and of being perpendicular to the optical axis of the telescope. The first condition is fulfilled when the two images of a star are seen on the same vertical. The second is obtained by auto-collimation with the aid of a mirror inclined at an angle of $\alpha - 90^\circ$ to the horizon and placed at M above the prism and the mercury trough. This adjustment must be made to within one or two minutes of arc.

Horizontality of the optical axis of the telescope and of the bisecting plane of the prism are not necessary conditions, and need only be approximately obtained.

MULTIPLICATION OF SIGHTS.

(18) The instrumental altitude is not known exactly *a priori*, but during the course of an evening's observations it remains constant within a margin of the order of 0.1 sec. of arc corresponding to about 0.01 sec. of time for stars 90° from the meridian. As there is only one time sight per star, the accidental errors are fairly large, much over 0.01 sec., and do not allow the full accuracy of which the apparatus is capable to be attained.

To obtain an accurate final result, a large number of stars must be observed or else several sights must be taken of each star.

The former procedure is used in observations with equilateral-prism astrolabes (*). The latter has the advantage of reducing the preparative calculations, and the working out of the observations, but it has the disadvantage of introducing prismatic sheets of glass, fixed or movable, which superimpose their angle one way or the other on that of the prism.

Is the constancy of the instrumental altitude, which is fundamental to the equal altitude method, as well maintained when this altitude is obtained by an assemblage of several component parts as when it is measured by the angle between two plane faces of a single block of glass ? Probably not, but experience seems to indicate that it is obtained with sufficient accuracy.

To increase the number of sights obtainable from a single star, the makers of the circumzenithal made use of the following process (Fig. 15) :-

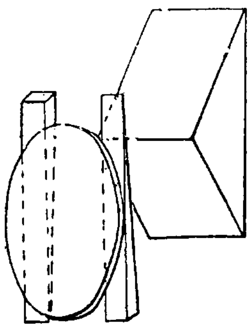


FIG. 15

On the face of the object-glass next the prism they placed two thin, flat wedges of glass, each covering one third of the object-glass at the sides, the central third being free. These wedges had angles of about 3 and 5 minutes of arc respectively, and one was fitted edge up, the other edge down, so as to cause deviations in opposite directions.

Thus, when observing a star, there are obtained in the telescope three once-reflected images and three twice-reflected images, all on the same vertical ; one of the groups moves upwards, the other downwards, across the field.

These six points of light enable up to 15 coincidences or symmetrical groupings of images to be obtained. The mean of the times of these various appearances of the field gives accurately the instant when the star crosses the altitude $180^\circ - \alpha$.

(*) TRÜMPLER, however, had tried to eliminate accidental observational error from the prismatic astrolabe. A set of wires was stretched horizontally in the focal plane of the telescope, and he observed the times of passage of the two images across the various wires. The time of coincidence is equal, within the limits of a small correction, to the mean of the two times of passage across any particular wire.

For stars near the meridian, a small correction must be applied to the observed times to allow for the fact that the variation of altitude of the star is not rigorously proportional to the time.

As the time stars cross the field fairly rapidly, one may rest content with a small number of observations, e.g. 9 groupings.

Quality of the images. — The horizontal plane through the edge of the prism, and the inner edges of the glass wedges, divide the object-glass into six parts, corresponding to the six images obtained.

The first division, which is also met with in the astrolabe, has the effect of lengthening the images along the vertical, which is a disadvantage from the point of view of the accuracy of the sights. The second method of division lengthens them along the horizontal, which is better but tends still further to reduce the brightness of the images.

(19) *Methods of computation.* — The authors prefer to calculate time and latitude separately, and the observations are consequently combined.

For working out the time they use pairs of time stars passing east and west of the meridian, either with neighbouring declinations (ZINGER's method) or at nearly symmetrical azimuths (method of corresponding azimuths).

Another method is to imagine the latitude to be exactly known and to use it to determine the altitude of observation (method of altitude corrections).

Finally one may suppose the altitude of observation and the absolute variations of refraction to be exactly known (method of absolute altitude). On account of altitude anomalies, this method could not be employed in the first work carried out with the circumzenithal.

Computation of altitude and latitude is done by GAUSS's method, which has the further advantage of giving the error of chronometer at the same time.

(20) *Results.* — The latitudes and chronometer errors deduced from the first observations were in satisfactory agreement, but the observed altitudes sometimes differed by several seconds.

This discrepancy is to a great extent explained as follows. The singly and doubly reflected images are formed separately by the upper and lower halves of the object-glass; also, the extension of the eyepiece for improving the focus as well as the deformation of the instrument under the influence of change of temperature entail a variation in the altitude of observation.

1905 MODEL.

(21) To remedy this fault in their instrument, the makers in 1905 produced a new type of circumzenithal in which the two images each came from a lateral half of the object-glass.

To obtain this the prism was superseded by two crossed mirrors, one each side of the vertical plane of symmetry of the telescope. They form an angle α with one another, and are arranged so that one reflects the rays of light from a star E on to the object-glass and the other reflects the rays falling on it after reflection in the mercury trough. (Fig. 16).

The rays of light reflected by these mirrors form two beams covering respectively the right and left halves of the object-glass.

The mirrors are fixed on a common mounting solid with the telescope.

Movable parts make it possible to make the adjustments, which are the same as with the prism, the edge of the latter being replaced by the imaginary line which forms the intersections of the planes of the two mirrors.

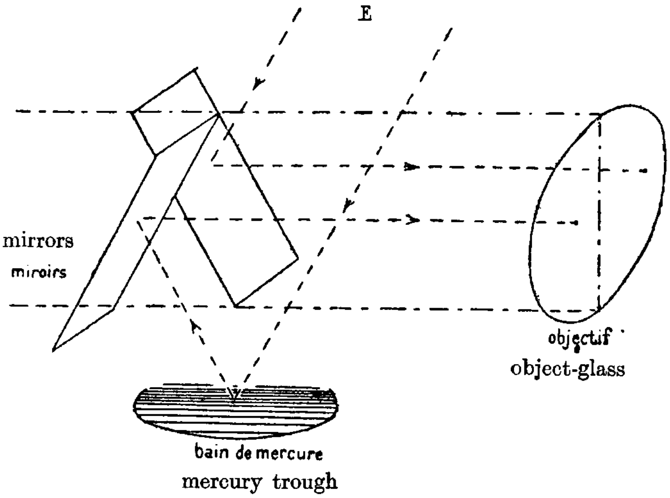


FIG. 16

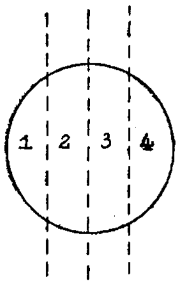


FIG. 17

This model still includes the optical device for multiplying the sights, though slightly modified. The thin glass wedges cover the two outer quarters only of the object-glass. Figure 17 shows the view through the object-glass. Part 1 is covered by a wedge which refracts the rays 50" in one direction; on part 4 is a wedge which refracts them 90" in the opposite direction.

Altogether this arrangement provides two singly reflected images, enabling seven coincidences or symmetrical groupings of images to be observed for a single star.

The performance of the instrument has been improved by introducing a thin wedge of glass in the path of the incident rays to bend them through a small angle ε. A simple device enables one to juggle with it or turn it 180° about a horizontal axis. Thus the same star can be observed at three adjacent altitudes in arithmetical progression: $h - \varepsilon$, h, and $h + \varepsilon$.

Results.

(22) The 1905 apparatus is appreciably more accurate than its predecessor. The division of the object-glass by a vertical plane has proved a beneficial factor in obtaining this result. Actually, owing to diffraction, the images of the stars take the shape of little horizontal, symmetrical dashes the coincidence of which in altitude is easy to appreciate.

Yet an analysis of some series of observations brought out a fact already noted in connection with the older model, namely that the mean error of a single complete passage of the star was generally much less than the mean error of the time determination resulting from this passage.

1922 MODEL.

(23) In the 1905 model, the inventors found that owing to the complexity of the arrangement for fixing the mirrors, the angle between the latter varied with the temperature. This angle also showed a slow variation as a function of age, due apparently to distortion gradually occurring in the threaded parts of the supports.

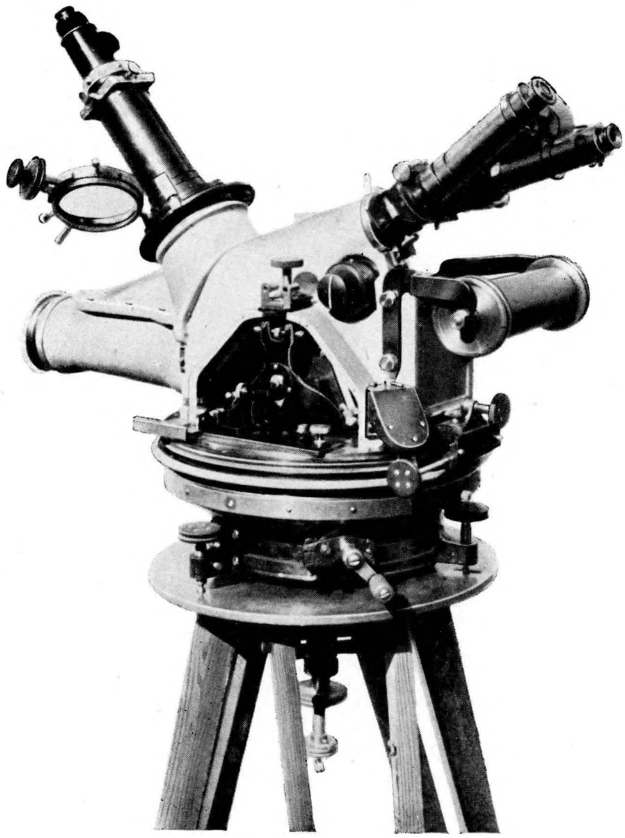


FIG. 18

CIRCUMZENITHAL (1922)
with rectifying device.

CIRCUMZÉNITHAL (1922)
avec dispositif de rectification

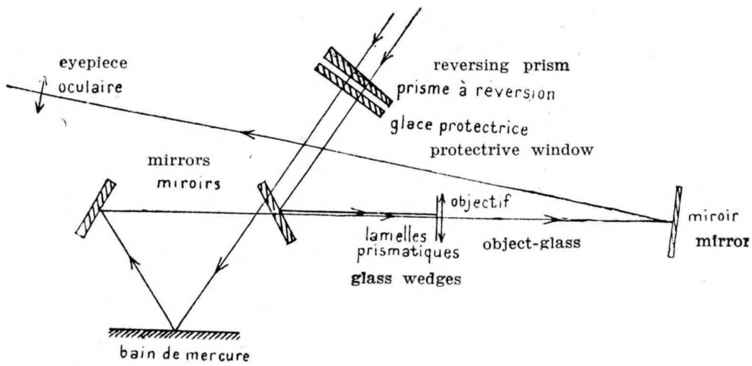


FIG. 19

So, when the Czecho-Slovak Geographical Service requested the construction of a circumzenithal apparatus for its work, one of the main modifications consisted of an alteration in the support of the mirrors.

Further, as the instrument ordered had to be transportable, it was necessary completely to reconstruct the apparatus so as to attach to it the mercury trough which previously had always been independent.

Description (FIGS. 18 & 19).

(24) The mercury lies in a flat-bottomed amalgamated trough. The depth of the trough is only three tenths of a millimetre, but its great diameter, more than 200 mm., makes it possible to have a reflecting surface of excellent horizontality. The apparatus is so made that the trough is completely enclosed, and screened from wind, dust and damp. In particular, the rays of light pass through a plane protective window to enter the instrument. The copper plate forming the trough is slid into the stand of the apparatus on a support whose height is adjustable, and which can be brought to a suitable position for observing by means of a crank.

The cast bronze *mirror support* is all in one piece, this arrangement being apparently the best for keeping constant the angle of 50^0 which has been adopted between the mirrors.

Opposed threads make it possible to turn the mirror supports through small angles in three planes at right angles to each other, so as to obtain the various adjustments.

A peculiarity of the new model lies in the arrangement of the mirrors; these are placed above the mercury trough, and their support is hollowed out so as to let the rays of light through.

The telescope, with a magnification of $\times$ 140, is bent and contains an auxiliary mirror *m*, so as to have, in spite of the relatively great focal length, an apparatus of reduced bulk and satisfactory stability.

The telescope and mirrors are fixed on a large diameter ball race, carried on a *tripod* with levelling screws, with a graduated limb. The support of the mercury trough also rests on this tripod, and when observing, the trough comes under the mirrors at the height of the ball race. The telescope and mirrors can thus be set over the trough at any azimuth.

As it is important that the observations should be made at the centre of the field, a *reticle* consisting of two vertical threads limits the region within which the coincidences correspond well with the instrumental altitude.

A *viewfinder* on the side of the telescope, with a reflecting prism, gives an image of a more extended part of the sky, but with the same centre as that observed in the instrument.

Multiplication of sights.

(25) The arrangements utilised in the 1905 model have been retained. Two glass wedges of 50" and 90" respectively before the object-glass open out the two images of the star reflected by the mirrors (Fig. 17).

Each wedge is fixed in a ring concentric with the object-glass and movable in its plane, so that the edges of the prisms can be made horizontal. This result is attained when the separated images are exactly one above the other.

A reversing prism in the path of the incident rays makes it possible to observe the same star at three altitudes, spaced 6' apart in pairs. In order that the prism should always return to exactly the same position during the observations, its mounting carries two steel studs which butt alternately into a slot on the instrument.

Adjustments. — First the auxiliary mirror *m* is adjusted by bringing into coincidence the images of the diaphragm of the eyepiece produced by the different spherical surfaces of the object-glass.

For the other adjustments one works the screw which moves the support of the mirrors. The operation is very handy, thanks to the addition of a special collimator which sends into the instrument the rays of light coming from an artificial fixed star formed by a lighted hole at its focus.

Results.

(26) Since the year 1924 the Czecho-Slovak Geographical Institute has determined numerous geographical positions with the aid of this instrument.

We have more especially examined the observations made from 1925 to 1927 at some 15 stations in sub-Carpathian Russia (*).

The series made at each station are relatively small in number — four to six as a rule for latitude, and three to four for time, each determination in itself resulting from the observation of about six stars.

The values deduced from each series differ from one another by slight quantities, rarely exceeding 0.7" for latitude and 0.15^s for longitude. In this latter difference allowance must be made for the fact that the observations were made by different observers with appreciably different personal equations.

The mean errors are $\pm$ 0.12" for latitude and $\pm 0.03^s$ for time.

In 1925-6 the latitudes were determined with a number of stars much above six (on one occasion as many as 37 stars).

The spread of the results and the mean difference obtained as a result of these long series are entirely comparable with those subsequently deduced from series of six stars. This number, then, appears sufficient; anyhow it corresponds to more than 100 points.

Recent observations, in 1932, have given even more accurate results.

Eight series were carried out at each station, each containing 7 stars for latitude and 6 for longitude.

The mean difference for a series is 0.24" for latitude. During the course of different evenings, the mean difference did not exceed 0.09" for latitude and 0.015^s for longitude.

D. *PENTAGONAL-PRISM ASTROLABE.*

(27) This astrolabe was designed in 1930 by Messrs. COOKE, TROUGHTON and SIMMS, the makers, to the specification of Captain T. Y. BAKER, R.N.

The inventor's idea was to substitute for the 60°-prism astrolabe an equal-altitude instrument which would fulfil the following additional conditions :-

(1) The direct image and that reflected in the mercury trough to be furnished by beams of light centred on the optical centre of the object-glass.

(2) Each star to be available for several sights so as to obtain sufficient accuracy with observations of only a few stars.

(3) The altitude of observation to be diminished so as to give a greater choice of stars to observe.

(*) *Annual Report of the Military Geographical Institute of Prague,* Vol. X.

Before going any further it may be noted that the last two conditions are to a certain extent contradictory, for obtaining several sights per star leads to the observation of a small number of stars, 12 or 16 at the most, and the frequency of stars observable in a 60°-prism astrolabe already enables one to attain this number in a reasonably short time. This increase in the number of stars observable is even less explicable if it is noticed, as we shall presently see, that the pentagonal-prism astrolabe enables stars of the 7th magnitude to be observed while on the other hand the uncertainty in connection with variations of refraction increases rapidly with the zenith distance of observation, which tends to lessen the accuracy.

Be that as it may, this third condition has appeared sufficiently fundamental to Captain BAKER for him to have adopted an angle of 45° and substituted for the equilateral prism a pentagonal PRANDL prism containing a right angle and four equal angles.

DESCRIPTION.

(28) Like all analogous instruments, this new astrolabe comprises essentially, besides the prism, a fairly high-power telescope and a mercury trough.

The whole is mounted on a support with three levelling screws and can turn round a vertical axis which can be adjusted to a sufficient degree of accuracy by means of a level; a graduated limb, which can be trained in the desired direction with the aid of a bearing plate, is used to set the telescope in azimuth.

Path of the rays (Fig. 20). — The telescope, which is inclined at an angle of 45°, has an aperture of 50 mm. and a magnification of × 36. In front of the object glass is fixed the PRANDL prism *ABCFG*. The faces *AG* and *CF* are silvered; *AG* is completely so, whilst the central part of *CF* is not a reflector. The rays from a star *S* at 45° from the zenith enter normally by the face *BC*, being reflected off *AG* and off the silvered part of *CF*, passing normally through *AB* and entering the telescope by the outer ring of the object-glass corresponding to the silvered part of *CF*.

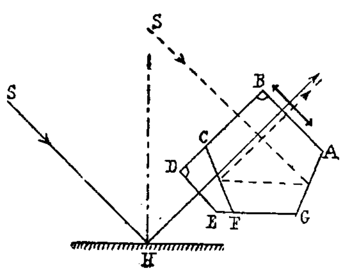


FIG. 20

An auxiliary prism *CDEF*, right-angled at *D*, is applied to the face *CF* in such a way that its face *DE* is parallel to *AB*. The rays of the star *S* falling in the mercury trough are there reflected, enter the block of prisms normally through *DE*, pass through the unsilvered part of *CF*, emerge normally through *AB*, and enter the telescope by the central part of the object-glass.

When the altitude of the star changes, the two images obtained cross the field of the telescope vertically and in opposite directions; they are in coincidence at the moment when the star passes exactly through the altitude of instrumental observation, which bears a simple relationship to the angle *B*.

As the axis of each of the beams passes through the optical centre of the object-glass, change of focus has no influence on the relative position of the images. This is one of the goals at which the inventor was aiming. It should be noted that this is not a peculiarity of the pentagonal prism; the use of two prisms in contact along a half-silvered face (concentrically or in strips) enables an astrolabe to be produced, according to the angles given to the prisms, with which observations can be made at any altitude, and where the two images are formed by beams centred on the optical centre of the object-glass. The PRANDL prism is only one of the elegant solutions of this problem.

MULTIPLICATION OF SIGHTS.

(29) Before being reflected in the mercury trough, the rays of light from the star are made to cross three thin prisms with horizontal edges. These prisms can turn through 180° round a horizontal axis so that their edges may be turned upwards or downwards with reference to the rays from the star.

The angles of these prisms are in the ratio 1 : 2 : 4. Let us call them α, 2α, and 4α.

The first observation is made with the three edges all the same way up; the deviation is then $\alpha + 2\alpha + 4\alpha = 7\alpha$.

The position of the prism of angle α is changed every sight, that of the prism of double angle every other sight, that of the prism of quadruple angle every fourth sight; thus 8 successive deviations are obtained in arithmetical progression with a difference of 2α, running from 7α to -7α.

The rotation of the prisms is managed without difficulty by a single knob, all that is required being to give the latter a complete turn after each sight. The direct image maintains its course; by actuating the knob the reflected image overtakes it by 2α.

The angle α has been made about 3', so as to leave time for the mercury to become perfectly still and for the observer to make sure of the adjustments for the next coincidence.

Owing to the relatively low magnification, the field of the telescope is of the order of a degree, and the eight coincidences take place in the central third of the field. With greater power, the field would be appreciably reduced, and to observe at an analogous rate in the central part it would be necessary to have the deviating prisms much larger, covering both the mercury trough and the entry face of the pentagonal prism, so that the direct and reflected images should be symmetrically deviated.

If the prisms are not exactly in the ratio 1 : 2 : 4, the eight sights none the less correspond to altitudes which are symmetrical in pairs with respect to the instrumental altitude, and their mean is not affected.

OBSERVATION.

(30) To facilitate observation with the pentagonal-prism astrolabe, the direct image is split by a thin prism of angle about 1', covering half the outer zone of the object-glass. Thus two direct images are obtained at the same altitude, subtending an angle of 35 to 40 minutes at the eye, and slightly elongated along the horizontal by diffraction.

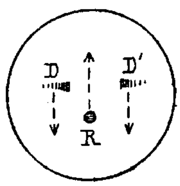


FIG. 21

The observation consists in making the image *R*, reflected off the mercury trough, pass between the two direct images *D* and *D'* and noting the instant when it crosses the imaginary line defined by these two images. The relative displacement in a horizontal direction of the reflected image with respect to the direct images, is obtained by slightly turning the prism round the optical axis of the telescope. If care be taken to make the reflected image pass as nearly as possible equidistant from the two direct images, it is not necessary to insist on the edge of the splitting prism being strictly vertical.

The mercury trough is sheltered from the wind by a parallel faced glass window, serving as a hood, and rigidly fixed to the mounting of the apparatus. If its faces are not rigorously parallel, things are so arranged that the prismatic effect would take place along the vertical.

For preparing the observations, the inventor recommends the use of diagrams. The reduction is analogous to that of observations with standard astrolabes. The graphical method may be used, or that of least squares. It is necessary, however, to introduce a small correction to allow for the fact that the movement of the star in the field is not uniform; a diagram has been made to give this term.

With this instrument, as with other equal-altitude instruments giving several sights per star, it is desirable in simultaneous determinations of time and latitude to observe stars at about 45° either side of the meridian, to northward and southward.

The pentagonal-prism astrolabe enables 7th magnitude stars to be observed, a praiseworthy result in view of the number of glasses and the comparatively low power.

For less accurate observations, the construction of a smaller model has been undertaken; the telescope must have an aperture of 35 mm. and no stars dimmer than the 5th magnitude will be observable. It may be asked whether the relatively high price entailed by the lay-out of this astrolabe is justified for observations in which considerable accuracy is not required.

RESULTS.

(31) Trials carried out by British Naval Officers during eight evenings at the same spot, and comprising 8 or 12 stars, have given quantities with maximum differences of 1.3" for latitude and $0^s,2$ for longitude, the mean error of a single determination being respectively $\pm 0.3''$ for latitude and $0^s,05$ for longitude.

E. *BENT TELESCOPE BY M. DE LA BAUME PLUVINEL* (1922).

(32) The bent telescope makes it possible to observe the time of passage of a star at a constant altitude of about 60°; though it uses the mercury trough as datum for the vertical, this instrument nevertheless differs in essentials from the instruments just described.

With the latter, the passage of a star at the altitude of observation is determined by the coincidence of two images of the star or by an analogous phenomenon, one of the images having been reflected in the mercury trough. These two images are produced by different portions of the object-glass, which detracts from their quality whatever optical combinations are used.

This fault does not exist in the bent telescope; the small circle of constant altitude which the stars cross at the instant of observation is as it were materialized by a horizontal wire in the focal plane of the object-glass. The image of the star furnished by the object-glass is reflected in the mercury trough and passes behind this wire.

DESCRIPTION.

(33) (*a*) The telescope support consists of a perfectly rigid metal frame in the upper surface of which are fixed on one side the object-glass and on the other side the reticle and the tube carrying the eyepiece. Its base acts as support for a trough containing the mercury, and the frame is enclosed so as to shelter the artificial horizon from wind and dust; it has a window through which the trough may be inserted and access to it obtained.

The optical axis of the telescope, on the object-glass side, makes an angle with the horizon equal to the instrumental altitude adopted.

The centre of the mercury trough is situated on the optical axis at a distance from the nodal point of emergence equal to half its focal length. The layer of mercury is extremely thin, and reflects the rays of light received by the object-glass.

The reticle consists of ten horizontal wires and occupies a symmetrical position in the focal plane of the object-glass with respect to the mercury trough.

The eyepiece enables the image of the star to be observed as it passes behind the wires of the reticles.

(*b*) The telescope just described can turn round a vertical axis borne on a tripod with levelling screws which is fitted with a graduated circle enabling the telescope to be placed in the azimuth of observation. Verticality of the axis of rotation is obtained with sufficient accuracy by means of a level.

A viewfinder is attached to the telescope; its axis is parallel to that of the eyepiece; a prism in front of its object-glass makes it possible to observe a more extended, but concentric, field of sky than that observed in the bent telescope.

(*c*) *Path of the rays.* — A star passing at an altitude of roughly 60° sends on to the object-glass of the telescope which is set in the vertical through the star, a beam of parallel rays which, on account of the interposition of the mercury trough, converge in the plane of the reticle. Under the effect of the diurnal movement, the image of the star passes behind the wires of the reticle, and the observation consists in noting the time of the various passages; the angle between the trajectory of the star and the line of the wires is equal to the angle at the star of the triangle of position.

Choice of instrumental altitude.

(34) M. de la Baume Pluvinel has adopted an altitude of 60°, but obviously bent telescopes could be made enabling observations to be made at a different altitude.

However, the altitude chosen is a convenient one, for at that zenith distance refraction is fairly small, and its variations, which directly affect the altitude of observation, can be taken as being negligible during the taking of a series.

By making observations nearer the zenith, the errors due to variations of refraction would be reduced, besides those, proportional to cos h, caused by variations in the level of the reflecting surface (*). But the number of stars observable diminishes rapi dly with the altitude, and there would be a risk of appreciably prolonging the duration of the series on account of the necessity for observing stars well distributed in azimuth.

An instrumental altitude of 60° has the further advantage of being the same as that of the equilateral-prism astrolabe, for which preparation tables or diagrams are already in being. These documents can be used, if not for the actual preparation of the observations with the bent telescope, at least for choosing the stars which may be observed.

Principle of the instrument.

(35) Let us suppose for a moment that the star we are observing is stationary; we shall see that, if certain conditions are fulfilled, the image will continue to be formed at the same point on the reticle when the apparatus is moved in the vertical plane passing through the star, and that consequently the altitude of observation depends solely on the construction of the telescope and not on the levelling of the vertical axis of rotation.

Translatory movements are clearly not of very much importance, for as the star is at an infinite distance they do not make the least difference to the path of the rays. Hence we shall only consider rotation in the vertical plane V through the star.

(*) See § 37 (b).

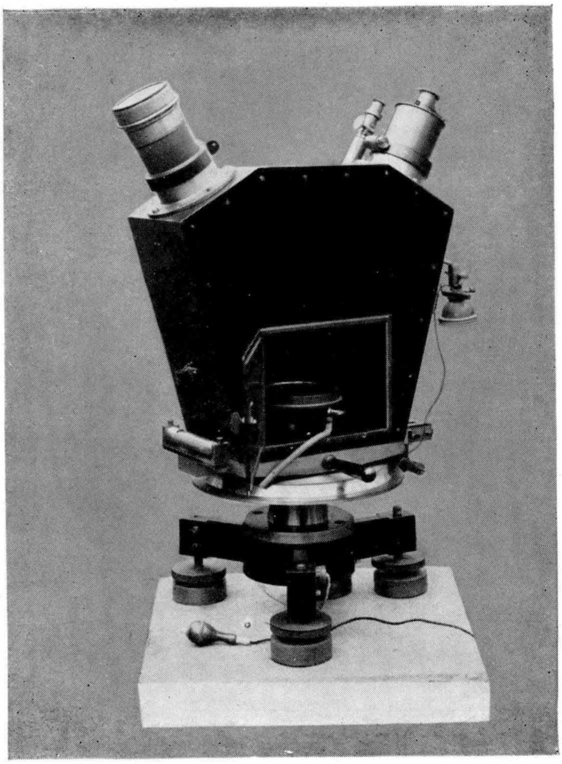


FIG. 22

BEN TELESCOPE — LUNETTE COUDÉE

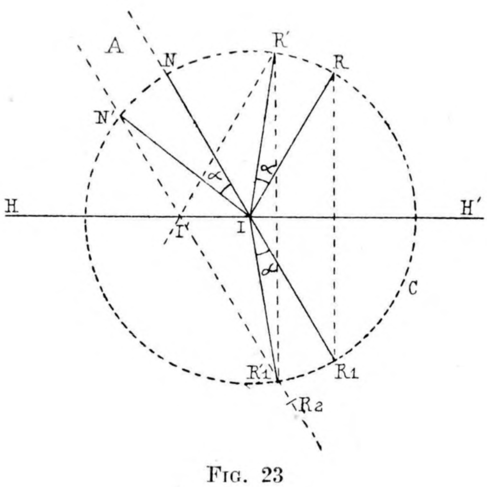


FIG. 23

Let us assume that the trough containing the mercury takes the shape of a cylinder of revolution with vertical axis, at least near the surface of the trough. When the instrument is tilted, this surface turns round the diameter perpendicular to the plane V; *this diameter thus remains permanently fixed with regard to the telescope mounting,* and we can treat it as the axis of rotation.

Let I be the point where it cuts the plane V which we shall adopt as the plane of the figure (Fig. 23).

Let N be the nodal point of emergence of the object-glass, and let it be assumed that the star A is at an altitude such that the axis of the beam of light from A is NI. Let R_1 be the point on the focal plane of the object-glass where the image of A would be formed. Let us finally assume as an hypothesis that the mercury trough has been so placed with reference to the object-glass that I is the mid-point of NR_1.

Owing to the presence of this trough, an image of A is formed at a point R on the reticle, symmetrical to R_1 with respect to the reflecting surface HH'. The three points N, R and R_1 are on a circle C with centre at I.

Let us turn the telescope through an angle α in the plane V round the perpendicular through I. N and R respectively come to N' and R'; N' and R' are on C, as also is R'_1, the point symmetrical to R' with respect to HH'.

The axis of the beam, coming from A and falling on the object-glass, is AN', parallel to AN; it cuts the surface of the mercury at I'. Without the mercury the rays would converge at R_2 such that $N'R_2 = NR_1 = F$, the focal length of the object-glass.

It is easy to see that $N'R_2$ passes through R'_1; consequently the image of the star is formed at the same point of the reticle as before the rotation. The distance $R_2R'_1 = NR_1\ (1 - \cos\alpha) = F\frac{\alpha^2}{2}$ is of the second degree in α, and so practically negligible; anyhow it is merely comparable with a slight error in focussing.

A tilting of the axis of rotation in a direction perpendicular to the vertical through the star only entails an error of the second degree in the altitude of observation. The error reaches 0.1" for an inclination of 5'.

Adjustments.

(36) (*a*) We have assumed that the mercury cuts the optical axis of the object-glass at an equal distance from the nodal point of emergence and the focal plane.

This condition is obtained with sufficient accuracy by the construction. To reset or improve it, use is made of a suitably arranged collimator throwing on to the reticle the image of a point of light at its focus. The telescope is then tilted by working the levelling screws, and by trial and error one arm is lengthened and another shortened until the image appears stationary on the reticle in spite of any changes of level on the part of the instrument. The wires of the reticle being somewhat thick (they correspond to an altitude of nearly 2"), it is best to check the fixity of the image with respect to some fault on the reticle, such as a stain, speck of dust, etc.

A metal rule with a cursor enables the distance between the reticle and the mercury trough to be measured to within 1 mm., which is sufficient for adjusting the bent telescope when no collimator is available.

If the lengths of the arms of the apparatus differ by dl, the error in the instrumental altitude h, when the telescope is tilted through α, is $dh = \alpha\frac{dl}{F}$. When $dl = 1$ mm. and $F = 600$ m., quite a noticeable inclination of 1' merely introduces an error of 0.1" into h.

(*b*) The horizontality of the wires of the reticle must also be adjusted. This is done by a photographic reduction on glass of a large scale drawing (Fig. 24). It comprises 10 parallel lines (*), in three groups, within which the lines are equidistant; the interval between the groups is equal to 1 ½ times the interval between the lines; the latter is thus a tenth of the interval between the outermost lines, which is about 5'. The circular rim of the reticle is graduated in 5 degree units, concentric with the field, which enables one to make the image of the star, the position angle of which will have been already worked out, pass through the centre of the figure. This eliminates the error which would otherwise enter into the observed altitude if the wires were not horizontal.

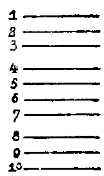


Fig. 24

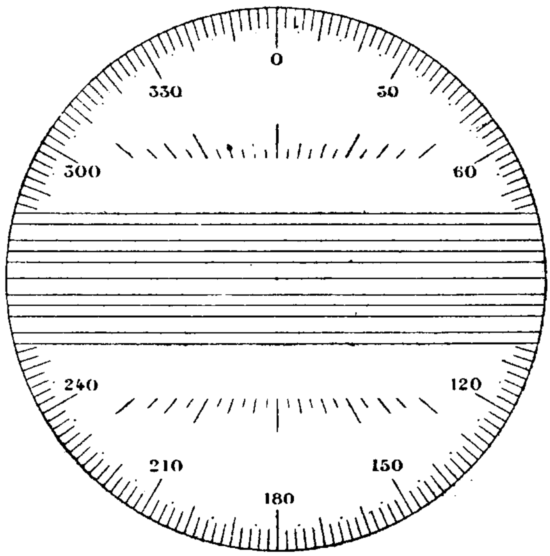


Fig. 25

For convenience, however, they are made horizontal in the following way. A star is made to enter in the field by the graduation A corresponding to its position angle. It leaves by the graduation $A' = A + 180^\circ + dA$. Then the reticle is turned in the proper direction through the angle $\frac{dA}{2}$, after which it must be verified that the star, entering by the graduation corresponding to it, actually leaves by that graduation which is diametrically opposite.

Instrumental conditions.

(37) (a) *Shape of the mercury trough.* One diameter of the surface of the bath must remain invariable when the apparatus is tilted.

For this purpose we have assumed that the trough had a cylindrical boundary. One might equally well assume that near the surface of the mercury it had a spherical boundary, the reflecting part occupying a diametrical plane of the sphere.

We shall see that a segment of a hollow sphere of large radius R could also be taken as a trough. Let e be the maximum depth of the space occupied by the mercury (Fig. 26).

(*) For some time M. de la Baume Pluvinel has been using a reticle having 10 wires spaced symmetrically about an eleventh, central wire (Fig. 25). The periphery is graduated in 2° units. The use of this reticle obviously makes no difference to the general reasoning, merely altering the coefficients of para. 39.

When the telescope is tilted, the surface of the mercury remains tangent to a sphere with the same centre O as the trough and of radius $R - e$.

The centre of the mercury no longer occupies the same position with respect to the remainder of the apparatus.

It assumes a distance $2(R - e)\sin^2\frac{\alpha}{2}$ from the initial surface line of the mercury, which we suppose to have been carried round by the rotation. The distance of nearly $R\frac{\alpha^2}{2}$ is negligible with the values of R adopted in standard practice (with a radius of 1 metre it is 0.04 of a micron for an inclination α of 1').

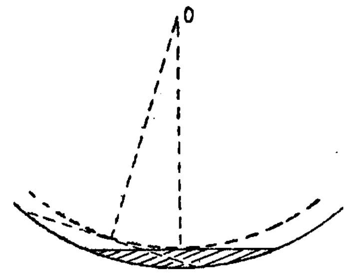


Fig. 26

It can be shown that a trough with a conical boundary near its surface would give an error of the same order.

(b) *Constancy of the level of the mercury.* If the reflecting surface rises or falls by an amount m, the image of a star seen at an altitude h is moved by $dh = \frac{2m \cos h}{F}$. The instrumental altitude changes by this quantity. When $h = 60^\circ$ and $F = 600$ m., we have $(dh)'' = 343\,m$ nearly, m being expressed in millimetres.

If $m = \frac{1}{343}$mm., or about 3 microns, the instrumental altitude varies by one second.

Thus, in the course of a series of observations, the position of the surface of the mercury with respect to the telescope mounting must have a degree of fixity of the order of a micron.

It is therefore absolutely essential not to touch the trough during observations, a precaution which is unnecessary with other equal-altitude instruments.

It should be noted that expansion of the telescope has only a secondary influence. In fact, if an increase of temperature makes the part of the mounting carrying the object-glass and the reticle move further from the bath, the distance between these two parts also increases in the same ratio, and the angle of observation does not alter appreciably.

On the other hand, it is possible that oxidation of the mercury may give rise to a film on its surface which will make the instrumental altitude vary slowly.

If the observations have been prepared for a given altitude, corresponding to a determined position of the reflecting surface, it is necessary, for carrying out the observations, to be able to bring the mercury to this position within a few tenths of a millimetre (a millimetre corresponds to about 6' of arc).

The arrangement of the trough enables this condition to be satisfied. (Fig. 27). Its sides are vertical, which is the most suitable shape for reducing the errors due to errors of levelling, and the centre part of its bottom is raised.

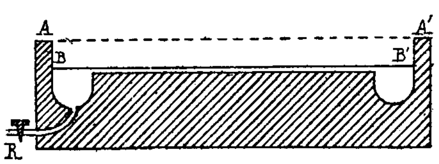


Fig. 27

Mercury is poured in up to the edge AA', the surface of the mercury is carefully cleaned, then, by means of the tap R, a determined quantity of mercury is removed, which brings the surface of the bath down to BB', very near the raised bottom of the trough and always practically to the same level.

Preparation of the observations.

(38) The bent telescope being an equal-altitude instrument, the taking and working out of the observations are broadly the same as for the instruments already discussed.

We shall therefore deal only with the peculiarities introduced by its use in the preparation and reduction of the observations.

For preparation, besides the azimuth of the star and its approximate time of passage, it is necessary to determine the angle at the star of the position triangle, knowledge of which enables us, as we have seen, to make the image of the star pass through the centre of the field. There is no difficulty about this calculation and an accuracy within one degree is sufficient.

On the other hand, the sets of readings need not be as crowded as for observations with the prismatic astrolabe. In fact the necessity for illuminating the reticle fairly strongly does not permit one to observe stars dimmer than the fourth magnitude, and multiplication of sights of a single image gives good results while limiting the series of observations to a dozen stars well distributed in azimuth.

Reduction of the observations. (*)

(39) For working out the observations, the unknown instrumental altitude is taken to be that of the imaginary mean wire, i.e. the mean of the altitudes corresponding to the ten wires of the reticle.

With the majority of stars, the time of passage across the mean wire is obtained by striking an average of the observed times of passage across the ten wires.

This procedure is not sufficiently accurate for stars observable near the meridian, for they do not cross the field at a uniform speed. It is, however, permissible to assume that this speed is uniformly accelerated, and a correction is made to the observed times of passage in accordance with this hypothesis.

Let us suppose that from observations of time stars the angular distances $y_1, y_2 \ldots\ldots y_{10}$ (expressed in seconds of arc) of the ten wires from the mean wire have been determined.

The correction to be applied to the time of passage across the nth wire, in seconds of time, is

$$dt_n = \frac{\sin 1''}{30} \; y_n^2 \; \frac{d^2H}{dh^2}, \text{ where } H \text{ is the hour angle,}$$

whence
$$dt_n = \frac{\sin 1''}{30} \; y_n^2 \; \frac{\cot Z \;\; \cot S}{\cos h \cos \varphi \sin Z}.$$

The correction is the same in magnitude and sign for wires which are symmetrical about the mean wire.

It is preferable to correct each passage separately, rather than to apply the mean correction to the mean time of the passages

$$dt_m = \sum_1^{10} y^2 \; \frac{\sin 1''}{300} \frac{\cot Z \; \cot S}{\cos h \; \cos \varphi \; \sin Z},$$

but the latter becomes necessary when, owing to the inclination of the apparent trajectory of the image, it has not been possible to observe its passage across all the wires of the reticle.

(*) For notation, see § 75.

Supposing the reticle which we have described to exist, let us denote the interval (in minutes of arc) between the extreme wires by 10 Δh.

$$\text{Taking } k = \frac{\sin 1''}{120} \frac{(\Delta h)^2 \cot Z \cot S}{\cos \varphi \;\cos h \;\sin Z},$$

the times of passage will be corrected:

at wires	1 and 10,	by	100 k;
—	2 and 9,	—	64 k;
—	3 and 8,	—	36 k;
—	4 and 7,	—	9 k;
—	5 and 6,	—	k;

If observations are made at all the wires, the mean of the times of passage must be corrected by 42 k; it must be corrected by 5 k or k respectively if observations are made at the four or the two middle wires only.

The omission of this correction results in an error equal to the product of

$$dh = \frac{\sin 1''}{8} \frac{(\Delta h)^2 \cot Z \cot S}{\cos h}$$

and the factor f which may have a value of 42, 5 or 1, according to the wires across which observations have been made.

This circumstance depends on the position angle of the star, and consequently on its azimuth, but it is also a function of the ratio of the interval between the wires to the diameter of the field of the telescope.

For Paris, and assuming a field of 60 Δh, the following are the values of k and of fdh:–

Z	100k	*Number of wires*	f	fdh	S
5°	— 4ˢ.985	4 middle wires	5	0.22"	196°
10	— 0. 603	10 wires	42	0.44"	211
20	— 0 .065	10 wires	42	0.08"	233
30	— 0 .014	"			249
40	— 0 .003	"			261
50	0 .000	"			270
60	+ 0 .001	"			278
70	+ 0 .001	"			286
90	0 .000	"			299
110	— 0 .004	"			312
120	— 0 .008	"			319
130	— 0 .020	"			326
140	— 0 .038	"			332
150	— 0 .099	10 wires	42	0.21"	339
160	— 0 .351	4 middle wires	5	0.06"	346
170	— 2 .897	4 middle wires	5	0.24"	353
175	— 23 .336	2 middle wires	1	0.23"	356.5

As Z varies between 0 and 180 degrees, the quantity k is negative, then positive, then negative again; but it is only of importance near the meridian. Further, it can always be considered as negative to the eastward and positive to the westward. It always increases the absolute value of the hour angle.

It will be seen also that the correction k increases rapidly as the meridian is approached, and particularly to the southward (in the northern hemisphere). On the other hand fdh remains small and does not attain 0.5" at 5 degrees from the meridian (*).

The error in the altitude of observation thus remains small, and the correction could be dispensed with, particularly if the stars observed were near the meridian both to northward and southward.

But if, with a view to determining the latitude, one associates circummeridians, say, to the southward, for which it would be necessary to make a correction, with stars to the northward which are far enough from the meridian for one to be able to neglect the correction, then failure to correct the circummeridians introduces a systematic error in the calculated latitude.

Thus it is safer to apply the corrections discussed above to the observed times of passage.

RESULTS — CONCLUSIONS.

(40) M. de la BAUME PLUVINEL has had two bent telescopes made with focal lengths of 0.6 m. and 0.95 m. and apertures of 0.041 m. and 0.075 m. respectively.

For several years, fairly numerous series of observations have been made with these telescopes, particularly at the Observatory of Paris by different astronomers.

Each series, on an average, comprised some ten stars. An examination of the results shows that the bent telescope gives the latitude within about

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(*) This result may be explained as follows :

Imagine that there is a very large number of wires in the reticle — one every second of arc, for example.

For the wire at p" from the mean wire, the correction would be

$$dt_p = \frac{\sin 1''}{30} \quad p^2 \quad \frac{\cot Z \ \cot S}{\cos h \ \ \cos \varphi \ \ \sin Z}.$$

The correction for the mean of the $2n$ wires crossed by the star would therefore be

$$dt_m = \frac{\sin 1''}{30} \frac{\sum_1^n p^2}{n} \frac{\cot Z \ \cot S}{\cos h \ \cos \varphi \ \cos Z} = \frac{\sin 1''}{30} \frac{n(n+1)\ (2n+1)}{6} \frac{\cot Z \ \cot S}{\cos h \ \cos \varphi \ \sin Z}.$$

If this correction be neglected, an error is introduced into the observed angle of

$$dh = \frac{\sin 1''}{12} \ (n+1) \ (2n+1) \ \frac{\cot Z \cot S}{\cos \ h}.$$

If n is large enough, this gives nearly

$$dh = \frac{n^2 \ \sin 1''}{6} \frac{\cot Z \cot S}{\cos h}.$$

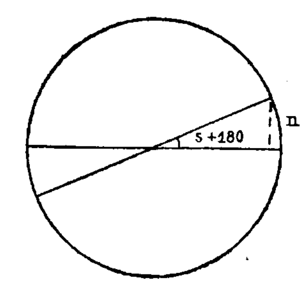

FIG. 28

But the number of wires crossed is a function of the angle at the star S and of the size of the field c (expressed in wire intervals) : $\tan S = \frac{2n}{c}$ (Fig. 28).

On the other hand, the declinations of the stars near the meridian do not differ very much and consequently $\sin Z$ varies nearly proportionally with $\sin S$. The same applies to $\tan Z$ which is thus sufficiently proportional to n.

Thus we can take, $\cot Z \ \cot S = \frac{q}{n^2}$, where q is nearly a constant. The result is that $dh = \frac{\sin 1''}{6} \ \frac{9}{\cos h}$ varies within very restricted limits.

0.5" and the time to within about 0.03^s (without allowing for the observer's personal equation) (*).

By observing a greater number of stars, the accuracy of the determinations would doubtless be increased.

Up to the present, the best graphical solutions obtainable do not appear more satisfactory than the best graphs furnished by observation, with the large model astrolabe, of a greater number of stars, indeed, but comprising a far smaller total of sights than with the bent telescope.

The altitude error of a star observation with this telescope is a little under 1". One would expect the fact of observing ten passages of a single star to enable one to obtain greater precision. The obstacle seems to be the width of the wires of the reticle, which is actually in the neighbourhood of 2".

By decreasing this, it would certainly appear that an improvement should be obtained.

In the course of the observations it was noticed that certain stars that seemed to have been well observed, both according to the appreciation of the operator and according to the comparison of the ten times of passage, gave an instrumental altitude differing by several seconds of arc (4 and even more) from that deduced from the other stars of the series.

The cause of these discrepancies remains unknown. Possibly it must be looked for in an accidental variation in the level of the mercury. We have seen that a vertical displacement of 3 microns in its surface led to an error of 1". A little dust, a little adhesion to the sides, a fleeting change in the surface layer, may at a given moment cause a change in the reflecting surface of the order of 0.01 mm., and so falsify the instrumental altitude by a few seconds.

Furthermore it has been found that observations made with a mercury trough prepared some days before gave a better mean difference for the calculated altitude than observations made with a trough which had only just been put in place.

These anomalies seem to have disappeared since the introduction of a trough with a flat amalgamated bottom and with conical non-amalgamated sides which M. DE LA BAUME PLUVINEL has recently designed.

Be that as it may, the bent telescope must figure among the equal altitude instruments capable of giving time and latitude with precision.

It will be well to add that its construction makes it very suitable for fitting with an impersonal micrometer, analogous to that of transit instruments.

F. *ACCESSORIES FOR OBSERVATIONS.*

(41) The precision of a determination of geographical position by the equal-altitude method depends not only on the instrument used for observation but also on the time-piece used, as well as the procedure adopted for noting the times of passage of the stars and the times of reception of wireless time signals.

It is not necessary to look for great precision in any one of these factors if the others do not enable a comparable degree of accuracy to be obtained. One must not, in fact, lose sight of the fact that an increase of accuracy

(*) Observations made at the time of the revision of longitudes in 1933 with the large-model bent telescope, fitted with a new micrometer and a new mercury trough, gave a mean error of 0.01^s in the time determination and 0.2" in the latitude determination.

entails as a rule an increase in the bulk of the material; the observations and computations also become longer and more delicate.

The methods in use for marking the time of passage of stars at the instrumental altitude are very varied. In practice that one will be used, as far as possible, which corresponds best to the ultimate object of the operation and to the material available.

The most rudimentary of them consists of a simple "stop" given by the observer to an assistant who reads the chronometer.

The most precise makes use of a *recording chronograph* which unrolls, at a speed of several centimetres per second, a sheet of paper on which are inscribed two series of electric signals, one controlled by a precision clock, the other by the observer by means of an appropriate switch.

Among the intermediate processes, that of the *eye and ear* was long in favour on account of its simplicity, but it requires a certain amount of training on the part of the observer; the latter listens to the ticking of the escapement of the time-piece, which is amplified if necessary by a microphone; he locates the instant of observation by estimating the fraction of the interval between the tick on either side of it.

Sometimes, *second-counters* are used; these are started at a determined second by the timepiece and stopped at the exact moment of the observation. The choice of secound-counters is of great importance, for they are of widely different sensitivities, varying from 1/4 or 1/5 to a few hundredths of a second.

With instruments for multiple sights, the various observations of a single star follow one another at rather short intervals, so that it is practically essential to employ a recording chronograph. Besides, this method of observation harmonises well with those equal-altitude instruments which are designed to obtain accurate results.

MANIPULATORS.

(42) In most of the recording processes the observer, at the instant when the phenomenon takes place, actuates an electric signal governing an inscribing stylus.

The manipulators used are generally analogous to the ordinary pear-shaped bell-push, whence the name *poire à tops* often given them. The observer opens or closes an electric circuit which contains the electro-magnet of the inscribing stylus by pressing with the thumb on a small bone button.

These manipulators must be well designed if great precision is desired.

In fact the travel of the button, before the electric phenomenon governed by it is released, is a primary cause of lag in the records.

There is another: before the button has got into motion, it has sunk into the observer's thumb, more or less according to its shape and the strength of the opposing spring.

These two lags in practice combine with the personal equation of the observer and risk being all the more variable the bigger they happen to be.

A manipulator closing a circuit generally requires a relatively long travel of the button, at least a few tenths of a millimetre, corresponding to several hundredths of a second. Preferable to this, therefore, is a circuit-breaking device, the effect of which is more immediate.

One can thus keep a certain amount of strength in the opposing spring, in keeping with the rather weak tactile sensitivity of the inner face of the thumb, and sufficient to avoid involuntary "stops" caused by false movements.

In particular, compression of the thumb will be avoided by making the button with a large surface area and slightly hollowing it out so that it roughly takes the form of the finger.

PART III.

ELIMINATION OF THE PERSONAL EQUATION.

A. *THE PERSONAL EQUATION.*

(43) It does not appear that, since the introduction of equal-altitude instruments, any very great importance has been attached to the measurement of the personal equations of the observers. True, there were only very vague notions about this systematic error, which is of the order of the tenth of a second of time for most observers. To eliminate this, it was thought sufficient to interchange the latter during determinations of differences of longitude made with prismatic astrolabes, which assumed implicitly that its variation with the time could be neglected.

From 1910, at which period the Bureau International de l'Heure was founded, B. BAILLAUD, Director of the Observatory of Paris, instituted frequent comparisons between the various transit instruments of the Observatory, so as to obtain greater accuracy in the determination of the time. Observations with the large model prismatic astrolabe were made on that occasion. In discussing 19 series of observations made in 1916 at the Observatory of Paris with this instrument, one of us stated categorically the existence of the personal equation and its order of magnitude and showed, from that moment, the necessity of adding to the astrolabe a special apparatus for measuring the observer's personal equation; (*) but circumstances did not allow such an instrument to be produced immediately, and it was only in 1921 that the first apparatus for measuring absolute personal equations was constructed to the plans of FAVÉ, Ingénieur hydrographe en chef. The death of FAVÉ in 1922, before the apparatus was completely ready, prevented its being brought into use.

At the time of the International Longitude Determination of 1926, an apparatus of which the principle was analogous to that of FAVÉ, was used at Dehra Dun by the officers of the Geodetic Survey of India.

It was only in 1927, in view of the new International Longitude Determination, and under the encouragement of General BELLOT, Director of the Army Geographical Service, in Paris, that a certain number of appliances for measuring the absolute personal equation were tested. Their construction lasted until 1933.

In 1932, M. BUCHAR, a Czecho-Slovak astronomer, also produced an instrument for correcting observations with the circumzenithal, in the workshops of the Military Geographical Institute at Prague.

(44) Before describing these instruments, we will try to clarify our notions somewhat on the subject of the personal equation, so as to bring out the conditions which they must fulfil and the proper method of using them.

The personal equation appears in the form of a difference between the time correction determined by equal-altitude observations and the most probable time correction, deduced from transit instruments fitted with impersonal micrometers. It is, on the average, fairly stable for a given observer always using the same recording material, and it generally corresponds to a lag in the observation.

At the time of a sight, the moment when the pen of the recording chronograph deviates is not exactly that at which the physical phenomenon to

(*) Rapport Annuel de l'Observatoire de Paris pour 1917, p. 9.

be marked takes place. The time interval separating these two instants depends both on the recording device and on the reaction of the observer; the latter is influenced probably by his physiological condition, by the speed of approach of the images, and by their brightness.

Various hypotheses have been propounded as to the part attributable to each of these various factors; but hitherto it has not been possible to verify them, observations of real stars containing too many different causes of errors for individual ones among them to be isolated. Observations of fictitious stars, repeated a great many times under identical conditions so as to eliminate accidental errors, are alone capable of making it possible to analyse the complex total constituted by a series of real stars.

The determination of the reaction of the observer as a function of all the variables which may influence him is of great interest, but it entails a large number of observations of fictitious stars. Knowledge of it would enable us to correct every sight on a real star, and no doubt in the result of a series thus treated one would find an improvement manifested in the diminution of the mean error of an observation, and in the elimination of the personal equation, this latter point being by far the more important.

If it is desired merely to get rid of it, the matter becomes simpler. In fact, the personal equation, as we have defined it above, apart from any other hypothesis, only affects the time correction which is itself deduced principally from the passage of time stars. We may therefore consider that the personal equation is equivalent to the mean lag arising in the observation of these stars, and consequently that it is sufficient, to obtain it, to determine the reaction of the observer corresponding to the time stars.

These two points of view have led to the production of two different types of apparatus for measuring the absolute personal equation; one type enables all the real observations to be reproduced; the others only give a fictitious star, the movement of which is analogous to that of the time stars.

PRINCIPLE OF THE APPLIANCES FOR MEASURING ABSOLUTE PERSONAL EQUATION.

(45) The common principle of all these instruments is as follows:–

An optical device comprising movable parts enables an observer to see the same phenomenon as if he were regarding a real star through an equal-altitude instrument (other than the bent telescope).

The observer first of all very slowly works the system moving the fictitious star until the two images appear to him as exactly as possible upon the same horizontal. He then makes an adjustable electrical contact, which is governed by the movement of the movable system, traverse until it acts when the position just found for the latter is reached. The position of adjustment of the contact is appreciated by the noise produced in the headphone or by a sharp deviation of the needle of a galvanometer. This contact then gives a material indication of the coincidence of the images; it forms part of the circuit of one of the pens of a recording chronograph, another pen being controlled by the observer. Its position, called the zero of the apparatus, must be obtained with an accuracy corresponding to 0.01^s for the speeds at which the apparatus is used. It must also be checked frequently.

When the zero setting has been made, the fictitious star is made to leave the field, then it is made to cross it at the apparent speed of a real star; the electric contact actuates its own pen at the instant of coincidence, and the observer marks the phenomenon on his part; the difference of time between these two "stops" is an isolated value of the personal equation. A minimum of some thirty measurements is required to obtain it to within

0.02^s. It is important that during these determinations the recording material, particularly the manipulator (*poire à tops*), should be the same as the one used for observations of real stars.

(46) Appliances for measuring the absolute personal equation fall into two categories.

In one, the fictitious star is seen through the equal-altitude instrument.

The fictitious star consists of a point of light either some metres from the instrument or at the focus of a collimator which directs a beam of parallel rays on to the instrument.

The point of light may also be either movable or stationary. In the latter case a movable optical device, which will be referred to as a *deviator*, enables the rays directed on to the equal-altitude instrument to be displaced.

The electric contact for marking the coincidences generally has one pole solid with the moving parts, the other being carried on the frame of the instrument.

This type of instrument calls for the following comments.

(*a*) The movements of the source of light or of the deviator are such that the beam falling on the equal-altitude instrument describes a plane passing through the optical axis of the telescope. Thus, by merely rotating the movable system, one can make the beam received by the telescope sweep any plane passing through its optical axis, and so obtain fictitious stars with an apparent motion analogous to that of any real star, whether circummeridian or time.

(*b*) If, for observing fictitious stars, the instrument be used with its mercury trough, the installation entails relatively heavy work. In fact, for a given position of the movable system and particularly for that corresponding to zero, the incident beam must make an angle with the vertical which will not vary by more than $0.1''$ during the course of the observations.

When the apparatus includes a collimator, the latter must be installed on a solid foundation; in the case of an apparatus with a distant point of light, the same applies to the mirror which directs the rays from the fictitious star on to the instrument.

It seems, however, that in appliances fitted with a collimator, the use of a solid base can be dispensed with by substituting for the mercury trough of the equal-altitude instrument a roughly horizontal mirror fixed on the frame of the collimator. The whole of the apparatus can move, but the constancy of the angle of observation is maintained if, in a given position of the movable system, the angle between the rays issuing from the collimator and the horizontal does not vary in the course of the experiments.

(*c*) The zero adjustment is carried out by bringing the movable system to such a position that the two images of the fictitious star seen in the instrument appear on the same horizontal. It is thus obtained with the same magnification as is used for observation, and generally one cannot make use of any greater magnification to obtain it.

(47) The other appliances for measuring absolute personal equation do not make use of the equal-altitude instrument.

The two images of the fictitious star are represented by two points of light which can move with symmetrical speeds along closely adjacent verticals. The dimensions of the appliance are so arranged as to give the operator the same impression of speed of approach, brightness and interval between images as when he is observing real time stars.

The observation is made with the naked eye or through a low-power telescope. The latter is better, for when observing with the naked eye the whole width of the pupil is utilised and the images are only good if the observer has no defect of eyesight.

The movable parts on which the points of light are fixed each carry one of the poles of the electric contact which automatically marks the coincidences.

In appliances of this type, the zero can be determined very efficiently; to make the adjustment the observer brings the two images on to the same horizontal, coming as close as possible to the instrument, which is equivalent to the use of a much higher power than that used for observation.

Doubts have been cast on the results obtainable from these appliances, the observations being made under unreal conditions. But it is only the physiological part of the personal equation which is in question, and the latter of necessity is reproduced identically when the appearances are the same, whatever may be the optics outside the eye and the method of producing the images. Experience has confirmed the legitimacy of this hypothesis, as is shown by the comparative table in § 15.

APPLIANCES USING A COLLIMATOR.

Favé apparatus.

(48) The apparatus designed by L. FAVÉ(*) for measuring absolute personal equations comprises a collimator of 0.5 m. focal length, inclined 30° to the vertical and arranged so as to direct rays of light on to the prism and the mercury trough of an astrolabe.

A lens gives the true image of a very small hole in a metal sheet, illuminated from behind, in the focal plane of the collimator.

A micrometer screw, driven by an electric motor, enables the perforated sheet to be moved in its own plane, and so to vary the direction of the rays of light from the collimator. The screw carries a drum fitted with a contact stud which completes an electric circuit when it passes across a flexible blade fixed to the frame of the apparatus. The drum is held by friction, so that the contact can be made to take place when the position of the lighted hole is such that the two images are seen on the same horizontal in the astrolabe.

This apparatus has not proved usable in practice, in spite of the makers' efforts, on account of the instability of the contact. This arose partly through vibration from the motor, the connection between which and the micrometer thread was too rigid. It should be added that the small dimensions of the collimator entailed a degree of mechanical precision for the movable parts which was difficult to obtain.

Survey of India apparatus.

(49) This apparatus, made in 1926 by Captain G. BOMFORD, is also intended to be used with a prismatic astrolabe.

It closely resembles the last instrument. The use of a larger collimator, differences in the transmission from the motor, in the motion of the source of light and in the making of the contact, resulted during the 1926 longitude determinations in a reasonably satisfactory performance. At the same time, it was found that there was some slip in the zero position during the experiments, which forbade absolute confidence being placed in the personal equations obtained. Captain BOMFORD is therefore designing a new appliance.

Apparatus by Colonel Hurault (S.O.M.).

(50) This apparatus was manufactured by the *Société d'Optique et de Mécanique* for the Army Geographical Service in Paris (Fig. 29).

(*) Manufactured by the *Société d'Optique et de Mécanique de précision* (S.O.M.).

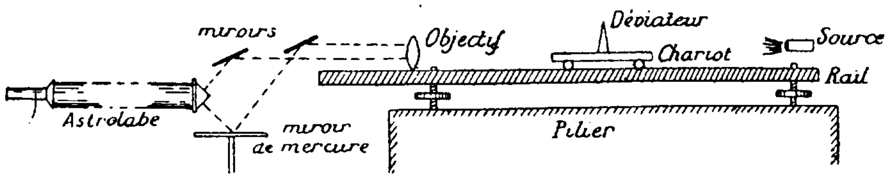

FIG. 29 — *Hurault-S.O.M. apparatus.*

The collimator, with a focal length of about a metre, is mounted horizontally on a masonry plinth. The object-glass and the source of light are fixed on a frame in the form of an optical bench, which in essentials comprises two horizontal steel rods, supported by four levelling screws. Sensitive levels enable the rods to be laid truly horizontal. The rods serve as rails for a trolley carrying the deviator, a wedge-shaped prism placed normally to the optical axis of the collimator. This prism can thus be moved parallel to the optical axis between the focus and the object lens of the collimator, the effect of which is to displace the rays from the collimator in a plane perpendicular to the edge of the prism. The prism is lightly held by friction in a cylindrical mounting, so that it can be slewed in any direction within a degree or so in its own plane.

At the end of the rails outside the collimator, two plain steel mirrors are arranged so as to reflect downwards, at 30° from the vertical, the rays coming from the collimator, which can thus be received in an astrolabe. The use of two mirrors enables the prism and mercury trough to be covered without having to make the object-glass of the collimator unduly large.

The contact automatically marking the coincidences is attached to the frame, along which it can be moved. A spur on the trolley opens it in the position of the deviator corresponding to the coincidence. There are actually two contacts, one for each direction of motion.

The zero setting is facilitated by reading a vernier carried by the trolley alongside a graduated scale fixed to the frame.

A simple electrical device enables also the speed of translation of the deviator during the experiments to be measured. This translation was originally done by means of an endless thread parallel to the optical axis of the collimator, turning in a nut to which the trolley was attached. Rotation was obtained by means of an electric motor through a geared drive.

Experience showed that the travel of the images appeared to be jerky; the screw was abolished and the trolley driven directly by a belt, geared down by the pulleys.

This transmission made the progression of the images considerably more regular; but the values obtained for the personal equations still presented numerous anomalies, showing that the instrument has not yet finally reached perfection.

Jobin-Yvon apparatus.

(51) This instrument can be installed at the actual site of observation; this has the advantage that fictitious measurements can be introduced without difficulty in the middle of an astronomical series. The only stars that cannot be observed are those few which cut the small circle 30° distant from the zenith at a distance of two or three degrees from the meridian.

On the pillar of the astrolabe itself, beneath the latter and in the plane of the meridian, is arranged a horizontal collimator of about 0.8 m. focal length, bent 90° on the object-glass side so that the rays leave it vertically upwards (Fig. 30).

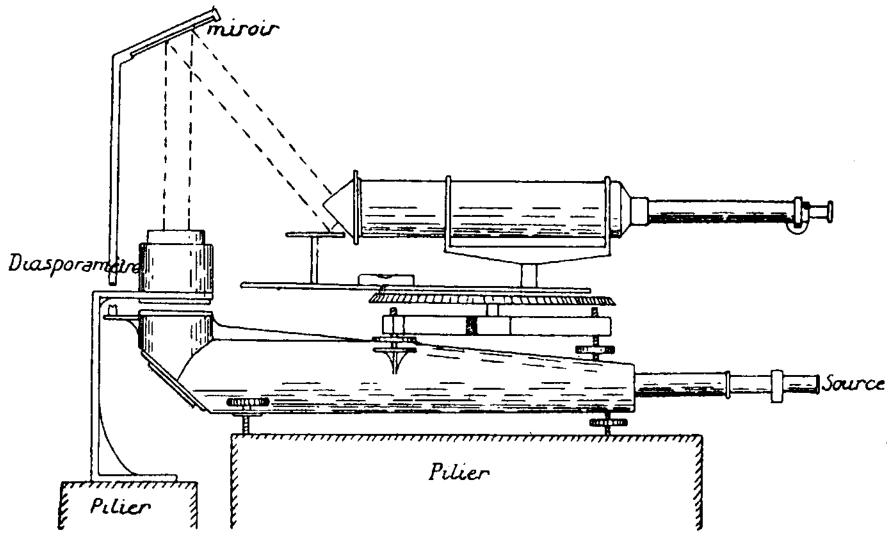

FIG. 30 — *Jobin-Yvon apparatus.*

Above the object-glass and quite near it is a diasporameter carried by an arm recessed into a pillar independent of that of the astrolabe.

The two thin prisms of the diasporameter are horizontal and fixed in cylindrical mountings which a suitably geared electric motor turns concentrically with themselves and in opposite directions, the prisms remaining in their own plane. The effect of this rotation is to displace the rays of light from the collimator in parallelism with the vertical plane which passes through the bisector of the edges of the prisms of the diasporameter. According to the initial orientation given to this bisector one can thus sweep any vertical; that which passes through the optical axis of the collimator and of the astrolabe corresponds to fictitious time stars.

The rays of light, leaving the diasporameter in an upward direction, are directed on to the astrolabe by an adjustable mirror fixed to the collimator.

The contacts marking the coincidences are two in number, for a complete turn of the diasporameter produces two coincidences. They are fixed to the mountings of the thin prisms; a spur on each of the turning parts actuates the corresponding contact; these are breaking contacts; they have been designed to obtain the maximum of precision in spite of the relatively slow movement of the spurs.

The source of light, in the focal plane of the collimator, is the reduced image of a small, brightly lighted hole. A micrometer device enables this image to be displaced in the focal plane to bring the images into coincidence in the astrolabe, at which time one of the spurs comes into such a position that it performs the contact-breaking functions for which it is responsible.

It is important to verify this adjustment, for after a series of measurements it is not uncommon to find it out of step, this error, translated into time, being capable of reaching a few hundredths of a second.

APPARATUS WITH DISTANT POINT OF LIGHT.

Apparatus by M. Buchar.

(52) This apparatus, constructed in the workshops of the Military Geographical Institute at Prague, is intended more especially for measuring the

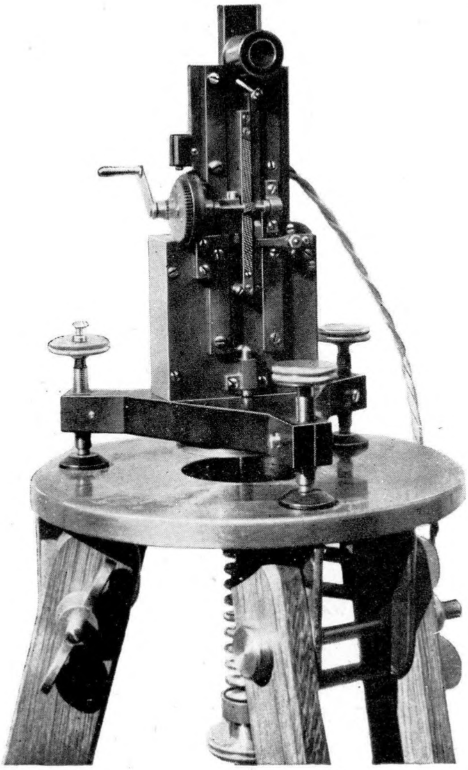

FIG. 31

BUCHAR APPARATUS.
APPAREIL BUCHAR

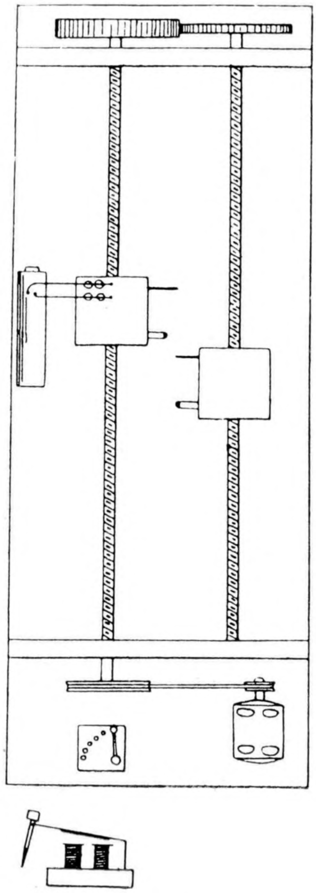

FIG. 32

HURAULT-CHASSELON APPARATUS.
APPAREIL HURAULT-CHASSELON

absolute personal equation of observers with the circumzenithal, but it could be associated with nearly all the equal-altitude instruments.

It comprises an artificial star consisting of a hole in a metal sheet, lighted from behind. The star is fixed on a rack, which can be moved vertically by a handle. The latter also works an ebonite disc carrying an electric contact. The whole is mounted on a support with levelling screws, on a solid tripod. (Fig. 31).

The horizontal rays coming from this apparatus are reflected on to the circumzenithal by a plane mirror, situated near the instrument of observation, and movable by means of a micrometer screw.

This screw is worked to bring the images into coincidence in the circumzenithal, when the artificial star is in the position in which closing of the contact takes place.

The artificial star is some forty metres from the instrument of observation. This distance is sufficient to make it possible to neglect the errors due to the fact that the rays coming from the star and falling on the apparatus are not rigorously parallel.

With this instrument M. BUCHAR has made numerous measurements with a view more especially to determining the laws by which the personal equation varies with the speed of approach of the images and with their brightness.

This apparatus should give good results if care be taken that the mirror which sends the rays from the artificial star back on to the circumzenithal makes a rigorously fixed angle with the vertical, once its position has been adjusted by the aid of the micrometer screw.

DIRECT OBSERVATION APPARATUS.

Apparatus by Colonel Hurault, manufactured by Chvsselon.

(53) This apparatus comprises a frame about 2 metres (6 ft. 7 in.) long, fixed vertically against a wall and supporting two fairly finely threaded vertical screws, which are turned by an electric motor in opposite directions (Fig. 32).

A guided nut, in which is a little hole lighted by an electric bulb, moves along each of them. The two points of light move in opposite directions along neighbouring verticals like the two images of a time star in the astrolabe.

The observer takes up a position some ten metres away and looks at these two points through a very low-power telescope. The speed of rotation of the screws and the distance of the observer have been worked out so that he has the same impression of the speed of the images as if he were looking at a time star in his astrolabe.

When the two lighted holes are at the same height, the ends of two flexible steel blades come into contact, thus closing the circuit which automatically marks the coincidences. The blades forming the contact are adjustable; their position is set for one direction of motion of the images, and for the other direction a correction must be applied to the automatic "stop" equal to the length of time taken by the points of light to travel the thickness of one of the blades.

Two electromagnets carried on one of the screws inscribe the automatic and the observer's "stop" on a sheet of paper fixed to the frame. Thus the use of a recording chronograph can be dispensed with.

This instrument has given excellent results; of the four appliances tested by the Army Geographical Service in Paris, this is the one which gave the smallest mean difference for an isolated point (0.03^s).

Apparatus by MM. Claude and Driencourt, manufactured by S.O.M.

(54) This differs from the last instrument chiefly in its much smaller bulk, which makes it easily transportable.

It comprises a vertical mounting, some 30 centimetres (11 3/4 in.) in height, acting as a guide for two racks moved by a handle in opposite directions at a speed of 1 cm. per revolution of the handle (Figs. 33 and 34).

On the racks, two holes 0.3 mm. in diameter, lighted by small electric bulbs, form the images to be observed.

The operator looks at them through a blackened cardboard tube set horizontally. He takes up a position one or two metres from the instrument, so as to obtain, for a rate of working of one turn of the handle per second, the same apparent speed of the images as that of the time stars in an astronomical instrument.

The setting is done by bringing the two holes to the same height; a vernier on one of the racks moves along a graduation engraved on the other, and in this way several readings are taken corresponding to the horizontal position of the line passing through the two points of light; the racks are then set at the mean reading with a grub screw.

Then the automatic contact for the coincidence, which functions by breaking, is adjusted. The contact is carried on one rack and the spur which opens it is on the other. There are two of these contacts, one for each direction of motion. They can be adjusted to within 1/20 mm., corresponding to 0.005^s — an amply sufficient degree of accuracy.

This is the apparatus which was adopted by the Bureau of Longitudes for determining the absolute personal equations of the observers with the prismatic astrolabe who took part in the International Determination of Longitude in 1933.

As in all appliances where points of light are observed without any interposed optical device, the images appear bad if the observer's eyesight is not perfect; so it is an advantage to look at them through a pin-hole of 3/4 to 1 mm. in diameter, while slightly illuminating the field so as to facilitate observation.

COMPARISON OF RESULTS.

(55) The following table summarizes the personal equations obtained by meansof the three appliances which we have more especially examined in collaboration with Commandant de VOLONTAT of the Army Geographical Service.

Observers.	*Jobin-Yvon Apparatus.*	*Hurault-Chasselon Apparatus.*	*Claude and Driencourt (S.O.M.) Apparatus.*
C	$+ 0.04^s$	$+ 0.02^s$	$+ 0.01^s$
V	+ 0.19	+ 0.13	+ 0.18
G	+ 0.12	+ 0.09	+ 0.10

N. B. — The observers all used the same manipulator.

The close agreement between the figures found by each separate observer show that the three appliances considered are equally suitable for measuring the absolute personal equation in observations with the prismatic astrolabe.

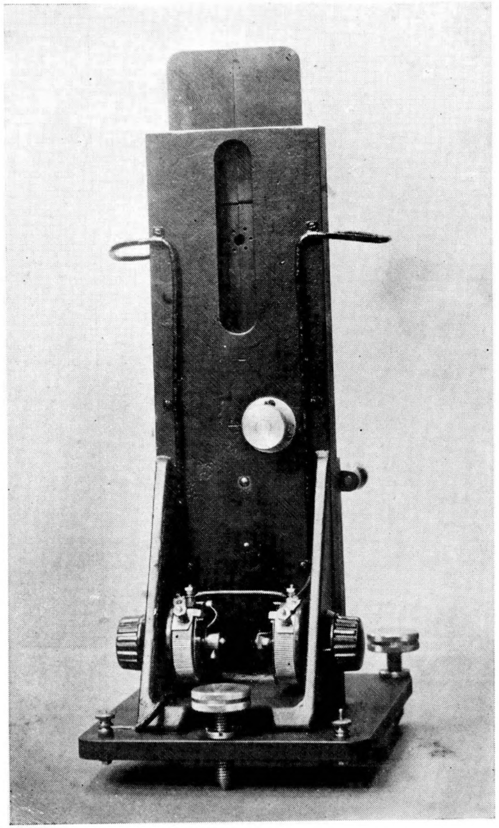

Fig. 33

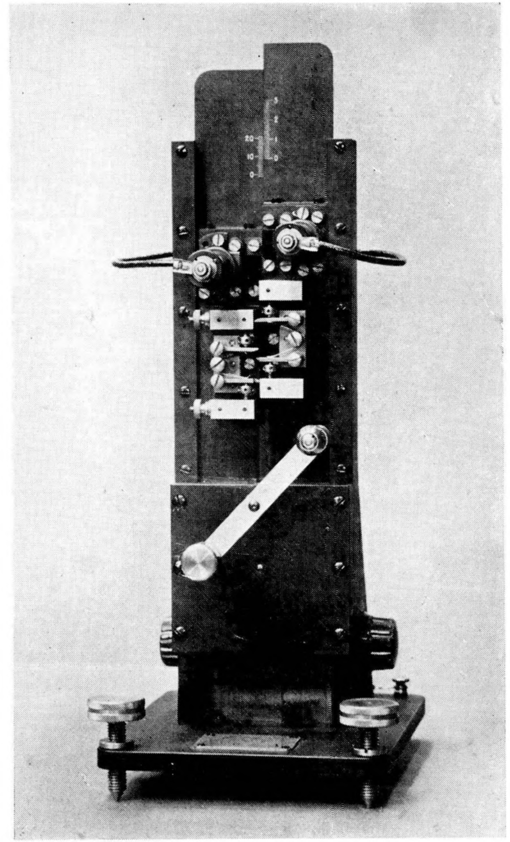

Fig. 34

CLAUDE AND DRIENCOURT APPARATUS. — APPAREIL CLAUDE ET DRIENCOURT

Thanks to their use the astrolabe will, from now on, be usable for determining the absolute time, and thus to check the results furnished by transit telescopes fitted with moving-wire micrometers.

An examination of the foregoing table further leads to the conclusion that for the pure and simple determination of personal equation, *appliances making use of direct observation are sufficient.*

The JOBIN-YVON apparatus has the additional interest of allowing a more complete study of the observer's reactions, and of establishing the law of variation of the personal equation of a given observer, as a function of the speed and the magnitude of the star.

PRECAUTIONS WHEN OPERATING.

(56) In measurements of the personal equation, it is necessary not only to use with the fictitious stars a method of notation of the times of passage identical with that adopted for the real stars, but also to use the same reading instruments (second-counter, manipulators, chronographs, etc.).

The notion of an observer's personal equation really groups together all the systematic errors affecting the determination of time, imputable both to the observer and to his observing material; in taking the measurements it is necessary as far as possible always to associate the observer with his usual material, particularly in the case of the manipulator.

The appliances with which personal equation is measured are usually designed to accompany precision observations, and always presuppose a recording chronograph. In general the same chronograph will be used for the measurements and for the observations.

But a traveller who does not carry a chronograph and who desires to correct his observations by the mean of his personal equation, *measured before and after his field work,* must proceed as follows. The exact instant of passage of the fictitious star is recorded on a chronograph concurrently with the ticks of a time-piece placed near it, preferably the one the observer is going to use.

The latter on his part determines the time at which he sees the fictitious star pass, with reference to this time-piece and by the same procedure as he uses in the field. The difference between the two times of passage thus obtained gives a value of the personal equation under observational conditions.

B. *IMPERSONAL APPARATUS.*

(57) It is evident that instead of measuring the personal equations of observers it would be more practical and more accurate to obtain observations which were free from this systematic error.

To make equal-altitude instruments impersonal, it is necessary either to provide the instruments with reticles with a moving-wire micrometer, such as the bent telescope, or else, in coincidence instruments, to place in the track of the rays of light a movable deviator the effect of which will compensate for the movement of the star in altitude, this deviator also completing an electric circuit when it passes through a well-determined position defining the instrumental altitude.

(58) Such appliances are hard to produce, mainly because the mechanical arrangements entail movable pieces which communicate vibration to the mercury trough fixed to the instrument.

In the early models of circumzenithal, the mercury trough was on a central pillar and the telescope was movable on another concentric pillar. One may yet be led to reverting to this arrangement in the construction of impersonal appliances.

That perfected by M. R. BAILLAUD, which seems to have assumed a more or less final form, carries on one same pillar a heavy mass of cast iron, serving as cradle for the telescope, and a thick concentric collar, also of cast iron, in which the mercury trough moves. This conception appears to be sufficient, at least as regards absence of vibration.

The use of synchronous electric motors or of clockwork movements, isolated simply from the apparatus by a rubber mat, would perhaps suffice to remove the mercury from the influence of vibration; the trough could also be supported by rubber. This solution would have the advantage of abolishing the delicate form of transmission necessitated by a motor placed on a separate frame.

(59) In an impersonal equal-altitude instrument, there must be a movable deviator, having attached to it several contacts corresponding to different altitudes of observation. It is thus an apparatus for multiple sights.

But as a result of the introduction of this deviator and the play entailed in its use, it is no longer possible, as regards the constancy of the instrumental altitude, to benefit by the perfection of the shape of the prism of the astrolabes. It seems preferable, then, not to retain the optical defect inseparable from the use of this prism, but to fall back on the solution of Messrs. NUŠL and FRIČ by using two crossed mirrors arranged as in the circumzenithal and giving horizontally deformed images in the telescope.

(60) It can further be shown that it improves the gear ratio, and consequently the accuracy, to place the movable deviator between the star and the mirrors. The axes of rotation of the deviator must also be very rigidly attached to the mirrors. Changes in the position of these parts may introduce considerable errors.

(61) As in transit instruments with impersonal micrometers, the range of speed required of the motion of the deviator must extend from dead slow up to that corresponding to stars of declination nil, observed from a point on the terrestrial equator.

All the same, in equal-altitude instruments, the mechanical part can be greatly simplified by confining oneself to observations of only those stars which are situated at about 45° on either side of the N.-S. meridian. The speed must be capable of variation in ratio from 1 to 2 for displacements in latitude between the equator and the parallels of 60°.

At a given station, differences of azimuth correspond to very slight changes of speed, 10 % for a difference of 5°.

APPLIANCES UNDER DESIGN.

(62) Messrs. NUŠL and FRIČ have tried to adapt to the circumzenithal a system of two lenses of low curvature sliding in their own plane under the action of a micrometer screw fitted with electrical contacts.

The first trials were hindered by a lack of solidarity between these lenses and the crossed mirrors.

At Nice, in 1924, M. MICHKOVITCH fitted a prismatic astrolabe with a moving-wire micrometer. With the wire horizontal, the observer bisected the rising image before the coincidence, the falling image after it. In spite of results which appeared satisfactory, the designer did not pursue the matter.

M. CHRÉTIEN has recently proposed the use as deviator of a diasporameter, placed normally to the rays coming from the star, and covering both the prism and the mercury trough of an astrolabe. It is also his idea to replace the prism by a system of two crossed mirrors, one being only half silvered in vertical strips.

APPARATUS BY M. RENÉ BAILLAUD.

Principle.

(63) Near the object-glass O of the telescope of an astrolabe, between the object-glass and the eyepiece, M. R. BAILLAUD has inserted two small panes of glass L, of the same thickness and coefficient of refraction, with plane-parallel faces (Fig. 35). They are perpendicular to the vertical plane through the optical axis A of the telescope; one of them covers the upper half, the other the lower half, of the object-glass. Each of them can turn round a horizontal axis a parallel to its own plane and perpendicular to the plane of the figure.

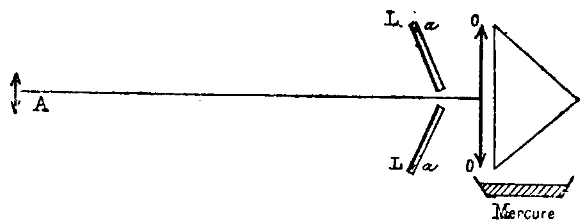

FIG. 35

Let us first make the two panes perpendicular to the optical axis; the position of the focal plane is not exactly the same as if the panes were removed; but when observing in the new focal plane the instant of coincidence of the two images formed by a star is not changed.

Let us turn the two panes in opposite directions through very small equal angles, and leave them fixed. Coincidence no longer takes place at the same instant, the rays of light which form the direct image are raised or lowered parallel to themselves, and those which produce the reflected image are lowered or raised by the same amount.

The time of coincidence of the two images is thus altered, as well as the apparent altitude of observation. At the same time the focus is slightly different, since the images are no longer formed in the same plane perpendicular to the optical axis; the position of this plane varies by a very small and almost imperceptible quantity, if the angle of rotation of the panes is very small.

In any case, the slight alteration in the instrumental altitude entailed by this alteration of focus is superimposed on the new apparent altitude of observation, and the sum gives an altitude which is the same for all stars observed in this position of the panes.

Let us now move the panes in opposite directions, with a uniform movement, and with a speed equal to the speed in altitude of the images in the field. For a given latitude, this speed depends solely on the azimuth of the star.

The two images then appear to be fixed in altitude, but their motion in azimuth is not altered.

If we start to turn the panes at the moment when the images are on the same horizontal, they thus remain there, for practical purposes, for the duration of the rotation.

The mechanical parts solid with the parallel-faced panes bear a series of electrical contacts, interposed in the circuit of a chronograph, which make it possible to record the time of passage of a star at a certain number of altitudes relative to well-determined positions of the panes, which are consequently always the same for the various stars. The mean of the times thus obtained corresponds theoretically to one and the same apparent altitude for all the stars of the series.

Description of the instrument.

(64) The equal-altitude instrument used (JOBIN medium model) was modifide as follows (*). A sort of rectangular box containing the panes was placed immediately behind the object-glass.

The thickness of the panes was made equal to 12 mm., following a calculation the object of which was to determine the value of this quantity corresponding to a minimum alteration of focus during the rotation, and so to a minimum of astigmatism.

The mounting of each pane is fixed to a very rigid and very long metal rod *T* (Fig. 36).

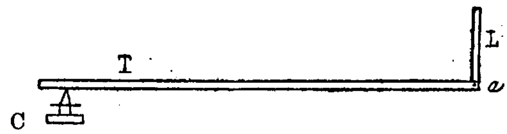

FIG. 36

In its movement about the axis *a*, perpendicular to the plane of the figure, the rod *T* moves in a vertical plane parallel to the optical axis of the object-glass.

The end of each of the rods *T* rests on a knife-edge *C* fixed on a nut *E* in which a driving screw *V* can turn (Fig. 37).

The parts of the screw actuating the two nuts are threaded in opposite directions. When the screw turns, the rods are moved in opposite directions; the two panes then turn through the same angle in opposite directions. They are adjusted also so as to remain symmetrical with respect to the horizontal plane of the optical axis.

The screw is driven by a motor fixed on a frame on the north side (†) of the pillar of the instrument, through a cardan shaft driving a pinion situated on the vertical axis of rotation of the astrolabe.

A governor enables the speed of the motor to be regulated and the screw to be given the desired angular velocity.

This motor is reversible, for the panes must move in opposite directions for easterly or westerly observations.

A mercury switch cuts off the motor current when the nuts are at the end of their travel.

A differential acting directly on the screw enables the position of the panes to be corrected during their rotation, so as to keep the two images constantly on the same horizontal.

This result, actually, is not rigorously obtained *a priori* for several reasons :–

(*a*) The movement of the star is not absolutely uniform;

(*b*) The motor is not started at the exact moment of coincidence.

(*c*) Finally it is impossible to give the motor the speed strictly corresponding to the movement of the star.

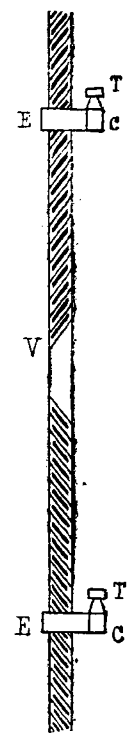

FIG. 37

On a drum on the screw is a stud which at each revolution makes the electric contact which closes the circuit of the chronograph. To avoid the functioning of the motor causing the apparatus and the mercury trough to vibrate, which would spoil the images, the support of the telescope has been

(*) Manufacturer, E. BOUTY.

(†) This position, as well as the south side of the pillar, causes the least interference with the observations.

made solid with the telescope itself; the latter is secured by two collars rigidly attached to the movable horizontal turntable. Further, M. BAILLAUD has completely separated the mercury trough from the body of the astrolabe; it can be moved on a heavy circular frame surrounding the socket of the instrument.

Possible errors.

(65) The errors to be avoided are more especially play of the panes in their mountings and play of the axes of the panes in their trunnions. The latter would have the same effect as an equal amount of play in the screw. The axes, the screw and the nuts must therefore be machined with the very greatest accuracy, and the panes made a very tight driving fit in their mountings. Also it is necessary to make every effort to determine the time lag, if any, of the screw. (*)

The use of plane-parallel faced panes of glass, interposed between the object-glass and the eyepiece, makes the beams of light which furnish the two images unavoidably astigmatic. But on condition that the inclination of the sheets be slight, M. BAILLAUD has shown that the astigmatism is negligible.

The *personal equation of horizontality* is easily eliminated, in time determinations, by the following procedure. For observations to the East, the rising image is always brought to the right, or else always to the left, of the descending image, while the reverse rule is followed for observations to the West. This personal equation is analogous to the *equation of bisection* in observations of stars' meridian passages made with a moving wire.

Method of observation.

(66) At the start of a series of observations, the vertical axis of the instrument is levelled and, with the aid of precision levels, the axes of rotation of the parallel-faced panes are made horizontal to within a few minutes of arc. Then the panes are brought into a vertical position, previously marked off, and the prism is adjusted by autocollimation.

About two minutes before the observation of a star, the pointer of the governor of the motor, calibrated by previous experiments, is put on the division corresponding to the azimuth of the star; the instrument is set, and the mercury trough placed before the object-glass. Also the levers are set at their full scope in the proper direction, according as the star is to the eastward or the westward.

When the images come into the field, the horizontality of the edge of the prism is adjusted, so as to make them travel along very close verticals, while still respecting the rule mentioned above (§ 65). At the moment when they cross the same horizontal, the circuit of the motor is completed; as the panes start to turn, the two images maintain relative immobility in altitude during the whole travel of the levers.

If the motor has not been started exactly at the moment of passage of the images over the same horizontal, the images will not be quite at the same level; this displacement is overtaken by manœuvring the differential attached to the screw which drives the panes. The horizontal wires of the reticle facilitate the operation.

In spite of the very careful rating of the motor, it is necessary to watch the horizontality of the line through the images for the whole duration of the

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(*) For the screws of transit telescopes, this systematic error is easy to measure; it is of the order of 0.02^s.

observations. With one hand the observer carries out this adjustment and with the other he works the azimuth screw so as to keep the images constantly between the vertical wires of the reticle.

Results.

(67) The following table contains a comparison of the time corrections determined with the aid of this instrument in the Spring of 1934, at the Observatory of Paris and the semi-definite time corrections *Cp* (*s.d.*) established at the Bureau International de l'Heure. The latter are, as is known, obtained by smoothing out the curve drawn from the time corrections taken from transit observations made with the GAUTHIER No. 381 longitude telescope and the BOUTY telescope; these two telescopes are fitted with self-recording moving-wire micrometers. With each of these three instruments, the time correction is taken from a series comprising some ten stars.

Instrument.	*Baillaud Apparatus.*	*Gauthier transit Telescope.*	*Bouty transit Telescope.*
Focal length	0.37 m.	0.84 m.	1.23 m.
Diameter of object-glass	40 mm.	75 mm.	102 mm.
Magnification of eyepiece	75	71	60

Dates.	*Cp(s.d)-Cp(Baillaud)*	*Cp(s.d)-Cp(Gauthier)*	*Cp(s.d)-Cp(Bouty)*
1934 Feb. 22.8	+ 0.043^s	— 0.044^s	+ 0.016^s
March 1.8	+ 16	— 2	— 2
3.8	+ 22	+ 6	— 7
15.8	+ 46	+ 3	+ 16
30.8	+ 19	— 12	
May 22.8	— 52	— 20	— 23
Mean difference	+ 0.016	— 0.012	0.000

Considering the small dimensions of the astrolabe used in comparison with those of the two transit telescopes, the results obtained may be considered very satisfactory.

C. *PHOTOGRAPHIC EQUAL-ALTITUDE INSTRUMENT (1923).*

(68) Before producing the impersonal apparatus just described, M. BAILLAUD had constructed in 1923 an equal-altitude instrument based on photo-

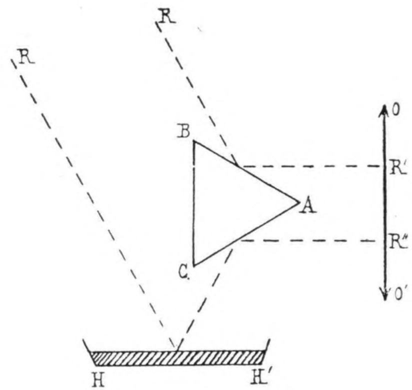

Fig. 38

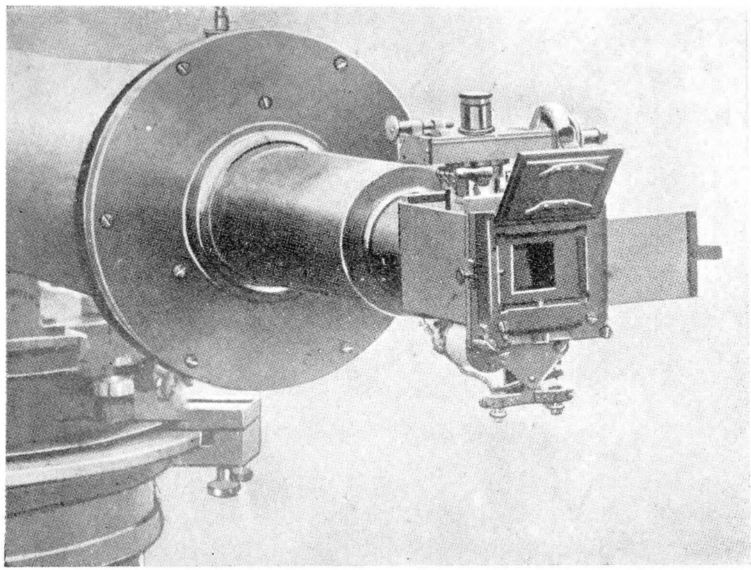

Fig. 39

Obturator and frame. *Obturateur et châssis*

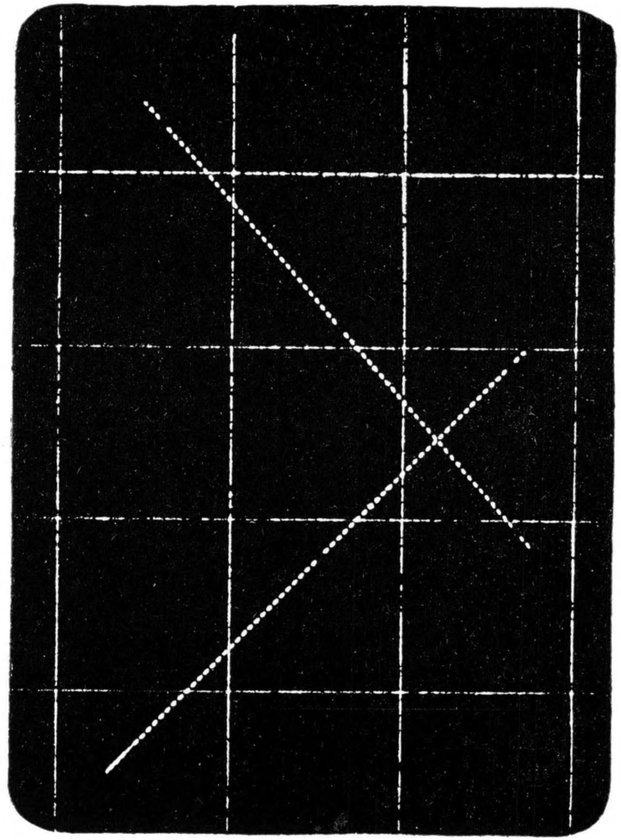

Fig. 40

α *Boötis — photograph taken at Nice, 10th March 1921.*

Magnification $\times$ 10.

α *Bouvier. — Photographie obtenue à Nice le* 10 *Mars* 1921.

Grossissement 10 *fois.*

Photographic Equal-Altitude Instrument — Instrument Photographique des Hauteurs égales graphic recording of the observations and consequently free from the observer's personal equation (*).

We have already seen (§ 11) the influence, on the instrumental altitude of a prismatic astrolabe, of the position of the plane of observation. It was to avoid variations in the position of this plane that it occurred to M. BAILLAUD to materialise it by placing the sensitive face of a photographic plate in the focal plane of the object-glass of an astrolabe.

The photographic process had the further advantage of giving impersonal results.

DESCRIPTION.

(69) Like all astrolabes, the instrument in essentials is composed of an object-glass, a prism and a mercury trough.

So as to be able to photograph a sufficient number of stars, it was necessary to make the instrument larger than the large model astrolabe. As it has not been possible up to the present to obtain perfectly homogeneous prisms bigger than 10 cm., the prism was arranged, using a process already put to the test by Messrs. NUŠL and FRIČ, in such a way that the horizontal edge A was towards the object-glass OO' (Fig. 38). The faces AB and AC are silvered and act as mirrors, off which the rays of light R from the star are reflected.

In the focal plane of the telescope is placed a photographic plate, and in front of this plate an obturator consisting of a shutter actuated by an electromagnet inserted in the circuit of a contact clock (Fig. 39). During the course of an observation, the shutter opens and closes alternately every full second, with an interruption at the 59th and 60th seconds. The image produced on the photographic plate is thus composed of two lines of small equidistant dashes (Fig. 40). Knowing the time of the start and finish of each dash, it is possible afterwards, with the aid of a measuring instrument, to determine accurately the positions occupied on the plate by the *corresponding points of the two lines.*

By a system of marking the instants when the shutter of the obturator functions on a pen chronograph worked by the clock, the opening and closing lags of this shutter are measured. Experience shows that they are practically equal to 0.05^s, and that they do not vary by more than 0.02^s.

Preliminary adjustment.

In front of the body of the camera is a small plane mirror, movable about a horizontal axis parallel to the plate, and above the mirror is an eyepiece mounted on a micrometer. When the mirror is inclined at 45° from the optical axis of the telescope, it reflects the rays of light from the star into the field of this eyepiece, which enables the apparatus to be used as a visual instrument. Thanks to this arrangement the operator, before starting the photographic recording, can verify all the adjustments, particularly that of the horizontality of the edge of the prism.

Mercury trough.

Having seen that the photographic images were absolutely blurred, although the mercury used was very shallow, it occurred to M. BAILLAUD to modify this component. After a few attempts he obtained reflected images as sharp as the direct images on the negatives, by making the mercury trough

(*) This apparatus was manufactured by the firms of JOBIN and BOUTY in collaboration.

float on a second, deeper trough. Vibrations from the ground are in fact only feebly transmitted to this second trough, and do not affect the first one which always remains steady.

Nevertheless it is well to set the telescope a fairly long time before photographing a star, for the shaking given the instrument at the moment of this operation stirs up the surface of the mercury.

Reduction of the measurements.

(70) The conditions of observation are not the same with this instrument and with the visual astrolabe, since use is made of the co-ordinates of the two images on the plate when they are not in the centre of the field. So the inventor, examining the influence exerted on the result by this displacement as well as by errors in the adjustment of the prism, has been led to introduce a certain number of instrumental constants into the reductions. It now becomes easy, if not quick, to conclude the exact time of coincidence of the two images by means of the measurements made on the corresponding points of the negative.

Knowing the times of coincidence of the images for several stars uniformly distributed to east and west, the time correction and latitude are computed by applying the general method of equal altitudes.

Two models of instrument.

(71) In the *small model photographic instrument*, the object-glass has an aperture of 10 cm. and a focal length of 1 m. The angle of the prism is very nearly 60°. Two of its faces are machined and silvered, the third is rough. Three adjusting screws on the rear face of the prism enable it to be directed perpendicularly to the optical axis of the object-glass. The mercury trough is a spherical segment of large radius, made of amalgamated copper.

With this instrument, stars of only the first three magnitudes can be photographed.

The object-glass of the *large model photographic instrument*, which was brought out later, is 18 cm. in diameter, and 1.9 m. in focal length. The prism is of crown glass; it has only two machined faces, the angle between them being 45°, and its edge is 20 cm. long.

There are definite advantages in the large model as compared with the small; there is an increase in the number of stars which can be observed, for the object-glass enables stars of the 4th and 5th magnitudes to be photographed, and, besides, the prism angle is smaller; further, there is more accuracy in the measurements made on the negatives, for the linear velocity of the images is nearly doubled, and the diffraction images are smaller.

Results.

(72) *Time determination.* — In February and March 1921, twelve evenings of observations, comprising each about 13 stars, enabled M. Baillaud to find that the mean difference between the individual *Cp's* and the mean *Cp* for any one series was less than 0.05^s. Differences reaching a tenth of a second were very rare. These figures are of the same order of magnitude as those obtained from observations with small transit telescopes.

From a comparison between these results and those furnished by the transit instrument, with which M. Fayet was observing on the same dates, the mean difference Cp (photographic instrument) — Cp (transit instrument) produced, as result, the value $+ 0.04^s$ (*). The mean difference between the individual differences and this figure is only 0.01^s, confirming that the time was determined with the two instruments with the same intrinsic precision, but showing also that there probably existed a systematic cause of error with one or both of the instruments.

Equation of magnitude. — A close examination of the various negatives then led M. BAILLAUD to consider the deformation of the images to be a possible cause of error. Each of these in fact only comes from one half of the object-glass and is elliptical in shape. Further, the images are larger or smaller according to the brightness of the star. It may thus be feared that for a brilliant star the coincidence may take place before or after the instant when it would have done so if the star had been dimmer. To this error, which would affect the time of coincidence of the two images, M. BAILLAUD has given the name of *instrumental equation of magnitude.*

With a view to easily ascertaining the repercussion of this error on the time correction deduced from the whole set of negatives taken in a single evening, he considered the particular case of one star being observed at both its eastern and western passages. If there is an equation of magnitude, it is the same in absolute value in these two cases. From a simple consideration of the opposite directions of the trajectories on the two negatives, it is easy to show that if the time arrived at for the coincidence to the eastward is too small, it is too large by the same amount to the westward, and vice versa. Consequently, the time correction Cp_2, concluded from the set of two observations, cannot be affected by the equation of magnitude.

A few series of observations made in December 1921 confirmed this hypothesis. In one of these in particular, the mean difference between each of the Cp's and their general mean is only 0.01^s. This order of precision is rare, even with the best transit instruments.

It proves, apart from the existence of the equation of magnitude, that measurements made on photographs are capable of great accuracy, and that the computation of the instrumental constants is rigorous.

This use of the photographic equal-altitude instrument cannot however be recommended for stations situated in a mediocre climate, nor for the summer period when the nights are short. The prism angle being 45°, it is necessary to wait more than eight hours between the two passages of a single star, and the total duration of a series of observations sometimes reaches twelve hours.

True, by using an equilateral prism one could reduce this duration.

Simultaneous observations with different instruments were organised afresh at the Observatory of Nice in 1924, and seven series gave a mean of $- 0.03^s$ for the differences Cp (photographic instrument) — Cp (transit instrument), whereas in 1921 the figure obtained had been $+ 0.04^s$. The difference between the individual differences and the mean was 0.04^s in 1924, a less satisfactory figure than that of 1921 which was equal to 0.01^s.

And yet M. BAILLAUD had taken the precaution of determining the time by photographic observation of passages of the same stars to the east and west. He had also replaced the trough of 200 mm. diameter which he had hitherto used, by a 300 mm. trough. So as to ensure shelter from the wind, he had set up the instrument in an excavation 2 metres (6 ft. 7 in.) deep.

(*) The figure given by M. BAILLAUD in his thesis (p. 91) is 0.07^s, but in the Report of the Observatory of Nice for 1924, M. FAYET states that it must be decreased by 0.03^s on account of corrections made to the polars used in the observations of the meridian passages.

But as the correction of the reference clock, in the comparisons made in 1921 and 1924, is from a single transit instrument, provided with a single level, and without checking of the inclination by nadiral sights, there is no proof that it was free from any systematic error; it is consequently impossible to form any formal conclusion from the above experiments with regard to the impersonality of the photographic equal-altitude apparatus.

(73) *Latitude determination.* — With regard to the determination of latitude, M. BAILLAUD has not published any results, but he has noted that the difference between the circle (of least squares) and the altitude vectors near the meridian is of the same order in graphs furnished respectively by visual and photographic instruments.

This would result, according to him, from the fact that phenomena of photographic irradiation cannot be eliminated in determination of latitude.

(74) *Constancy of the prism angle.* — It is appropriate to draw from M. BAILLAUD's study some conclusions also on the stability of the prism angle.

They have been taken from an investigation of a series of 24 stars photographed in an interval of six hours on 10th March 1921; allowance has been made for the variation of refraction due to change of temperature. From the table containing the values of the altitude relative to each observation, we deduce that the mean difference between the 24 individual altitudes and the mean does not attain 0.3", and further that the various differences do not show systematic change between the beginning and the end of the series.

In spite of the good results with which it has provided its inventor, the photographic equal-altitude instrument does not seem called upon to enter, in its present state, into current practice, on account of the length of time required for series of observations and of the very numerous precautions to be taken.

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PART IV.

PREPARATIVE AND REDUCTION CALCULATIONS FOR EQUAL-ALTITUDE OBSERVATIONS.

A. *NOTATION.*

(75) Problems on observation by equal altitudes introduce the triangle of position, formed by the pole of the celestial sphere, the zenith of the place of observation and the observed star.

MM. CLAUDE and DRIENCOURT have adopted a special system of notation bringing out the sides of this triangle. Several authors have followed their lead more or less completely (BOSSERT; Observatory of Madrid; Fr. DE LA VILLEMARQUÉ).

These notations, attractive in some respects, nevertheless cut across already well-established customs from which it is difficult to break away.

In dealing with geographical positions, it is always the latitude which is considered and not its complement. The catalogues give the declination and not the polar distance of the stars. The refraction tables always use the altitude as argument.

Furthermore recent international congresses have adopted a certain number of notations with the object of attaining agreement and simplification. We therefore think it more rational to follow them and conform to their decisions.

The only exception we shall make to this is the azimuth, for if astronomers have always been accustomed to reckon it from South, the practice among geodesists and seamen is to take the North as its origin.

Here, then, are the notations which we shall use.

φ Latitude reckoned from 0° to 90°, positive towards north, negative towards south.

δ Declination reckoned from 0° to 90°, positive towards north, negative towards south.

h Altitude reckoned from 0° to 90°, starting from the horizon, positive towards the zenith.

α Right ascension reckoned from 0 h. to 24 h., starting from the vertical circle through ♈, in the direction opposite to the diurnal motion.

H Hour angle reckoned from 0 h. to 24 h., starting from the meridian of the place, in the direction of the diurnal motion.

t Local sidereal time, hour angle of ♈ at the place; $t = H + \alpha$ for a given star.

Z Azimuth reckoned from 0° to 360°, from north towards east (the geodesists' azimuth).

A The astronomers' azimuth, generally reckoned from 0° to 360° starting from south towards west; $Z = A + 180°$.

In addition, we shall call S the angle at the star, reckoned, like the Z azimuths, from the vertical circle of the star.

B. *PREPARATIONS FOR EQUAL-ALTITUDE OBSERVATIONS.*

(76) The preparative work consists in choosing the stars which it is proposed to observe during the course of the evening, and determining the approximate time of their passage at the instrumental altitude as well as the corresponding azimuth.

It is necessary to know the approximate values h_0 and φ_0 of the altitude of observation and of the latitude of the station. As the times of passage are expressed in local sidereal time, it is also necessary to have an approximate value of the error of the hack-watch to which the observations are referred.

(77) *Stars observable.* — The stars which it is possible to observe are limited in declination by the parallels $\varphi_0 + 90° - h$ and $\varphi_0 - (90° - h)$. The limits of Right Ascension of these stars are the intended times of observation, expressed in local sidereal time, the initial time being decreased and the final time increased by thequantity H_0 given by the relation $\sin H_0 = \dfrac{\cos h_0}{\sin \varphi_0}$. H_0 is the greatest of the time intervals which, at the latitude φ_0, separate the stars' meridian transits from their transits at the instrument.

A list is drawn up of these stars, the positions of which are obtained from the catalogue in use at the time in question by astronomers. In this list all stars are struck out which are too dim to be observed with the instrument available.

The catalogue usually adopted nowadays is EICHELBERGER'S. Its main advantage is that the various astronomical ephemerides give the apparent positions of the stars appearing in it for every ten days, which greatly simplifies the reduction work. Also, the number of stars in this catalogue is amply sufficient to enable one to make as complete equal-altitude observations as may be desired, in most latitudes.

As a rule the following form is adopted for the list of stars observable (List 1).

Number or Name.	*Magnitude.*	*Declination.* δ	T_2	*Right Ascension.* α	H	T_1	Z_2	Z_1

The columns H, T_1, T_2, Z_1 and Z_2 are reserved for the later part of the preparation work. The four other columns are taken from the star catalogue. As it is unnecessary to go beyond the first decimal place for accuracy in magnitude, the nearest minute of arc in declination and tenth of a minute of time in Right Ascension, one need only consult the table of mean positions in the catalogue.

If the catalogue used is too old, the positions of the stars will be referred to a new epoch near that of the observation by means of a simple computation or an easily drawn-up diagram. If this precaution has not been taken, one must apply at the end of the preparation the formulae which we shall give presently for bringing a table of old settings up to date.

(78) The hour angle and azimuth are determined for each of the stars by solving the star's position triangle, of which the three sides are known: polar distance of the star, approximate co-latitude of the station, and zenith distance of observation.

If the list of stars is a long one it is preferable to have recourse to a *table of interpolation*, giving, for the latitude and instrument in question, the values of the hour angle and azimuth corresponding to conveniently spaced values of the declination. Then one interpolates the elements contained in this table for the declinations of the various stars. According to the degree of accuracy required in the preparation work, there are several solutions to choose from.

(79) (*a*) The most rigorous consists in directly calculating the table for values of δ spaced so that Z varies by about 2° at a time. It is not a bad thing first to calculate the azimuth Z, and from it to deduce the hour angle H by the sine rule. The formulae used may differ according to the means available (logarithmic tables, tables of natural values of trigonometrical ratios, or calculating machines). The relation

$$\cos Z = \frac{\sin \delta - \sin \varphi_0 \sin h_0}{\cos \varphi_0 \cos h_0}$$

lacks something of precision when Z is near 0° or 180°; it can then be replaced by

$$2 \sin^2 \frac{Z}{2} = \frac{\cos (h_0 - \varphi_0) - \sin \delta}{\cos \varphi_0 \cos h_0} \text{ for } Z \text{ nearly } 0°,$$

and by

$$2 \cos^2 \frac{Z}{2} = \frac{\cos (\varphi_0 + h_0) + \sin \delta}{\cos \varphi_0 \cos h_0} \quad \text{for } Z \text{ nearly } 180°.$$

The work is arranged in columns, each line corresponding to a value of δ; there will thus be a hundred or so lines. The determination of the azimuth is limited to the nearest tenth of a degree, and that of the hour angle to the nearest tenth of a minute of time. Four- or five-figure logarithms suffice.

(80) (*b*) Instead of working out a complete table of interpolation they may be extracted from certain known tables.

In the first place, all the navigation tables are suitable, since they enable a large number of spherical triangles to be solved. Their use is more or less convenient according to their arrangement.

General preparatory tables have been established for the prismatic astrolabe. A table for the station in question can be obtained from them by interpolating the hour angles and azimuths between the two tables which give the even latitude on either side of the station.

In azimuths near the meridian, the interpolation is done for the same values of $\delta - \varphi$; in the other azimuths it is done for the same values of δ.

The tables published at Zi-Ka-Wei by Fr. DE LA VILLEMARQUÉ lend themselves especially well to this work. They extend from 0° to 56° of latitude (north or south).

In the Year-Book of the Observatory of Madrid, similar ones are to be found for latitudes of interest to Spain and the Spanish Possessions.

Those figuring in MM. CLAUDE and DRIENCOURT's book give less strictly accurate results, especially for azimuths near the meridian, but they have the advantage of being contained within a few pages and of including all latitudes

(81) (*c*) Whether it is directly calculated, or whether it is taken by interpolation from existing documents, the table of interpolation takes the following form.

Declination. δ	*Difference.* $\Delta\delta$	*Hour angle.* H	*Difference.* ΔH	*Azimuth.* Z	*Difference.* ΔZ

Only the hour angles corresponding to the westerly passages are entered, and the azimuths between 0° and 180° corresponding to the easterly passages. The values of the declination are very close together at the start and at the end of the table, when Z is near 0° or 180°; but towards the middle of the table it is sufficient to vary δ a degree at a time.

(82) (*d*) When a medium degree of precision is sufficient, it is convenient in practice to use diagrams in place of tables of proportional parts. Any diagram which solves spherical triangles, such as that of ROLLET DE L'ISLE and FAVÉ (diagram for finding the position at sea), could be used, but it seems preferable to have recourse to those which have been drawn up specially, as has been done for observations with the prismatic astrolable.

The diagram by Colonel LABORDE, recently published by the Army Geographical Service, is at present the most accurate. It is in the nature of a large slide-rule giving the hour angle H as a function of δ, φ_0 and $\delta - \varphi_0$ by the formula

$$2 \sin^2 \frac{H}{2} = \frac{\sin h_0 - \cos(\delta - \varphi_0)}{\cos\delta \cos\varphi_0}$$

The $\delta - \varphi_0$ scale is graduated from the values of the function $\sin h_0 - \cos(\delta - \varphi_0)$. A simple auxiliary diagram gives the azimuth.

The older diagram of the Naval Hydrographic Service comprises a board on which are drawn the parallels and meridians of the celestial sphere in the system of projection of the plane chart. The altitude curve, which in this projection is identical with itself at all latitudes, is drawn on a transparency and graduated in azimuth. It is applied on the board with its centre at the latitude of the station. The azimuth is read on the altitude curve at its intersection with the parallels of declination. The hour angle is that of the meridian passing through the point of intersection. Interpolation is done by sight.

This diagram, which is far from sufficient for passages near the meridian, has been completed by a nomogram specially drawn up for circummeridian observations by PERRET. The argument of interpolation is $\delta - \varphi_0$.

The relation between Z, δ and φ_0 is expressed as

$$\tan \varphi_0 \, [\cos (\delta - \varphi_0) - \sin h_0] - \cos h_0 \cos Z + \sin (\delta - \varphi_0) = 0.$$

This equation can be solved by a nomogram with aligned points; the φ_0 and Z scales are rectilinear, the $\delta - \varphi_0$ scale consists of two symmetrically curved arcs.

This nomogram furnishes azimuths from 0° to 20° and from 160° to 180°. The hour angles are obtained from the general diagram, but starting from the value found for the azimuth.

(83) (*e*) With the aid of the tables or diagrams, list 1 is completed by entering in columns H and Z_1 the absolute value of the hour angle and azimuth corresponding to the declination of each star.

Then the times of easterly passage $T_1 = \alpha - H$ are calculated, and the times of westerly passage $T_2 = \alpha + H$. The azimuth Z_2 of the westerly passage is equal to $360° - Z_1$.

(84) Next the stars must be arranged in the order of their times of passage. If the list includes a fairly large number, an intermediate stage is gone through of using slips divided into three columns: number (or name) of the star, time of passage, numerical order. Each slip corresponds to a certain interval of sidereal time, half an hour, an hour, two hours, etc. according to the number of stars in the list. This interval will be so chosen that not more than some thirty passages are entered on any slip. The times of passage taken from list 1 (columns T_1 and T_2) are entered in their columns. The classification number indicates the consecutive order of the passages on each slip.

(85) It is then easy to draw up the *table of settings* as follows :—

Number or Name.	*Magnitude.*	*Time of passage.* T	*Azimuth.* Z	*Remarks.*

The elements are taken from list 1, the slips serving solely to establish the order according to the increasing values of T.

An English book published in 1917 by BALL and KNOX SHAW gives a series of settings for the prismatic astrolabe. These tables are established for each degree of latitude between 55° north and 55° south, and utilize the bright stars of the *Nautical Almanac*, to the number of about 300.

The Geographical Society of New York recently published an analogous book, but on a much larger scale. In it will be found 121 lists of settings for the prismatic astrolabe, one for each degree of latitude between 60° north and 60° south. These lists include all the stars whose apparent positions appear in the *American Ephemeris*, the *Nautical Almanac* and the *Connaissance des Temps*. They are further arranged so that it is easy to interpolate the elements for setting to the latitude of the station. The interpolation is done by sight for the time stars; a computation, though a simple one, is necessary if some degree of precision is required for the circummeridians.

With these two books, and particularly the second, it becomes easy to draw up quickly a table of settings for the prismatic astrolabe.

(86) From the table of settings, one finally extracts the *field book of settings*, arranged exactly like the table but in which are entered only those stars which it is proposed to observe, taking care that they are suitably spaced in azimuth and that the interval of time separating two consecutive passages is sufficient for the observer to change the setting of the instrument and make the necessary adjustments.

(87) The procedure to be used for the preparation work, and the degree of accuracy to be maintained all through the operation, depend essentially upon the instrument of observation used and the accuracy desired for the results.

The length of the preparation work varies with the method adopted.

The most rigorous preparation, which we shall call *meticulous preparation*, concerns precision observations made with an instrument for single sights.

Observations made with an instrument for multiple sights, or field observations made with instruments for single sights, are subjects for *rapid preparation.*

Meticulous preparation.

(88) Precision observations with an instrument for single sights actually comprise some forty stars per series. As one is dealing with work of a final character, as a rule one possesses a good estimated position for the station.

It is a further advantage that the series should last as short a time as possible, for the common reduction methods presuppose that the following will be constant during the period of the series :—

The altitude of observation, a function of the instrument and of refraction, and consequently depending on the atmospheric temperature and pressure ;

The rate of the hack-watch, also influenced by these factors;

The personal equation of the observer, liable to change as the latter becomes fatigued.

One can, however, allow for variations of refraction and, in some cases, for variation in the rate of the hack-watch, in the reduction work.

The first condition to be obtained for carrying out a fairly long series in the shortest time is that the azimuths should be known to within one or two tenths of a degree and the times of passage to within one or two tenths of a minute of time.

One thus avoids hit and miss attempts in azimuth, mistakes in the stars, premature or tardy settings, etc. These mishaps usually cause considerable loss of time; they are all the more appreciable in precision observations in that the instrument in use is high-powered and consequently of restricted field.

As in these operations the observer generally has an assistant who looks after the recording of the times of passage, the interval of time between two successive observations can be greatly reduced. Its minimum is some fifty seconds for a trained observer; that is the time necessary for noting the observation just made, moving to the next setting, letting the mercury settle, and making the readjustments for the fresh observation.

Finally, since in precise determinations it is well to take several series of observations, it will be realised that one is justified in devoting a relatively large amount of time to their preparation.

For choice, therefore, a table of interpolation will be used for the determination of the hour angles and azimuths, calculated as shown above.

By keeping, throughout the computation, an accuracy of a tenth of a degree for the azimuths and a tenth of a minute for the passages, a sufficient degree of accuracy will be obtained in the field book of settings.

In the case of the few stars which pass less than ten degrees from the meridian, it will be more advantageous to solve the triangle of position directly.

If the observations are to extend over a fairly long period, the field book of settings will be so drawn up that for a given period of time, half an hour or an hour, the distribution in azimuth will be the best possible. The series can thus be commenced at any time without worrying about the choice of stars.

The field book of settings has the advantage of enabling the same stars to be observed in the course of consecutive evenings, which considerably simplifies the reduction work.

None the less, when the sky is cloudy, it is well to use the table rather than the field book of settings, so as to profit by breaks in the clouds. It is rare, also, that such conditions of observation lead to excellent results.

Rapid preparation.

(89) The procedure is the same as for meticulous preparation, but the operations are not so long, for they are done for a smaller number of stars and a lesser degree of accuracy is admissible.

(90) *Observations in the field.* — Actually, in observations in the field with an instrument for single sights, only fifteen to twenty stars per series are observed. If the tim e of passage is somewhat uncertain, the observer takes the precaution of setting the instrument two or three minutes beforehand; he thus need not be afraid of missing the star, for with a few attempts in azimuth, thanks to the relatively wide field of the instrument, he can sweep an extensive a rea of sky. It is prudent also only to observe fairly brilliant stars, both on account of the frequently rather low power of field instruments, and of avoiding confusion of stars resulting from inaccuracy in the elements of the setting.

It is not always possible, in the field, to know the estimated latitude of the station closer than to a few minutes of arc, and consequently it is difficult (at all events in a first series) to avoid uncertainty in the elements of setting, which may attain considerable values, especially with circummeridian stars.

It is thus unnecessary for the intermediate stages of the preparation to be done with extreme precision.

If a prismatic astrolabe is available, the best procedure is to use the tables of the Geographical Society of New York. It can even, if one is pressed for time, be used on the spot without other intermediary. Then it is convenient if an assistant chooses the stars according to their azimuths during the observations, and does the interpolating for the elements of the setting.

This book in our opinion is the indispensable complement of the prismatic astrolabe for observations in the field. Without it, the preparative work is appreciably lengthened. It is therefore necessary to follow the procedure we have indicated for precision observations, but using diagrams instead of tables of interpolation for working out the hour angles and azimuths.

(91) *Instruments for multiple sights and impersonal instruments.* — As only a small number of stars are observed with these instruments (a dozen per series), fairly large time intervals may separate the passages, and, as in the case of field observations, the elements of setting may be fairly rough approximations; it is thus sufficient to use a rapid method of preparation. However, in the settings for stars of small magnitude, rather more accuracy must be sought for, if it is desired to avoid confusion between stars.

On the other hand, the methods of observation adopted in the use of these instruments enable one to reduce the number of stars to be prepared. With certain instruments it is an advantage to determine the latitude and the time separately; with impersonal instruments, to avoid too great differences between the vertical velocities of the images in the field, it is evidently advisable to determine these quantities by observations made in azimuths 45° from the meridian (within a few degrees).

Hence it is possible to contract considerably the limits of declination of the stars observable, which are thus reduced to a small number. This circumstance appears especially interesting when the altitude of observation differs from 60°; for then the tables and diagrams of preparation published for the equilateral-prism astrolabe are not applicable, and it becomes necessary to have recourse to computation.

Correction of tables of settings.

(92) One may find oneself called upon to remake a table of settings, either because the latitude of the station differs from that adopted for the calculations, or because it has been established with out of date star positions.

If the errors are small enough for one to be able to deduce the new settings from the old by a simple differential correction, it is a good thing to work out this correction as a function of the elements of setting themselves and of the two constants of preparation, latitude of the station and altitude of observation. It is always possible to do this, for these two constants, together with the elements of setting for a star, define the angle of position of this star and its position with reference to the meridian of origin, being equivalent to the co-ordinates of the star together with the two constants.

(93) *Alteration of estimated latitude.* — In observations in the field, the observer may have to take a series at a point where a preliminary observation has shown him that the estimated latitude was considerably in error.

Let $\Delta\varphi_0$ be the error, in minutes of arc, in the latitude; the elements of setting will be corrected by ΔT (in minutes of arc) and ΔZ (in degrees), given by:

$$\begin{cases} \Delta Z = \left(\dfrac{\tan h_0}{\sin Z} - \cot Z \tan \varphi_0\right) \dfrac{\Delta\varphi_0}{60} \\ \Delta T = - \dfrac{\cot Z}{\cos\varphi_0} \dfrac{\Delta\varphi_0}{15} \end{cases}$$

In these formulae φ_0 is taken as the mean of the two values, old and new, of the estimated latitude.

These expressions are easily tabulated as functions of the azimuth for, say, every ten degrees.

We may also take

$$\Delta Z = -\frac{\cot H}{\cos \varphi_0} \frac{\Delta \varphi_0}{60}$$

and may, in drawing up these tables, take H as a function of φ_0 and Z from the tables on pages 244 to 247 of CLAUDE and DRIENCOURT'S book, if observing with the prismatic astrolabe.

It will readily be seen from these formulae that ΔZ and ΔT give rise to symmetrical corrections for the easterly and westerly passages, that ΔT cancels out and changes in sign at 90° and 270° azimuth, and that ΔZ passes through a minimum in absolute value, namely in latitudes between h_0 and $-h_0$.

But in the case of the circumpolar regions (more than h_0 from the equator) and in that of circummeridian observations, i.e. when the parallel of the star and the small circle of instrumental altitude cut at a fine angle, the corrections assume large values and are no longer given with sufficient precision by the above expressions. It then becomes necessary to revert to calculation for the elements of setting for the new latitude; not a considerable undertaking, for circumpolar observations are exceptional, besides which the stars observable a few degrees from the meridian are few in number.

Bringing an old table of settings up to date.

(94) If at a certain period a table of settings for precision observations has been established, after a few years the position of the stars will have changed owing to the precession of the equinoxes and the settings will no longer be sufficiently accurate.

In practice, observations made at intermediate periods will have made it possible to rectify, little by little, the elements relative to a certain number of stars, but one may find it necessary to work out corrections for the whole table of settings.

As none of the classic treatises contain a demonstration of the correction formulae, it will be well to give one here.

The meridian of ♈ is tangent to the path described by the pole on the celestial sphere. We may thus assume, to within the second degree, that during the time dt the pole moves along the meridian of origin from P to P', along an arc $PP' = ndt$. During this time the meridian of origin moves from PM to $P'M'$, the dihedral angle formed by these two planes being equal to mdt. The quantities m and n are the constants of precession (Fig. 41).

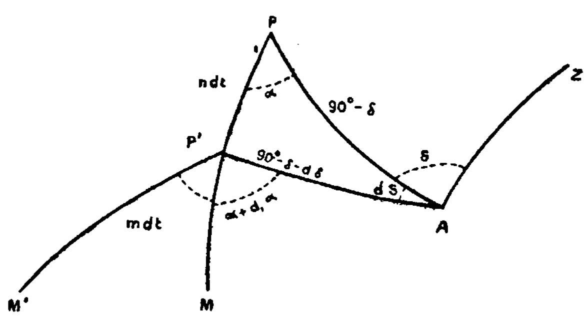

FIG. 41

Let α and δ be the co-ordinates of a star A with reference to the pole P, and S the azimuth at A of a direction invariably joined to A.

Let $\alpha + mdt + d_1\alpha$, $\delta + d\delta$ and $S + dS$ be the same elements at the end of the time dt, when P has moved to P'. We then have the following relations in the triangle $PP'A$, of which the first two are well-known.

For declination,

$$\sin(\delta + d\delta) = \sin\delta \cos ndt + \cos\delta \cos\alpha \sin ndt$$

$$\sin\delta + d\delta \cos\delta = \sin\delta + ndt \cos\delta \cos\alpha$$

$$(1) \qquad \frac{d\delta}{dt} = n \cos\alpha$$

For right ascension,

$$\frac{\sin(\alpha + d_1\alpha)}{\cos\delta} = \frac{\sin\alpha}{\cos(\delta + d\delta)}$$

$$\sin\alpha + \cos\alpha\, d_1\alpha = \sin\alpha\,(1 + d\delta \tan\delta)$$

$$d_1\alpha = d\delta \tan\delta \tan\alpha$$

$$\frac{d_1\alpha}{dt} = n \tan\delta \sin\alpha$$

The total variation of the right ascension is thus

$$(2) \qquad \frac{d\alpha}{dt} = m + n \tan\delta \sin\alpha$$

For azimuth,

$$(3) \qquad \frac{dS}{dt} = n\frac{\sin\alpha}{\cos\delta}$$

Let us consider the point B on the celestial sphere which coincides with the zenith at the place of observation, where the star A passes at a distance $90^\circ - h_0$ from this zenith. The co-ordinates of B are T and φ_0, the azimuth of BA being Z.

After the time interval dt, the co-ordinates of B have become

(formulae (1) and (2)) $\quad T_1 = T + mdt + ndt \tan\varphi_0 \sin T$

$$\varphi_1 = \varphi_0 + n \cos T\, dt\,;$$

and the azimuth of BA (formula (3))

$$Z_1 = Z + n\frac{\sin T}{\cos\varphi_0}\, dt.$$

But the distance BA has remained equal to $90^\circ - h_0$; consequently T_1 and Z_1 are, at the epoch $t + dt$, the elements of setting for a star A seen from a point of observation on the parallel of latitude φ_1. On the parallel of latitude $\varphi_0 = \varphi_1 - n \cos T\, dt$, the elements at the same epoch are :

$$(4) \qquad \begin{cases} T' = T_1 - \dfrac{dT}{d\varphi} n \cos T\, dt = T + m\, dt + n\, dt \tan\varphi_0 \sin T - \dfrac{dT}{d\varphi} n \cos T\, dt \\[2ex] Z' = Z_1 - \dfrac{dZ}{d\varphi} n \cos T\, dt = Z + n\dfrac{\sin T}{\cos\varphi}\, dt - \dfrac{dZ}{d\varphi} n \cos T\, dt. \end{cases}$$

But we have seen that $\begin{cases} \dfrac{dT}{d\varphi} = -\dfrac{\cot Z}{\cos\varphi_0} \\[2ex] \dfrac{dZ}{d\varphi} = \dfrac{\tan h_0}{\sin Z} - \cot Z \tan\varphi_0. \end{cases}$

We will therefore pass from the elements of setting T, Z at the epoch t to the elements $T' = T + \Delta T$, $Z' = Z + \Delta Z$, at the epoch $t + dt$, by making the corrections

$$(5)^* \quad \begin{cases} \Delta T = dt\,(m + n \tan \varphi \sin T + n \cos T \dfrac{\cot Z}{\cos \varphi}) \\ \Delta Z = dt\,(n \dfrac{\sin T}{\cos \varphi} + n \cos T \cot Z \tan \varphi - n \cos T \dfrac{\tan h}{\sin Z}). \end{cases}$$

If dt be expressed in years, ΔT in minutes of time and ΔZ in degrees, then in the first relation we take $m = 0.0512$, $n = 0.0223$; and in the second, $n = 0.00557$.

Thus, from the table of settings established for latitude φ_0 and altitude of observation h_0, with star co-ordinates relative to the epoch t, we can deduce an analogous table for the epoch $t + dt$ by applying to the elements of setting the corrections given by formulae (5). These corrections will be given by a double entry table of T and Z.

(*) Formula (5) can be found directly by noting that it is only change of declination that affects the hour angle and azimuth. Consequently

$$\begin{cases} \dfrac{dT}{dt} = \dfrac{d\alpha}{dt} + \dfrac{dH}{d\delta}\dfrac{d\delta}{dt} \\ \dfrac{dZ}{dt} = \dfrac{dZ}{d\delta}\dfrac{d\delta}{dt} \end{cases}$$

These relations are obtained by applying formulae (1) and (2) and by noting that

$$\frac{dH}{d\delta} = \frac{\cot S}{\cos \delta} \text{ and } \frac{dZ}{d\delta} = \frac{1}{\sin H \cos \varphi}:$$

$$(6) \quad \begin{cases} \dfrac{dT}{dt} = m - \dfrac{n}{\sin Z \cos \varphi}(\sin \alpha \sin S \sin \delta + \cos \alpha \cos S) \\ \dfrac{dZ}{dt} = \dfrac{n \cos \alpha}{\sin H \cos \varphi} \end{cases}$$

Let us consider the point Q where the meridian of γ cuts the vertical through the star; from triangles PAQ and PAZ (Fig. 42) we have

$$\cos Q = -(\sin \alpha \sin S \sin \delta + \cos \alpha \cos S) = \sin T \sin Z \sin \varphi + \cos T \cos Z$$

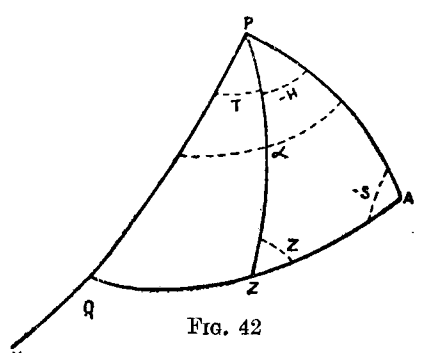

FIG. 42

Also,

$$\frac{\cos \alpha}{\sin H} = \frac{\cos (T - H)}{\sin H} = \cot H \cos T + \sin T,$$

and in the triangle PZA

$$\cot H = \frac{\sin \varphi \cos Z - \tan h \cos \varphi}{\sin Z}$$

Substituting in equations (6), we find ourselves back at formulae (5).

In the form (4), it will be seen that these corrections have the same disadvantages for the circumpolar regions and at all latitudes for the circum-meridian stars as corrections of elements of setting as a function of a change of latitude.

(95) As we have already mentioned, the tables of the Geographical Society of New York completely solve the expeditious preparation of observations with the prismatic astrolabe.

It would be worth considering the publication of a book in which the preparative operations were less thorough but which on the other hand would give greater accuracy.

Each page (or part of a page, according to the format) would be devoted to a star from a certain catalogue. One would find there the times of passage of a star in an equilateral-prism astrolabe, and the corresponding azimuths for a hundred or so values of latitude. These latitudes would be selected so that the azimuths would change by about two degrees from one value to the next. They would be fairly close together for the parallels where the star is observable near the meridian, but spaced about a degree apart for the mean values. Of course the latitudes chosen would be even degrees or simple fractions of degrees.

Four supplementary columns would be reserved for secular variations in the elements of setting as a function of the precession of the equinoxes. They would thus enable the book to remain up to date for a sufficient term of useful life. The star co-ordinates adopted for the calculations would also be referred to an equinox later by 10 or 15 years than the intended date of publication.

The stars would be arranged in order of declination.

To draw up a table of settings, the elements of setting for the star would be read on each page, with the limits of declination corresponding to the latitude of observation, and interpolating by sight for this latitude. They would be noted on a slip with the number and the magnitude of the star. The whole set of slips, classified in chronological order, would form the desired table of settings.

Each page of the work would look like this :—

Number of the star — its name — ephemeris giving its apparent positions — magnitude — position at the equinox of.................. $\alpha =$ .... $\delta =$ ....

Latitude	*Elements of setting.*						*Secular variation.*			
φ	T_1	dT	T_2	Z_1	dZ	Z_2	ΔT_1	ΔT_2	ΔZ_1	ΔZ_2

The latitudes would be given to a minute of arc, the times of passage to a tenth of a minute of time, the azimuths to a tenth of a degree, and the secular variations of these quantities to one decimal place more.

To calculate the secular variations, it is preferable to make use of the following formulae, more practical than formulae (5) when the star is given and the latitude is variable.

$$\left\{\begin{aligned} \frac{dT}{dt} &= m + n \sin\alpha \tan\delta + n \frac{\cos\alpha}{\cos\delta} \cot S \\ \frac{dZ}{dt} &= \frac{n \cos\alpha}{\cos h_0 \sin S} \end{aligned}\right.$$

C. *REDUCTION OF EQUAL-ALTITUDE OBSERVATIONS.*

(96) In the case of an instrument for single sights, the first result of a series of observations is the list of times of passage at a determined altitude. These times, referred to the same hack-watch, will have been obtained to within 0.1^s or 0.2^s in rapid work and 0.01^s in meticulous work.

With instruments for multiple sights, the times corresponding to each sight will have been collected, but reduction is necessary to bring the result to the above form, i.e. to determine the time of passage of each star observed at the instrumental altitude. Ordinarily this altitude is considered to be the mean of the altitudes corresponding to the various sights. The time of passage at the mean altitude is not the strict mean of the times of the various sights; the latter must be modified by a correction, which is nil when the star is observed in the prime vertical or at the maximum digression, but which is fairly large for the circummeridians.

If n sights are taken at altitudes differing by y_1, y_2............ y_n seconds of arc from the instrumental altitude, the value of the correction in seconds of time is

$$dt = \frac{\sum_1^n (y^2)}{n} \frac{\cot Z \cot S \sin 1''}{30 \cos h \cos \varphi \sin Z}.$$

In the case where the sights correspond to regularly spaced altitudes, it is preferable to correct each sight by

$$dt = y^2 \frac{\cot Z \cot S \sin 1''}{30 \cos h \cos \varphi \sin Z};$$

the times of passage obtained then differ from one another by a quantity the constancy of which enables the quality of the observation to be appreciated; their mean is the time of passage at the instrumental altitude.

(97) The object of the calculation of the observations is to deduce the latitude and time corrections of the times of passage of a certain number of stars at an altitude assumed to be constant. As this altitude is unknown, it is usual to determine it also.

This problem contains three unknowns; three equations, and thus three measurements are sufficient to solve it.

Both Gauss and Cagnoli have treated this simple case directly by spherical trigonometry. We shall not expatiate on the elegant solutions reached by them, for in practice, to avoid errors affecting the data, many more measurements are always made than there are unknowns, and we must have recourse to indirect methods of solution of the of superabundant systems equations.

These indirect methods have the further advantage of enabling supplementary unknowns to be introduced, such as the rate of hack-watch, if the latter is too uncertain.

Anger and Knorre were the first to recommend the use of indirect solutions in the calculation of equal-altitude observations.

To each measurement there corresponds an equation furnished by spherical trigonometry. We take as new unknowns the small quantities which separate the unknowns from sufficiently close values of the quantities to be determined, obtained by a rapid process. We substitute, and make the equations linear, assuming that the powers of the new unknowns are negligible.

In the present case, the linear equation at which we arrive is that of the position line of altitude which navigation has rendered classic.

The unknowns are the latitude φ, the instrumental altitude h, and the time correction Cp of the clock at a given instant t_0, e.g. at the mean time of the observations.

Further, let m be the rate of the chronometer, assumed to be known with reasonable precision.

The star, of co-ordinates α and δ, observed at the instant t, leads to the equation:

$$\cos\delta\cos\varphi + \sin\delta\sin\varphi\cos\left[\alpha - t - Cp - m\,(t - t_0)\right] - \sin h = 0.$$

We then choose approximate values φ_0, Cp_0, and h_0 for the unknowns, the same, of course, for the whole series of observations, and adopt as new unknowns

$$d\varphi = \varphi - \varphi_0, \quad dCp = Cp - Cp_0 \quad \text{and} \quad dh = h - h_0.$$

We determine the altitude h_1 at which the star is seen from the position in latitude φ_0 at the instant $t + Cp_0 + m\,(t - t_0)$, and put $h_1 - h_0 = \delta h$.

We thus arrive at the linear equation with three unknowns,

$$d\varphi\cos Z + dCp\sin Z\cos\varphi - dh + \delta h = 0.$$

In the coefficients of the unknowns we can replace φ by φ_0 and use a value of the azimuth deduced from the approximate elements. It is also more practical to take $dCp\cos\varphi_0$ as an unknown, and, taking the three unknowns $dCp\cos\varphi_0$, $d\varphi$ and dh as x, y and z, to write the equation in the form:

$$x\sin Z + y\cos Z - z + \delta h = 0.$$

Each star observed thus leads to an equation in x, y, z.

The system of equations with three unknowns corresponding to the series of observations can be solved either by the method of least squares or by the CAUCHY-TISSERAND method (quicker than the former), or finally by a graphical method derived from the method of solution of the position at sea and introduced by Admiral PERRIN.

We thus see that the calculation of a series of observations is composed of two quite distinct parts:

Establishment of the equations of condition, the important point of which is the working out of δh for each observation;

Solution of the system of equations of condition.

COMPUTATION OF δh AND Z.

(98) In precision observations, the approximate elements are always sufficiently known; the degree of approximation is in general a few seconds in the latitude and altitude of observation and a few tenths of seconds of time in the time correction.

In the field, if these data are too uncertain, one uses roughly approximate elements to make a preliminary calculation for four stars, well observed and suitably spaced in azimuth; the result of this calculation gives sufficiently precise elements to start with.

The approximate altitude is itself the algebraical sum of the approximate values of the instrumental altitude and the corresponding refraction.

Cp and *m* are the error and the rate of the hack-watch with respect to sidereal time. If a hack-watch regulated by mean time is in use, *m* is the sum of the rate of this instrument with respect to mean time and the rate of the mean time with respect to the sidereal time. Reception of wireless time signals enables us to know the rate of the hack-watch with precision.

If it is small, it is worked out for each hour, as also are the intervals of time separating the observations from the instant t_0 to which the time correction of the clock is referred; if the rate is large, the unit adopted is the minute.

In setting out the work, the case of observations in the field must be distinguished from the case of precision observations made in temporary or permanent observatories but generally comprising several series of observations for a single determination of the geographical position.

The apparent positions of the stars are taken from the astronomical ephemerides giving the apparent co-ordinates for every tenth day.

For observations in the field, the Right Ascensions are calculated to within 0.02^s or 0.03^s, and the declinations to within 0.2", by a rapid interpolation.

If greater accuracy be desired, we keep to a thousandth of a second of time in Right Ascension and a hundredth of a second of arc in declination. In that case the second differences must be allowed for, as well as the interval between the time of the observation and the time when the star passes the meridian adopted in the ephemeris (*). This interval of time, which is expressed in tenths of a day, is practically equal to the longitude of the place of observation with respect to the meridian of reference, plus the hour angle of the star observed. It is easily tabulated as a function of the azimuth of observation; for convenience we determine only the azimuths for which the correction in question is equal to an odd multiple of twentieths of a day.

It is only in high precision observations that the short period terms of the notation need be taken into account.

We next calculate the approximate hour angle H by subtracting from the Right Ascension of the star the time of passage corrected for rate and approximate error.

As, in observations in the field, we have not been observing the same star during several days, h_1 is calculated by one of the following formulae:

$$\sin h_1 = \sin \varphi_0 \sin \delta + \cos \varphi_0 \cos \delta \cos H,$$

or $$\sin^2 \left(45^\circ - \frac{h_1}{2}\right) = \sin \left(M + \frac{\varphi_0 + \delta}{2}\right) \sin \left(M - \frac{\varphi_0 + \delta}{2}\right),$$

in which M is an auxiliary angle given by

$$\cos^2 M = \cos \delta \cos \varphi_0 \cos^2 \frac{H}{2}.$$

The calculation is done with the aid of 7-figure logarithms for the first formula and 6-figure logarithms for the second.

The azimuth is given by $\sin Z = \dfrac{\cos \delta \sin H}{\cos h_0}$, which is worked out with 5-figure logarithms.

Then we form the difference $\delta h = h_1 - h_0$.

For the equilateral-prism astrolabe, there is an auxiliary table giving δh from a rapid computation with 6-figure logarithms. The idea of this table is

(*) All the ephemerides have now adopted the meridian of Greenwich.

due to Captain JORDAN, who transformed the first of the formulae giving h_1 as follows :—

$$(\delta h)'' = 2\,\frac{\sin^2\left(45^\circ - \dfrac{h_0}{2}\right) - \sin^2 \dfrac{\delta - \varphi_0}{2}}{\cos h_0 \sin 1''} - 2\,\frac{\cos \delta \cos \varphi_0 \sin^2 \dfrac{H}{2}}{\cos h_0 \sin 1''}$$

The table gives, in tenths of a second of arc, the first term of δh as a function of suitably spaced values of $|\delta - \varphi_0|$. The second term is readily calculated by noting that $\dfrac{2}{\cos h_0 \sin 1''}$ is constant for the prismatic astrolabe and that $\dfrac{2 \cos \varphi_0}{\cos h_0 \sin 1''}$ is constant for any one station.

At stations where the same star can be observed in the course of several series, the previous methods, which entail solving the triangle of position of the star every time, are not employed ; having made the original computation, a differential method is subsequently used.

The formula used is then the following :

$$d(\delta h) = dh_1 = \cos S\ d\delta + \cos \varphi_0 \sin Z\ dH$$

Ordinarily an even declination δ_0 is chosen, and the three angles H_0, Z and S of the triangle of sides $90^\circ - \delta_0$, $90^\circ - \varphi_0$ and $90^\circ - h_0$ are calculated by applying the formulae of BORDA. Only H_0 must be worked out to within 0.01^s of precision. For Z and S, which only enter in the coefficients of differential formulae, a tenth of a degree suffices.

In this triangle, $h_1 = h_0$ and consequently the appropriate formula to apply for calculating the successive observations may be written :

$$\delta h = (\delta - \delta_0) \cos S + (H - H_0) \cos \varphi_0 \sin Z$$

As h and δ are expressed in arc and H in time, the coefficient of $H - H_0$ must be multiplied by 15, and the final result is

$$\delta h = (\delta - \delta_0) \cos S + (H - H_0)\ 15 \cos \varphi_0 \sin Z$$

If we have available tables of natural values of trigonometrical ratios to 8 decimal places, it will be possible to use the following formulae, which are a little more expeditious, instead of BORDA's formulae.

$$\sin^2 \frac{H_0}{2} = \frac{\cos (\varphi_0 - \delta_0) - \sin h_0}{2 \cos \varphi_0 \cos \delta_0}$$

$$\sin S = \frac{\cos \varphi_0}{\cos h_0} \sin H_0. \qquad \sin Z = -\frac{\cos \delta_0}{\cos h_0} \sin H.$$

There can be no indecision as to the values of S and Z, which are always known approximately beforehand.

Finally, in a *permanent observatory*, instead of solving the foregoing triangle for each of the stars met with in the course of the observations, we construct a table, giving H_0, Z, $\sin Z$, $\cos Z$, $\cos S$ and $15 \cos \varphi_0 \sin Z$, for the values φ_0 and h_0 characterizing the station and for fairly closely spaced values of δ.

This document is usually given the name of *table of hour angles*, for H is the only element which needs to be known with much precision. The use of

8-figure logarithms for drawing up this table enables us to ensure an accuracy of 2 or 3 thousandths of a second of time for the hour angle.

The values of δ may be spaced 1' apart in the greater part of the table but must approach 30'', then 10'', for values of δ near $\varphi_0 \pm (90^\circ - h_0)$.

Let a star be of declination δ, for the hour angle of which we have obtained the value H; we enter the table with the nearest even declination δ_0, and by simple reading obtain H_0 and the two coefficients of the differential formula enable us immediately to calculate δh. The values of $\sin Z$ and $\cos Z$ are the coefficients of the equation of condition.

The use of a table of this sort thus considerably shortens the calculations for reduction of equal-altitude observations.

We give below an extract from the table drawn up for the observations with the equilateral-prism astrolabe made at the Observatory of Paris ($\varphi_0 = 48^\circ 50' 10''$).

$\delta_0 = 24^\circ$

'	0'' H_0 -1^h	30'' H_0 -1^h	*cos S*	15 *cos* φ_0 *sin Z*	*sin Z*	*cos Z*	*Z*	'
0	$25^m 42^s,481$	$25^m 46^s,472$	0,8768	6,588	0,667	-0,745	138° 8'	0
1	50 ,459	54 ,443	8764	597	668	744	4	1
....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....
....								....
30	29 35 ,873	29 39 ,663	8652	6,845	693	0,721	136 7	30
31	43 ,450	47 ,234	8648	853	694	720	136 3	31
32	51 ,014	54 ,791	8644	861	695	719	135 59	32

If, for example, we have $\delta = 24^\circ 31' 41'',07$ this value being taken from the ephemeris, we enter the table with $\delta_0 = 24^\circ 31' 30''$ and find $H_0 = -1^h 29^m 47.234^s$; $Z = 136^\circ 0'$; $\cos S = 0.8646$; $15 \cos \varphi_0 \sin Z = 6.857$; $\sin Z = 0.695$; and $\cos Z = -0.719$.

N. B. — The elements included in this table relate to observations made to the eastward; when they are made to the westward, then for Z we take the complement to 360° of the figure in the table and change the sign of H_0, $\sin Z$ and $15 \cos \varphi_0 \sin Z$.

Solution of the equations of condition.

Graphical Method.

(99) As the equations of condition only contain two parameters, δh and Z, an infinite number of graphical solutions can be imagined. The simplest *a priori* appear to be to take Z as abscissa and δh as ordinate or else δh on the vector radius in a direction Z, and to draw the mean curve passing

as closely as possible through the points obtained. But this sine curve or Pascal's limaçon is difficult to draw or to make use of.

There exists, on the other hand, a very elegant graphical solution, arrived at either from consideration of the actual form of the equation of condition or by geometrical interpretation of the equal-altitude method.

The equation of condition is, in effect, that of a line of bearing $Z + 90^o$, passing at a distance δh from the origin of co-ordinates, and also tangential to the circle of centre x,y and radius z.

On the celestial sphere, let us consider the co-ordinates φ_0, t_0 (zenith of the estimated position) and the stars observed, of which the Right Ascensions will have been increased by $(1 + m)(t_0 - t)$. The spherical distances of the stars from the estimated position are then $90^o - h_0 - \delta h$. If we draw the small circles whose centres are the stars and whose radii are $90^o - h_0$, they will all be tangents (internally or externally) to a small circle whose centre will be at an equal distance from all the stars, and whose radius will be the difference between this distance and $90^o - h_0$.

The centre of the circle is consequently the exact zenith of the point of observation; its radius is the correction to be made to h_0 to get the altitude of observation (*).

If we represent the portion of the celestial sphere near the zenith on a plane, taking the estimated position as origin of co-ordinates and the meridian of that point as y-axis, a circle of altitude, in view of its small curvature, may be taken as equivalent to its own tangent the equation of which is identical with the equation of condition.

The graphical method consists then in adopting two rectangular axes of co-ordinates, of drawing the altitude lines (bearing $Z + 90^o$ and passing at a distance δh from the origin) and of constructing the circle tangent to these lines.

The ordinate of the centre is the correction to be made to the estimated latitude; its abscissa, divided by $15 \cos \varphi_0$, gives the small term which rectifies the approximate time correction; finally, the radius of the circle is the correction to the estimated altitude.

According to the degree of precision required, we adopt a scale varying from 1 to 4 mm. per second of arc. It is well in drawing the circle for its radius to be at least 40 mm.; thus we may add to all the δh's an arbitrary constant which need only be subtracted from the value of the radius of the circle to obtain the correction to the estimated altitude.

On account of the various errors of observation or in the position of the stars, of variations of refraction and of faulty rate of the hack-watch, the altitude position lines are not rigorously tangent to the circle to be drawn. The operation is effected by trial and error, first determining the radius of the circle at the mean of the lines relative to the circummeridian stars, then the centre is found. It is convenient to draw several circles of very nearly equal radii and to try them in succession on the graph.

The lines of altitude corresponding to observations which are judged to be only passable are drawn with broken lines; in finding the circle they are given less weight than the lines corresponding to good observations which are drawn with full lines.

(*) If the altitude of observation were exactly known, each of the circles of altitude would furnish a *geometrical locus* of the zenith at the station. The position of this zenith would be selected for the best from among the intersections of all the geometrical loci corresponding to the various observations.

Our ignorance of the exact instrumental altitude results in these loci being approximate. We seek the quantity by which they must all be shifted parallel to themselves, so that they should give the best intersection; this is equivalent to seeking the most probable enveloping circle for these loci.

(100) When the series comprises more than some thirty stars, the graph is confused and it is difficult to take all the lines of altitude into account in looking for the solution.

We can then group neighbouring lines of azimuth in a single mean line, whose δh and Z are the mean of the δh's and Z's. These mean lines are weighted according to the number of lines which they replace. We can then weight the mean line itself, so as to allow for appreciations made at the time of observation.

The enveloping circle of the altitude lines must pass the nearer to a line according as its weight is greater.

Let us try to evaluate the error resulting from the introduction of a mean line.

Suppose we wish to establish the mean of the n lines whose equations are

$$x \sin Z_1 + y \cos Z_1 - z + \delta h_1 = 0,$$
$$x \sin Z_2 + y \cos Z_2 - z + \delta h_2 = 0,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$x \sin Z_n + y \cos Z_n - z + \delta h_n = 0.$$

The corresponding azimuth will be $Z_0 = \frac{1}{n} \Sigma Z$.

The tangent of the enveloping circle for the azimuth Z_0 passes at a distance d from the estimated position such that

$$x \sin Z_0 + y \cos Z_0 - z + d = 0.$$

The mean line is parallel to it and passes at a distance $\frac{1}{n} \Sigma \delta h$ from this point.

The displacement from this line is therefore

$$k = \frac{1}{n} \Sigma \delta h - d = x (\sin Z_0 - \frac{1}{n} \Sigma \sin Z) + y (\cos Z_0 - \frac{1}{n} \Sigma \cos Z).$$

Let δZ_1, δZ_2.............. be the differences between the various azimuths and their mean.

Neglecting the third powers of these differences, and noticing that their sum is zero, we may write

$$k = (z \sin Z_0 + y \cos Z_0) \frac{1}{2n} \Sigma (\delta Z^2).$$

The expression in brackets is equal to $z - d$, whose maximum is the distance D between the centre of the circle and the estimated position.

Let V be the difference in azimuth of the extreme lines of the group; $\Sigma (\delta Z)^2$ does not exceed $\frac{nV^2}{4}$, and consequently k at its greatest is equal to $\frac{DV^2}{8}$; but generally it is much less than this quantity, on account of the distribution in azimuth of the lines inside the pencil of amplitude V and also of the value of $z - d$.

Further, if the pencil is fairly narrow, $\frac{DV^2}{8}$ is fairly small; for example, if $D = 5''$ and $V = 16^\circ$, k at the most is equal to $0.05''$.

Graph of Observations
made at San Diego
on 17th November 1933 :

S. O. M. Prismatic Astrolabe (large model) N° 1.

Graphique des Observations
effectuées le 17 Novembre 1933
à San Diego :

Astrolabe à prisme grand modèle S. O. M. N° 1

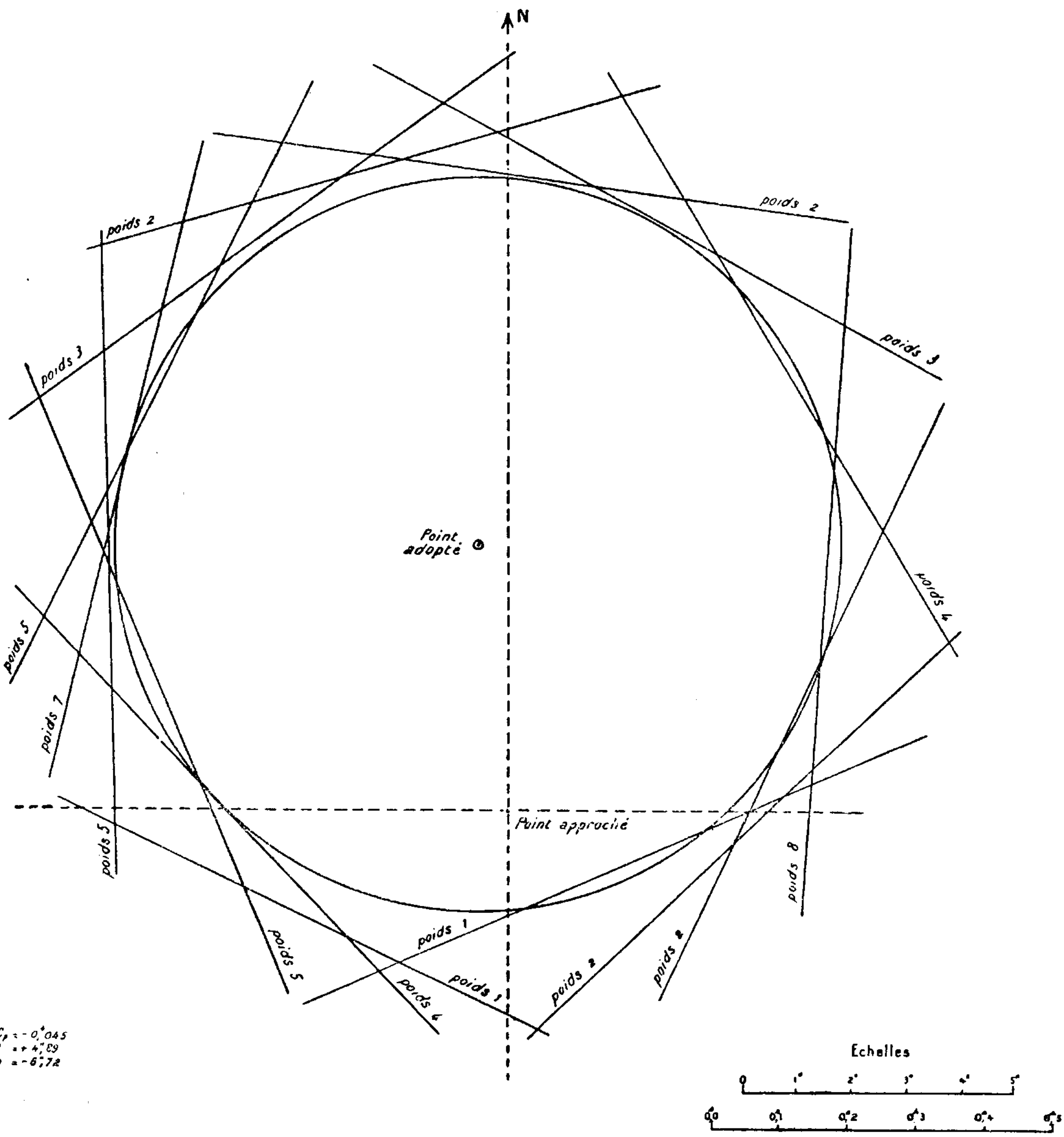

Fig. 43

Note. - *Each altitude line has been weighted by the number of real altitude lines of which it is the mean line* (see § 100).

Remarque : *Pour chaque droite de hauteur, le poids a été pris égal au nombre de droites de hauteur réelles dont elle est la droite moyenne* (voir § 100).

This simplification is also of great advantage in series containing numerous stars. The latter are classified by increasing azimuths, and a mean is taken of the δh's and Z's by pencils of 15 to 20 degrees, or more, according to the value of D and the error tolerated in the mean lines. Thus only some fifteen lines of altitude are obtained, sufficiently accurate to make it possible to make a graph on a scale of double the usual one. We have used 10 mm. for 1" of arc, with series of sixty odd stars taken with a prismatic astrolabe of $\times$ 120 magnification (Fig. 43).

Methods of Computation.

(101) Graphic solution has great advantages over mathematical methods. It is appreciably quicker; one can appreciate from a glance the relative value of each observation, show up the abnormal discrepancies, and eliminate defective elements at need, without reverting to lengthy calculation.

On the other hand, it includes a degree of personal interpretation, while the other methods lead different computers to identical results, which facilitates checking but does not give any greater accuracy; the tentative attempts of the graphic method correspond to the uncertainty of the result.

The *method of least squares* is the most rational, but, as it necessitates long calculation, it will only be used exceptionally, for instance in high precision observations. It enables us to obtain readily the values of the mean errors, without entailing the computation of the residuals.

To avoid errors, the computation must be methodically set out. We shall not give the proper arrangement here, as it figures in the text-books.

The CAUCHY-TISSERAND method is quicker than the foregoing, besides being appreciably more accurate.

To form the final equation relative to an unknown, we first suppress the equations of condition in which the coefficient of this unknown is less in absolute value than one-third of the greatest of them; then we change the sign of the equation retained, where the coefficient of the unknown is negative; finally we sum the equations retained.

In the equation thus obtained, the coefficient of the unknown in question is the largest possible.

We then solve the system of final equations by common algebraic methods.

For the unknown z of our equations of condition the coefficient is constant and equal to -1. The corresponding final equation is the sum of all the equations.

(102) Before dealing with the equations of condition by a method of computation, it will be prudent to look summarily over the result of working out the δh's.

For this purpose we mark, on a graph, a point of abscissa Z and ordinate δh corresponding to each star, and draw as well as possible the mean sine curve passing through these points.

The stars whose differences of ordinates between themselves and the curve are more than 2 to 4", according to the magnification of the instrument and the method of observation, will be eliminated, if they have been noted as mediocre at the time of observation. If the annotation has been good, we shall look for a material error in the later part of the operations, ensuing from the times of passage or from the calculations.

If no fault is found, we shall eliminate the star, attributing the discrepancy to unknown causes, for example an unnoticed disturbance of the mercury at the moment of coincidence.

But we must be very circumspect in doing this. The sine curve is in effect difficult to draw, if there are few points and if they are badly spaced in azimuth. Further, the points, even when numerous, are always somewhat scattered along the curve, especially when the conditions of observation have not been excellent, also we must not eliminate except for the precise reason that the star has fallen definitely outside the zone of dispersion.

Comparison of Methods of solution.

(103) We have used the three methods, graphical, least squares and CAUCHY-TISSERAND, to deal with ten series made at the Observatory of Paris in October 1926, during the revision of the world longitudes. The instrument used was a JOBIN prismatic astrolabe, large model, and each series comprised an average of 37 stars.

The latitudes obtained for the south face of the Observatory were:

by least squares	$48°50'$ $11.39'' \pm 0.14''$;
by CAUCHY-TISSERAND	$11.44'' \pm 0.14''$;
by graph	$11.29'' \pm 0.23''$.

For the time corrections we got as a mean of the differences as compared with the rectified corrections published by the Bureau International de l'Heure, the following values.

by least squares	$0.030^s \pm 0.009$;
by CAUCHY-TISSERAND	$0.027^s \pm 0.008$;
by graph	$0.028^s \pm 0.015$.

All these figures are comparable, and if the graphical solution leads to slightly larger mean differences, it is none the less very acceptable on account of its rapidity, simplicity and elasticity, and it is worthy of use in the majority of cases.

If, however, one insists on a mathematical method, preference will be given to that of CAUCHY-TISSERAND, which is shorter than, and as accurate as, that of the least squares.

Correction for aberration.

(104) The diurnal aberration has the effect of decreasing the true altitude of a star by $0.311'' \cos \varphi \sin Z \sin h$, or, expressed in time, $0.021^s \cos \varphi \sin Z \sin h$.

We must therefore reduce the calculated altitude h, by this quantity, and the equation of condition will be written

$$d\varphi \cos Z + (dCp - 0.021 \sin h) \cos \varphi \sin Z - dh + \delta h = 0.$$

For a given type of instrument, $0.021 \sin h$ is a constant; thus the computation will be performed without taking it into account, but it will be added to the Cp obtained.

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CONCLUSIONS.

(105) Before concluding this study it seems advisable to make a summary comparison of the instruments which we have described and try by this juxtaposition to explain why a method which appears so simple as the method of equal altitudes has led to so many types of apparatus.

The method consists in noting the instant at which a star reaches a determined altitude, equal to half the angle between the rays of light from the star and the rays from its reflection in a bath of mercury.

The equal-altitude instruments at present in use, ignoring the bent telescope, include an optical device enabling these two beams of light to be directed into a telescope. When they become parallel the images coincide, and the star is at a well-determined altitude depending on the optical device in use.

The accuracy of the results furnished is thus a function of three different elements :

(1) Accuracy in the reference to the vertical given by the mercury trough;
(2) Constancy of the altitude of observation in the course of a single series;
(3) Accuracy in the recording of the instant when the star reaches this altitude.

The errors due to the mercury trough are small when care is taken to clean it frequently, so that the images may be as sharp as possible. It is also desirable that the layer of mercury should be sufficiently thick to ensure the surface being truly horizontal.

Constancy of the altitude of observation.

(106) The altitude of observation is not solely the instrumental altitude resulting from the construction of the apparatus ; it must be reduced by the amount of the atmospheric refraction, which depends chiefly on the temperature. Further, factors of an external nature such as the imperfect horizontality of the trough or short-period variations in the refraction, can also influence it ; none the less their effect is only shown by an increase in the size of the accidental errors.

The fact of receiving two beams of rays in the same telescope necessarily entails cutting the object-glass into two parts, each one being reserved for one of the beams. The quality of the images always suffers from it ; but the faults differ according to what line of separation is adopted. In attempting to remedy the fault which they thought was the most important, the constructors have chosen various methods of separation, and consequently have produced various types of optical devices. It is the choice of the optical part which enables one to differentiate between the equal-altitude instruments used nowadays.

In the French astrolabe, as the object-glass is cut horizontally by the prism, the images are elongated in a vertical direction, and a change of focus of the eyepiece (*) modifies the altitude of observation. In a medium model apparatus, for example, a displacement of 1/100 mm. of the eyepiece with reference to the object-glass entails a variation of more than a tenth of a second of arc in the altitude observed.

The British 45° astrolabe (without the device for multiplying the sights) escapes this defect, but one of the images is given by the marginal part of the object-glass ; it is mediocre and more spread out than that furnished by the central part. Further, the prism, consisting of two prisms stuck together, probably entails the altitude being less strictly preserved (†).

(*) Which may be accidental or caused by a variation of temperature.

(†) The same comments might be made about the instruments designed by various experimenters, which include two prisms adhering along a half-silvered face. Any arrangement of half-silvering, assuming the centre of the stuck face as centre of the figure, will give images furnished by beams centred on the object-glass, the coincidence of which will not change with the focussing of the eyepiece ; in particular, if the half-silvering is circular, we find ourselves back at the arrangement of the British astrolabe.

If the half-silvering admits a horizontal plane of symmetry, the images will be elongated horizontally, and the coincidence in altitude will be independent of the focus ; the silvering of the half of the adhering face situated on one side of the vertical plane of symmetry of the apparatus corresponds, from the point of view of the images, to one of the crossed mirrors of the *circumzenithal*.

If, finally, the half-silvering is done in parallel vertical strips, the images are elongated horizontally and entirely independently of the extension of the eyepeice, but on account of diffraction at the edges of the silvering, they will be all the more misty as the strips are more numerous.

In the circumzenithal (without the arrangement for multiplying the sights) the images are better, being equal and elongated horizontally, but the angle of observation is obtained by two mirrors fixed on a metal frame. In spite of the care taken in its construction, this heterogeneous assembly seems less advantageous for constancy of angle than a single block of material.

In these various instruments it has been found, by analyzing the results, that the optical part used ensures amply sufficient constancy for the altitude of observation.

In the bent telescope, the idea of which is a little different, the stellar image given by the entire object-glass is very good, but the altitude of observation only remains constant if the mercury maintains its level to within a few microns.

Accuracy in making the observations.

(107) We shall leave aside the physiological part of the observation, also its mechanical part, which depends on the *poire à tops* and chronograph used; they are the cause of a systematic error, the personal equation (*), and of accidental errors which must be added to the other errors of observation.

We shall deal here with the optical part of the observation only; it depends on the focal length of the object-glass and on the quality of the images, and above all on the magnification employed at the eyepiece. By increasing the latter, experience has shown that the accuracy of an isolated sight becomes greater.

On the subject of the magnification used in equal-altitude instruments, the following remark (†) always holds good: on account of the relative motion of the images, the instant of coincidence is determined with twice the precision that would be obtained from the passage of a single image behind a fine wire in the same telescope. Everything thus takes place as if the magnification employed at the eyepiece were doubled.

The precision of the sight, however, remains considerably less than that associated with the instrumental altitude of observation. If they were equal, three or four stars would suffice in theory to determine the time and latitude with the same precision as that of the instrumental altitude. In practice, however, it would be necessary to double or treble this number of stars, so as to eliminate as far as possible accidental errors arising from short-period variations of refraction, uncertainty in the position of the stars and in the greater or less perfection of the horizontality of the surface of the mercury at the moment of observation.

But on account of the appreciable difference between the accuracy of an isolated sight and that of the instrument itself, it becomes necessary greatly to increase the number of sights, though this should not be overdone, for the conditions of observation change with the time; on account of instrumental deformations, of the variation of the observer's personal equation under the effect of fatigue, and of the fluctuations in the rate of the hack-watch, there is no object in unduly prolonging the series, and some forty stars constitutes the maximum which it is merely wasteful to exceed.

Multiplication of sights.

(108) As numerous observations lead to long calculations of preparation and reduction, certain constructors have had the idea of improving, not the accuracy of an isolated sight, but that of an observation, by multiplying the sights on each star.

The procedure used exposes one, however, to loss of a little precision in the constancy of the altitude of observation. It consists in effect of adding

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(*) Which we can measure with the aid of an auxiliary appliance.

(†) Clearly not applicable to the bent telescope.

thin auxiliary prisms, fixed or movable, to the main prism, enabling the passage of a star to be observed at a certain number of adjacent altitudes, ten or so for example, these being always the same and as far as possible equidistant and symmetrical in pairs with respect to the instrumental altitude (*).

When these deviators are movable, it is necessary that they should always take up their position within a few hundredths of a millimetre, so as to maintain a sufficient degree of precision in the definition of the instrumental altitude.

Though these material improvements may be of the same order of precision as the construction of the micrometer of a transit instrument, it remains necessary to observe the greatest prudence in their use; the errors which they can introduce in the results do not become evident separately but add their effect to other errors arising from various causes, which the observations, even when repeated, do not always make it possible to disengage from each other.

With appliances for multiple sights, the degree of accuracy of the observations is shown by the mean differences between the sights relative to each star, as well as by the mean differences of the altitudes observed during the series, provided the number of stars be at least ten or so.

Observation of stars very near the meridian is of no advantage, for their movement in altitude is slow and does not vary uniformly with the time. For this reason it is well, in determining the latitude, to use stars sufficiently far from the meridian, which may be convenient also for determining the time. The most rational proceeding consists in observing stars at 45° or so from the meridian, to northward and southward; two or three stars in each quadrant seem sufficient in this case.

(109) To sum up, equal-altitude instruments — like all man-made works — are not perfect, and, alongside their numerous advantages, have certain inevitable defects. The latter, however, are small; the errors arising from them are less than those which were accepted, not so long ago, in the majority of the instruments which aimed at the same goal.

They thus mark definite progress in the domain of observation, and there is every reason to hope that after the present transitional period, in which we measure the personal equation, it will become possible to eliminate this error during the actual course of the observations; these instruments will then be capable of contributing not only precise determinations of time and latitude, but also improvements to the right ascensions and more especially the declinations of the stars.

(*) See the circumzenithal apparatus and the pentagonal-prism astrolabe.

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