THE USE OF AVIATION IN SURVEYING.

by

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In *The Hydrographic Review*, Vol. VIII, No. 2, November 1931, pp. 11 et seq., we showed a method of determining the inclination of a photograph by a comparison between the angles of the triangles formed by three points of known position on the ground and by their images on the photograph. Mr. E. WOLF, Technical Adviser to the Army Geographical Service of Rio de Janeiro, has discussed the same question in the *Rivista Militar Brasileira*, No. 2, 1930, and in *Bildmessung und Luftbildwesen*, March 1933, pp. 10-20; September 1934, pp. 128-142, and December 1934, pp. 190-195.

Neglecting deformations of the second degree due to the inclination of the photograph, which must not exceed a few degrees, he works out this inclination (as well as the orientation) by means of linear equations. The method which we ourselves indicated, and which is almost entirely graphical, seems to us to give the required precision more rapidly on condition that the angular differences be very carefully determined. It is advisable, with this object in view, to calculate the angles on the photograph by means of the co-ordinates of the apices of these angles, measured with a comparator, and to obtain the angles on the ground from the results of the triangulation.

With Mr. WOLF's studies as a basis, but keeping as much as possible to graphical methods, here are three other methods for obtaining the inclination and direction of photographs which are only slightly tilted.

(a) *Method employing comparison of length of sides.*

Let $A'B'$ be the image on the photograph of a side ab, Q the foot of the perpendicular dropped on to this line from the principal point O, and N the point of symmetry of the point Q with respect to the middle of $A'B'$; we have called this point N the *characteristic point* of the segment and have shown (*Annales Hydrographiques* for 1917, pp. 72-76) that the length of the line $A'B'$, after rectifying through an angle i (in radian measurement), will be increased algebraically by the product of the distance between the point N and the axis of rotation and the quantity $i \dfrac{A'B'}{f}$, where f is the focal length of the object-glass. The correction is positive when the effect of the rotation is to increase the distance between the characteristic point and the centre of perspective (see Fig. 1).

The axis of rotation is a perpendicular to the line of greatest slope passing through the isocentre I; but we shall make it pass through the principal point O which is only a very short distance from it. This consequently only alters the 2nd degree terms of the correction, and we neglect them here where we are supposing that the inclination i is a very small quantity of the 1st degree.

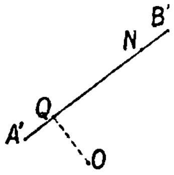

FIG. 1.

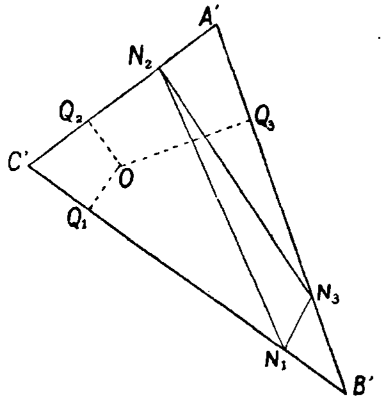

FIG. 2.

If, on the photograph, l_1, l_2, l_3 are the lengths of the sides $B'C'$, $C'A'$, $A'B'$ (see Fig. 2), and d_1, d_2, d_3 the distances from the axis of rotation of their characteristic points N_1, N_2, N_3, the lengths of the sides after correction will be

$$l_1\left(1 + i\,\frac{d_1}{f}\right); \qquad l_2\left(1 + i\,\frac{d_2}{f}\right); \qquad l_3\left(1 + i\,\frac{d_3}{f}\right).$$

All three of these lengths must be proportional to the lengths of the corresponding sides on the ground, if the rectification has brought the plate parallel to the plane through the three points on the ground. Using the co-ordinates of the apices, measured with a comparator, to calculate the ratios $\frac{l}{\lambda_1}, \frac{l}{\lambda_2}, \frac{l}{\lambda_3}$ of the lengths l_1, l_2, l_3 to the corresponding lengths on the ground, calculated from the results of triangulation, we can thus write :

$$(1) \qquad \frac{1}{\lambda_1}\left(1 + i\,\frac{d_1}{f}\right) = \frac{1}{\lambda_2}\left(1 + i\,\frac{d_2}{f}\right) = \frac{1}{\lambda_3}\left(1 + i\,\frac{d_3}{f}\right);$$

whence, by eliminating i, we get the relation :

$$(2) \qquad d_1(\lambda_2 - \lambda_3) + d_2(\lambda_3 - \lambda_1) + d_3(\lambda_1 - \lambda_2) = 0.$$

Let us consider the factors of the quantities d as parallel forces applied to the points N_1, N_2, N_3; their total resultant is nil and one of them of is contrary sign to the other two. It is easy to calculate the point of application on one of the sides of the triangle $N_1N_2N_3$ of the resultant of the two forces of similar sign; it is only necessary to join it to the opposite vertex of this triangle to have the direction of the axis of rotation. A parallel to this direction, taken through O, will enable us to measure graphically the lengths d_1, d_2, d_3; then to calculate i by means of formulae derived from equations (1) :

$$i = f\,\frac{\lambda_1 - \lambda_2}{\lambda_2 d_1 - \lambda_1 d_2} = f\,\frac{\lambda_2 - \lambda_3}{\lambda_3 d_2 - \lambda\, d_3}.$$

The points I and V will be on the line of greatest slope, perpendicular to the axis of rotation, drawn through O, at distances

$$OI = \frac{OV}{2} = f\,tg\,\frac{i}{2}\,.$$

The angle i, thus determined, is the angle which at the moment of exposure the plane of the photographic plate made with the plane through the three points a, b, and c on the ground. It is only the angle with the horizontal plane if these three points are in a horizontal plane.

The altitude H of the aeroplane above the plane abc will be given by the three values

$$H = \frac{\lambda_1 f}{1 + i\dfrac{d_1}{f}} = \frac{\lambda_2 f}{1 + i\dfrac{d_2}{f}} = \frac{\lambda_3 f}{1 + i\dfrac{d_3}{f}},$$

which must be equal for practical purposes if the inclination is small enough and the approximation close enough.

(b) *Conjugate photographs — Comparison of angles.*

It is known (see *The Hydrographic Review*, Vol. VIII, No. 2, pp. 17-19) that two conjugate photographs can be placed in the relative positions which they occupied in space at the moment of exposure by the consideration of five common points identified on the two photographs, without any point of reference on the ground being known.

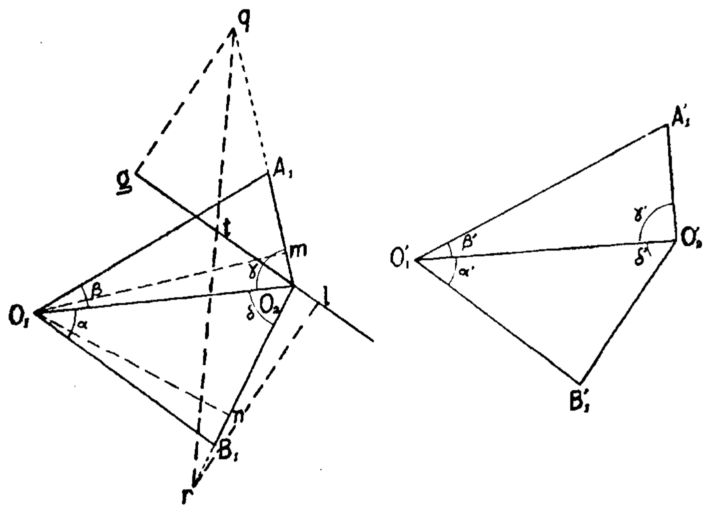

FIG. 3.

It is sufficient to consider only four points if it is known, in addition, that these four points are on the same plane on the ground, without their positions having to be known as well. This case will hardly arise unless the ground photographed is plane; and one can then choose the four points in such a way as to make the solution particularly simple. Let us take the points O_1 and O_2, O'_1 and O'_2 (Fig. 3) corresponding to the principal points of the two photographs, and any two other points A_1, B_1, A'_1, B'_1. We will assume as before that the negatives are only very slightly inclined from the plane of the ground and that terms in the second degree are negligible.

On the first photograph the angles α, β of apex O_1 are equal to the corresponding angles on the ground ; on the second, the same applies to the angles γ', δ' of apex O'_2. The differences $\beta - \beta'$ and $\alpha - \alpha'$ are thus due to the inclination i' of the second negative from the ground. The differences $\gamma' - \gamma$ and $\delta' - \delta$ are due to the inclination i of the first negative from the ground.

Applying the method shown in *The Hydrographic Review*, Vol. VIII, No. 2, we shall draw, through O_1, parallels O_1q and O_1r to the sides O_2A_1 and O_2B_1, equal respectively to the distances of these sides from O_1. The point t which divides the segment rq proportionally to the differences $\gamma' - \gamma$ and $\delta' - \delta$, when joined to O_1 gives the direction perpendicular to the line of greatest slope. The construction is quicker if it is transferred to O_2, as has been done in Figure 3.

We shall then have

$$i = f\frac{\gamma' - \gamma}{qg} = f\frac{\delta' - \delta}{rl}.$$

The inclination of the second negative with the ground will be determined in the same way.

Only the differences of angle need to be very exact, and they will be deduced from the angles calculated by means of co-ordinates measured with a comparator. For the rest, graphical construction is sufficient. If the points O_1, O'_1, O_2, O'_2 cannot be sufficiently clearly identified, they can be replaced by clearer points in their immediate vicinity.

The ratio of the altitudes $\frac{H}{H'}$ of the aeroplane above the plane of the four points on the ground will be equal to the ratio of the lengths of the two corresponding sides on the photographs after rectifying for the inclinations i and i'.

These inclinations and altitude ratios correspond to a horizontal plane if the plane through the four points on the ground is horizontal.

(c) *Conjugate photographs — Comparison of lengths.*

Let us take, as above, the two principal points O_1, O_2, O'_1, O'_2 (or points in their immediate vicinity), and let us choose two other points A_1, B_1, A'_1, B'_1 such as to be, for practical purposes, on the circles described on O_1O_2, $O'_1O'_2$ as diameters (see Fig. 4).

Let l_1, l_2, l_3, l_4, l_5, l'_1, l'_2, l'_3, l'_4, l'_5 be the lengths on the photographs of the four sides of the quadrilateral and of the diagonal passing through the principal points.

The characteristic points of the segments l_2, l_3 and l_5 are at O_2 (or very near O_2 if the angles A_1 and B_1 differ slightly from right angles) ; their factors of magnification $1 + i \frac{d_2}{f}$ will then be the same (d_2 being the distance between the point O_2 and the axis of rotation).

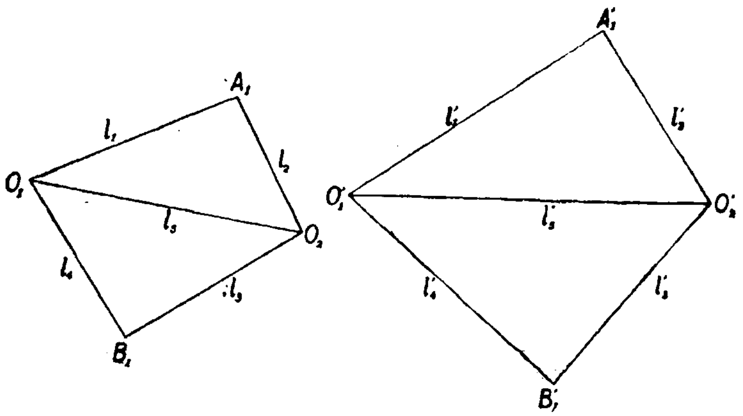

FIG. 4.

On the second photograph, the side l'_2 has its characteristic point at A'_1, at a distance d'_2 from the axis of rotation, the side l'_3 at B'_1, at a distance d'_3 from the axis of rotation, and the diagonal l'_5 at O'_1, at a distance d'_5 from the axis of rotation. We shall thus have between the lengths on the two photographs the relations

$$(3) \qquad \frac{l'_2}{l_2}\left(1 + i' \frac{d'_2}{f}\right) = \frac{l'_3}{l_3}\left(1 + i' \frac{d'_3}{f}\right) = \frac{l'_5}{l_5}\left(1 + i' \frac{d'_5}{f}\right);$$

from which, denoting the ratio $\frac{l}{l'}$ by λ, we get

$$(4) \qquad d'_2 (\lambda_3 - \lambda_5) + d'_3 (\lambda_5 - \lambda_2) + d'_5 (\lambda_2 - \lambda_3) = 0.$$

This equation is analogous to equation (2). We shall treat it in the same way, applying to two of the vertices of the triangle $A'_1B'_1O'_1$ the two parallel forces of similar sign and determining the point of application of their resultant; the line which joins this point to the third vertex gives the direction of the axis of rotation which is a parallel to this direction through O'_2; and the distance of this parallel from the vertices A'_1, B'_1, O'_1 gives the quantities d'_2, d'_3, d'_5; whence equations (3) enable us to determine i'. In the same way the characteristic points of the segments l'_1, l'_4, l'_5 are at O'_1; and those of the segments l_1, l_4, l_5 are at A_1, B_1, O_2, at distances d_1, d_4, d_5 from the axis of rotation. We shall thus have the relations

$$\frac{l_1}{l'_1}\left(1 + i \frac{d_1}{f}\right) = \frac{l_4}{l'_4}\left(1 + i \frac{d_4}{f}\right) = \frac{l_5}{l'_5}\left(1 + i \frac{d_5}{f}\right);$$

from whence we get the equation

$$(5) \qquad d_1\lambda_1 (\lambda_4 - \lambda_5) + d_4\lambda_4 (\lambda_5 - \lambda_1) + d_5\lambda_5 (\lambda_1 - \lambda_4) = 0.$$

We shall apply to two of the vertices of the triangle $A_1B_1O_2$ the two parallel forces of similar sign, determine the point of application of their resultant, and obtain the direction of the axis of rotation by joining this point to the third vertex. To find i we shall continue as we have just done for i'.

The ratio of the altitudes H and H' of the aeroplane at the moment of the exposure will be given by

$$\frac{H}{H'} = \frac{1}{\lambda_5} \frac{1 + i' \frac{d'_5}{f}}{1 + i \frac{d_5}{f}}.$$

The inclination and altitudes are referred to the plane through the four points on the ground; they refer to a horizontal plane if this plane is horizontal.