bottom edge, coincide with the meridian. This operation is so familiar to seamen that we shall not describe it here, but Fig. 3 shows the great benefit of the anticlockwise division of the protractor, whichever quadrant is concerned.

Having thus obtained the line representing the azimuth it is only necessary to determine the intercept on the latitude scale of the plot. For this purpose, returning to Fig. 2, this distance will be laid off along the latitude scale, 25' in the case under discussion, and by drawing through the point 25 of this scale a line parallel to *CA*, we obtain the intercept $A\delta$ which we need only lay off in the required direction, with a pair of dividers, before drawing the altitude line perpendicular to the azimuth.

Suppose after drawing two altitude lines (Fig. 3) we wish to determine the latitude and longitude of the fix *P*. The longitude (121°23') is measured directly on the plot by the fixed longitude scale of the protractor; to obtain the latitude of the point *P* we can use the PRATT protractor by placing it as shown in Fig. 3, reading the latitude (32°21') on the latitude scale of the protractor at its intersection with the line *PM* drawn parallel to *AC*.

H. B.

A.M.L. POSITION LINE SLIDE RULE

(Bygrave Slide Rule)

(Communicated by HENRY HUGHES & SON, Ltd., sole licenced manufacturers, 59 Fenchurch Street, London, E.C.3.)

This slide rule has been designed to calculate the altitude and azimuth of a celestial object as it would be seen from a given point on the earth's surface at a given time.

Theory of the Method

The three points — the Pole, the observer's position, and the sub-solar (or sub-stellar) point — determine a spherical triangle, sufficient elements of which are known to enable an unique solution to be obtained; this triangle is usually solved by direct logarithmic calculation or by the use of special tables based on logarithmic functions.

In order to solve the triangle by a slide rule it was necessary to re-arrange the formulae involved in the solution so that each step involved not more than four logarithms or three numbers.

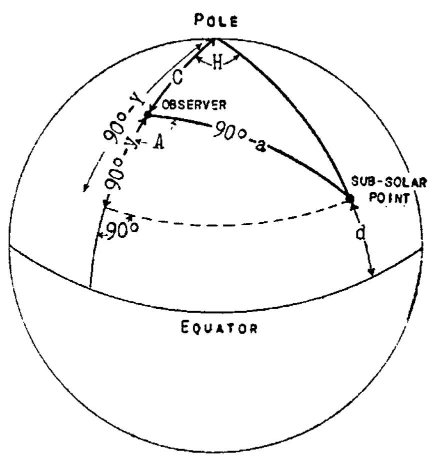

FIG. 1

H = Hour Angle. C = Co-Latitude.
A = Azimuth. d = Declination.
l = Latitude. a = Altitude.

Dotted Line — Perpendicular from Sub-Solar point to Observer's Meridian.

The method adopted is illustrated in Fig. 1, which shows the theoretical diagram and the formulae employed. From the sub-solar point a perpendicular is drawn to the observer's meridian forming two right-angled spherical triangles. These two triangles have the perpendicular and the right angle at the meridian in common; the angles opposite the common side are the Hour Angle and the Azimuth respectively; the sides opposite the right angle are the complements of the declination and the altitude; the parts of the meridian forming the third sides are the complements of the auxiliary angles y and Y respectively, the difference of these being the co-latitude.

By applying Napier's Rules of Circular Parts to the triangle containing the Hour Angle we get

$$\tan y = \frac{\tan d}{\cos H}$$

enabling the second auxiliary angle Y to be obtained by the use of the co-latitude.

The tangent of the perpendicular drawn, the common side of both triangles, can now be determined in terms of the elements of both triangles and these values equated to each other, giving

$$\tan A = \frac{\tan H \cos y}{\cos Y}$$

From the triangle containing the azimuth angle A the relation $\tan a = \cos A \tan Y$ is derived.

On examination of the original spherical triangle it will be at once seen that the altitude could have been found equally well if the perpendicular had been drawn from the observer's position to the meridian through the sub-solar point, in which case the corresponding angle to the azimuth in the above case would be meaningless as regards the Position Line Problem. This enables a check to be applied to any problem worked on the slide rule by interchanging the latitude and declination in which case the final altitudes should agree, although the second false azimuth of the check must be disregarded. As the true azimuth was used to determine the altitude, this has been checked by checking the altitude.

Principle of the slide rule

The azimuth A and the altitude a can be found by three settings of the rule, all three arising from the formula

$$\frac{\tan p}{\tan q} = \frac{\cos m}{\cos n}$$

or $\log \tan p - \log \tan q = \log \cos m - \log \cos n$.

The rule consists in the main of two scales and a cursor:

(1) A fixed scale AB (see Fig. 2) with zero O, on which is a graduation for log cos (in the figure, $ON = -k \log \cos n$ and $OM = -k \log \cos m$).

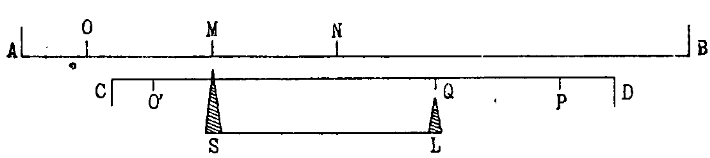

FIG. 2

(2) A movable scale CD with zero O', on which is a graduation for log tan (in the figure, $O'P = k \log \tan p$ and $O'Q = k \log \tan q$).

(3) A cursor with the marks S and L at a fixed distance apart.

We place S at the value m on the scale AB and move the scale CD until the value q on this scale falls on the mark L. Than the cursor is moved until the mark S

coincides with the value n on the scale AB. The mark L will have moved, as a result of this movement, along the scale CD, and comes over the point P on this scale.

S will have moved from M to N, L from Q to P, and so $MN = PQ$.

$$MN = ON - OM = -k \log \cos n + k \log \cos m = k \log \frac{\cos m}{\cos n}.$$

$$PQ = O'P - O'Q = k \log \tan p - k \log \tan q = k \log \frac{\tan p}{\tan q}.$$

PQ and MN being equal, it follows that $\dfrac{\tan p}{\tan q} = \dfrac{\cos m}{\cos n}$.

To be able to get clear readings to within 1', the scale must be very long. If a distance of 1 m. is selected as unit for the graduation (the value k in the relationships used above), then on the scale CD, for 45°, 1 mm. represents 4'.

If it is desired to be able to read every angle between 0°30' and 89°30', the scale must be roughly 4 m. long.

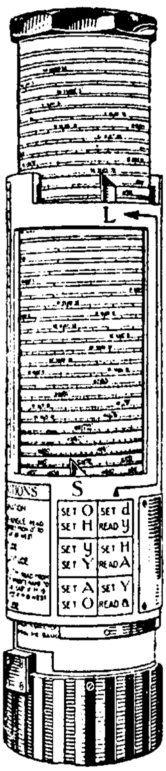

FIG. 3

The scale of the BYGRAVE Slide Rule is 24 ft. long, so that values on the scale CD may be found and read to within one minute.

To find room for this length of scale, the scales AB and CD are drawn as spirals on two cylinders, this having no effect at all on the method of use. Thus the model shown in Fig. 3 is obtained.

Description of the Slide Rule

Two scales are printed on two cylinders sliding with reference to each other. The inner cylinder, on which all results are read, is graduated with log tangents and the spiral scale is about twenty-four feet long. It is divided into minutes of arc throughout its length, and the smallest degree division, occurring at the middle of the scale, is over an inch in length. The outer scale is graduated with log cosines. Two pointers are provided, one for each scale, and are attached to a sliding ring, a stop being provided to register the cosine pointer on the zero of its scale. The pointer, which has to be used for each setting, is clearly marked, and the full instructions for dealing with all possible cases are printed at the bottom of the outer cylinder. After a little practice, the calculation can be performed in about two minutes, and the result should be accurate to one minute of arc, with careful use.

A third cylinder sliding inside the others is used for carrying notes, secured by rubber bands, and can be partly withdrawn from below.

On the back of the slide rule are given scales of dip, refraction, moon's parallax, and for converting time into arc, so that the whole of the reduction of a sight can be done without any reference book whatever, other than the abridged Nautical Almanac.

Directions for use

The data required are:

The Dead Reckoning or Assumed Position of the observer.

The exact Time of the observation.

The Right Ascension and Declination of the body observed.

From these the Local Hour Angle is calculated by the usual methods. Turning to the Slide Rule and holding it by the lower corrugated fibre ring at the bottom in the left hand, the value of y is found by the following operations.—

Set pointer S to zero.

Set pointer L to the declination by sliding and turning the inner cylinder secured to the top end cap of the slide rule,

Set pointer S to the Hour Angle H by turning and sliding the outer cylinder.

Read off the value of y by the pointer L. As two figures (supplementary) are shown to each mark on the scales it is necessary to remember that y is greater or less than 90° according to whether H was greater or less than 90°.

Next form Y from y and c, the co-latitude, adding y to c if the declination and latitude are of the same name and subtracting if they are of opposite names.

The Azimuth A is now found from the Slide Rule by setting the pointer S to y, then the pointer L to the hour angle H, the pointer S to the value of Y, and then reading off the azimuth from the pointer L. Here again we have the rule that A is greater or less than 90° according to whether Y is greater or less than 90°. The Azimuth is noted for future use and the altitude found from the Slide Rule by a third series of settings.

Set pointer S to the Azimuth A, pointer L to the value of Y, return pointer S to zero, and read off the altitude by pointer L.

The calculation is now complete, having found both Azimuth and Altitude.

As described in the Theory of the Method, the calculation can be checked by re-working with the declination and latitude interchanged, in which case the new azimuth is to be rejected but the altitude will be correct.

The Observed Altitude

Before the observed altitude can be compared with the calculated altitude, it must be corrected for Dip of the Horizon (height of eye), Refraction, semi-diameter, and Paral lax, and this is done either by the usual nautical tables or by the auxiliary tables found on the slide rule; while the latter are sufficiently accurate for aircraft observations, for marine work the nautical tables are to be preferred.

Here it is essential to note that if the observations were made with a bubble or artificial horizon, then the corrections for Height of Eye (Dip) and semi-diameter must not be applied.

The Azimuth and the difference of the calculated and corrected observed altitudes are now used to draw the position line.

Calculation of Great Circle Courses

Reference to the figure shows at once that the Slide Rule can be used to calculate Great Circle Courses by the following method :–

The latitude and longitude of the point of departure are used in place of the Dead Reckoning Position.

The latitude of the point of arrival is used instead of the declination.

The difference of the longitude S is used for the Hour Angle.

The Azimuth so found is the Initial Great Circle Course, and the altitude found is the length of the Great Circle Course which, when reduced to minutes of arc, is the length in Geographical Miles.

NOTE ON THE INTERNAL READING MICROSCOPE FITTED TO AZIMUTH CIRCLE N° 13.

COMMUNICATED* BY THE INSTRUMENT SECTION OF THE ARMY GEOGRAPHICAL SERVICE, PARIS.

For taking exact readings of graduated circles it is usual to make use of microscopes fitted with micrometer eyepieces.

In this case, the eyepiece bears a reticle in the plane of the image of the division. The lines of the reticle can be made to coincide with this image of the division by a translatory motion governed by a micrometer screw.

(*) *In French.*