ELLIPSOIDS AND TABLES

by

Ingénieur Hydrographe Géneral P. DE VANSSAY DE BLAVOUS, *Director*

By issuing Circular-Letters N° 18-H and N° 20-H of 1927, the International Hydrographic Bureau has been able to collect certain information with reference to the dimensions of the ellipsoids used in the geodetic operations with which hydrographic surveys are connected, and as to the tables based on these dimensions which have been calculated either for geodetic work or for drawing up graduation sheets on Mercator's Projection.

This information is given on the following pages in which the notation set out below is used :—

a major axis of the terrestrial ellipsoid

b minor axis » » »

$\alpha = \dfrac{a - b}{a}$ compression

$e = \sqrt{\dfrac{a^2 - b^2}{a^2}}$ excentricity

N radius of curvature of the perpendicular section

R radius of curvature of the meridien

φ latitude

$\nu^2 = 1 + e^2 \cos^2 \varphi$

Although the earth's sphericity had been recognised in the far past, the dimensions of our globe were measured but relatively recently with any degree of accuracy.

The Academy of Sciences in Paris, which had just been founded, instructed the astronomer PICARD, in 1665, to measure the length of a degree of the meridian. In 1669, PICARD produced a value, remarkably accurate for the time, by means of which NEWTON verified his law of attraction which, until then, had appeared to him to be in contradiction with the facts because of the erroneous dimensions of the earth accepted.

Then, from 1680 to 1718, CASSINI and LAHIRE undertook to measure the *Méridienne de France*, from Dunkerque to Perpignan; in 1739 these measurements were taken up again by LACAILLE and CASSINI DE THURY, and again in 1792 by DELAMBRE and MÉCHAIN. who extended the *Méridienne* as far as Barcelona.

The compression of the earth at the poles had been propounded by NEWTON in his book *The Principles of Natural Philosophy*, published in 1687; and it was to determine the existence of this that in 1736 the Academy of Sciences caused measurements of arcs of meridians in Lapland and Peru, to be made under the direction of CLAIRAUT and MAUPERTUIS in one case and of BOUGUER and LA CONDAMINE in the other.

The *Commission des Poids et Mesures*, instructed in 1791 to determine the unit of length, the latter being fixed at the tenmillionth part of a quarter meridian, calculated this length from the measurements of the *Méridienne de France* and of the arc in Peru; it adopted the ellipsoid :—

$$(1799) \quad a = 6\,375\,739 \qquad \frac{1}{\alpha} = 334.29$$

Later BIOT and ARAGO, by checking the calculations of these arcs, were led to adopt the dimensions :—

$$(1810) \quad a = 6\,376\,986 \qquad \frac{1}{\alpha} = 308.64$$

which were adopted for the calculations of the map of France on the scale 1/80.000, called the "Carte de l'Etat-Major" (General Staff Map) and of the map of Belgium.

But an error had crept into these calculations, and the correct figures were given by PUISSANT in his *Traité de Géodésie.*

$$a = 6\,376\,951 \qquad \frac{1}{\alpha} = 309.61$$

It was at about the same period that BEAUTEMPS-BEAUPRÉ, then commencing the survey of the coasts of France, adopted in 1816, the dimensions of the ellipsoid which ever since that time have been used for the construction of the Hydrographic charts of these coasts :—

$$(1816) \quad a = 6\,376\,522 \qquad \frac{1}{\alpha} = 308.65$$

The manuscript tables, calculated for French areas, have not been published.

Measurements of arcs of meridians and researches on the shape of the earth multiplied rapidly, and as a result, numerous authors put forward dimensions of the terrestrial ellipsoid which will be briefly summarised, a few of the tables which have been drawn up in order to facilitate their use, being mentioned.

DATE.	AUTHOR.	a (*)	$\frac{1}{\alpha}$	LENGTH OF THE QUADRANT IN METRES.
1819	WALBECK	6,376,895	302.78	10,000,268
1828	SCHMIDT	6,376,959	297.65	10,000,075
1830	EVEREST	6,377,276	300.802	
1837	AIRY	6,377,541.45	299,32	10,000,976
1839	FRANCŒUR	6,377,116	305	
1841	BESSEL	6,377,397.15	299.1528	10,000,855.76
1847	EVEREST	6,376,634	311.04	
1858	CLARKE	6,378,294	294.26	10,001,984
1863	PRATT	6,378,245	295.26	10,001,924
1866	CLARKE	6,378,206.4	294.98	10,001,888
1868	FISCHER	6,378,338	288.50	10,001,714
1872	LISTING		289	10,000,218
1877	SCHOTT	6,378,054,3	305.48	10,002,232
1880	CLARKE	6,378,249.17	293.465	10,001,869
1882	GERMAIN	6,378,284	294.3	10,001,965
1907	HELMERT	6,378,200	298.3	
1909	HAYFORD	6,378,388	297.0	

WALBECK (1819) $a = 6\,376\,895 \quad \frac{1}{\alpha} = 302.78$

These dimensions have been used for the maps of POLAND.

AIRY (1837) $a = 6\,377\,491 \quad \frac{1}{\alpha} = 299.32$

These data are used by the Ordnance Survey for the maps of the UNITED KINGDOM. This Department has published tables which give the values of $\log \frac{1}{N \sin 1''}$ and of $\log \frac{1}{R \sin 1''}$ to 10 places of decimals. (Principal Triangulation, page 675).

FRANCŒUR (1839) $a = 6\,377\,116 \quad \frac{1}{\alpha} = 305$

This compression, given by FRANCŒUR in his *Géodésie*, is deduced from

(*) Slightly different figures were sometimes given by the authors in their publications.

lunar inequalities. These are the dimensions employed by BÉGAT in his *Traité de Géodésie à l'usage des Marins* where the values of: $\log N$, $\log \frac{1}{N \sin 1''}$, $\log R$, $\log \frac{1}{R \sin 1''}$, $\log \nu^2$, are given for every 30', between 0° and 90°, to 7 places of decimals.

BESSEL (1841) $a = 6\,377\,397.15 \quad \frac{1}{\alpha} = 299.1528$

These dimensions were used in many European countries which still continue to use them in their geodetic work, among others: DENMARK, GERMANY, GREECE, HOLLAND, ITALY, (since 1922) JAPAN, NORWAY, SPAIN, SWEDEN.

The UNITED STATES OF AMERICA have employed them; later on it will be seen that at present they use CLARKE'S ellipsoid (1866). Until 1897, JAPAN had employed the ellipsoid of BESSEL; then it used that of CLARKE (1866). This country now employs that of BESSEL.

The tables relative to BESSEL'S ellipsoid are to be found in: JORDAN-*Vermessungskunde Teil III. Anhang.* He gives therein the values of $\log N$, $\log R$, $\log \sqrt{1 - e^2 \sin^2 \varphi}$ $\log \frac{1 - e^2 \sin^2 \varphi}{1 - e^2}$, for every minute between 0° and 90°, to 7 or 8 places of decimals. They are also to be found in ALBRECHT-*Formeln und Hilfstafeln für geographische Ortbestimmungen,* which gives $\log N$, $\log R$, and $\log \sqrt{RN}$ between 0° and 65°.

In *Astronomisch-Geodätische Hilfstafeln* by Doctors L. AMBRONN and J. DOMKE, Berlin, 1909, the values will be found in Table 27, of a second of a meridian and of a parallel for every degree from 0° to 90°, to 3 places of decimals; and, in Tables 28 and 29, the values of $\log R$, $\log N$, $\log \sqrt{RN}$, $\log\sqrt{1 - e^2 \sin^2 \varphi}$, $\log \frac{1}{N \sin 1''}$, $\log \frac{1}{R \sin 1''}$ at every 10', are given to 8 places of decimals from 0° to 72° and to 7 places of decimals from 72° to 90°, with several other tables simplifying the calculations.

The Hydrographic Office of GREECE uses tables published in Athens in 1898 by Admiral MATHEOPOULOS, which give :—

(1) The values of $\log N$, $\log \frac{1}{N \sin 1''}$, $\log R$, $\log \frac{R \sin 1''}{1}$ $\log \nu^2$, for every 30' between 0° and 90° to 7 places of decimals;

(2) The length of minutes of meridian and of parallel for every degree to 2 places of decimals.

The *Traité des projections des cartes géographiques* par A. GERMAIN, Paris, Arthur Bertrand, gives the values of $\log N$, $\log R$, N & R for every degree between 0° and 90° to 7 places of decimals for the logarithms, and to the nearest metre in the values, as well as the values of a degree of a meridian and of a parallel to one place of decimals.

CLARKE (1858) $a = 6\,378\,294 \quad \frac{1}{\alpha} = 294.26$

These values are used for the maps of BRITISH TROPICAL AFRICA.

CLARKE (*1866*) $a = 6\,378\,206.4 \quad \frac{1}{\alpha} = 294.98$

These data are used by CANADA, JAPAN (from 1897 to 1922), PORTUGAL, and the UNITED STATES OF AMERICA, in their geodetic operations and for the construction of their terrestrial maps. In the *Smithsonian Geographical Tables* by R. S. WOODWARD (Washington, 1906), the values will be found of *log N*, *log R*, to 7 places of decimals, for every 1' between 0° and 90° (expressed in feet) as well as the lengths of degrees of longitude and latitude, to 3 places of decimals, for every degree between 0° and 90° based on these data.

In 1922, the *Japanese Hydrographic Department* published the *Tables used in Hydrographic Surveying* which give, in Table *M*, the lengths of degrees of latitude to 1 decimal, for every 10' from 0° to 90°; and those of minutes and seconds of latitude, of the degrees, minutes and seconds of longitude to 2 decimals; in table *ZMZ* the values of $log \frac{1}{N \sin 1''}$, $log \frac{1}{R \sin 1''}$ for every minute between 0° and 60° to 7 places of decimal and several other results which facilitate calculations and in table *K* the values of *log N* and *log R* for every degree between 0° and 90° to 7 places of decimals.

CLARKE (*1880*) $a = 6\,378\,249.17 \quad \frac{1}{\alpha} = 293.465$ or $\frac{1}{\alpha} = 293.4663$

This ellipsoid is used by the Geographic Services of FRANCE and of the FRENCH COLONIES (1), by the French Hydrographic Office for the maps of CORSICA, MOROCCO, ALGERIA, SYRIA and INDO-CHINA.

The same ellipsoid is used by the BRITISH and BELGIAN offices also for all geodetic work in AFRICA. It is on this system that the arc of the 30th meridian of Gill in SOUTH AFRICA and the new equatorial arc (Arc de Pérou) have been calculated.

The Royal Geographical Society of London has just issued tables based on this system, giving the lengths of arcs of meridian to 3 places of decimals, starting from the equator, for every 10', with their variation for differences of 1 % in the value of $\frac{1}{\alpha}$, as well as the values of *log N* and *log R*, to 9 places of decimals.

The *Nautical Tables* by the Rev. James INMAN, D. D. London, 1913, give, in feet to 1 decimal, the lengths of the minutes of latitude and longitude for every degree on this system.

---

(1) The Geographic Service has issued the following publication: N° 724, *Eléments de l'ellipsoide de Clarke* (1880), Paris, autographed tables giving the value of *log N*, *log R*, to 8 places of decimals for every *centesimal* 10' between 0° and 58 grades, the values of $log \frac{1}{2NR \sin 1''}$, to 6 places of decimals, as well as the lengths of arcs of parallels and meridians to 3 places of decimals.

The Hydrographic Office published, in N° 949 of *Annales Hydrographiques*, 1911, page 122 Paris, a table of the values of *log N*, *log R*, $log \frac{1}{N \sin 1''}$, $log \frac{1}{R \sin 1''}$ for every 30' between 8° and 22° of latitude, to 7 places of decimals.

GERMAIN (1882) $a = 6\ 378\ 284 \quad \frac{1}{\alpha} = 294.30$

Ingénieur-Hydrographe GERMAIN in his *Traité d'Hydrographie-Tables*, Paris, 1882, gives the values of $log\ R$, $log\ \frac{1}{R\ sin\ 1''}$, $log\ N$, $log\ \frac{1}{N\ sin\ 1''}$, $log\ \nu^2$, to 7 places of decimals at every 30' between 0° and 90°, as well as the lengths of minutes of meridian and parallel to 2 decimals for every degree between 0° and 90° on this system.

These tables are used by the French Hydrographic Office for geodetic calculations in TUNIS and in MADAGASCAR.

For calculating the earth's dimensions, HAYFORD applied the theory of isostasy, which presumes the existence of an isostatic compensation at a depth of about 100 kilometres. He has proposed several dimensions for the ellipsoid at various times, of which only the following will be mentioned :—

HAYFORD (1909) $a = 6\ 378\ 388 \quad \frac{1}{\alpha} = 297$

The compression $\frac{1}{297}$ was adopted by the International Conference on Ephemerides in 1911, then by the *Connaissance des Temps*, the *Nautical Almanach*, *l'Almanaque Nautico* and *the Astronomical Union*. In 1924, the International Conference at Madrid recommended the use of this ellipsoid in order to standardise geodetic calculations. It is used by FINLAND for these calculations.

The *International Geodetic and Geophysic Union* has just completed the calculation of tables relating to this ellipsoid, giving the values of $log\ N$, $log\ R$, $log \sqrt{NR}$, $log \frac{1}{1 - e^2 sin^2 \varphi}$, to 10 places of decimals for every 1' between 0° and 90°, as well as the values of the minutes of parallels to 5 decimal places and the lengths of the arcs of meridians starting from the equator to 3 decimal places.

The tables published by the Royal Geographical Society of London likewise give these lengths of arc of meridian to 3 places of decimals, but according to the Hayford (1909) system. These values, as well as those of $log\ N$, $log\ R$, to 9 decimal places, will also be found in the *Geodetic Tables based on Hayford's figure* by VAISALA (*Veröff. des Finnischen Geod. Inst. Helsinki, 1923*) but only between latitudes 59° and 71°.

The advantage to be gained by using the same ellipsoid is very great, for it simplifies the connection between various geodetic systems and the study of the anomalies in the direction of the plumb-line.

This advantage outweighs those which would be gained by the adoption of an ellipsoid the dimensions of which would be continually undergoing change in order to take account of new measurements.

It is known that the geoid, the surface at sea-level assumed to be produced underneath the continents by canals, is not a perfect ellipsoid. Several proposals have been made to employ geometrical figures more closely approximating to the form of the geoid.

JACOBI showed in 1834 that mechanical law permitted the adoption of an ellipsoid having 3 unequal axes for the shape of the earth. In 1860, General VON SCHUBERT proposed one, the equatorial ellipse of which had a compression of $\frac{1}{8885}$, the majors axis being in the meridian 76°24' E. of Gr. In 1866 CLARKE proposed another one, the equatorial ellipse of which had a compression of $\frac{1}{3281}$; then HELMERT in 1915 and Dr HEISKANEN in 1924 proposed similar figures.

BOWDITCH, CLARKE, PANCKER and RITTER also proposed surfaces of revolution, the generatrice of which were other curves which differed from an ellipse.

E. FERGOLA (1874-75) put forward the hypothesis that the earth might be an ellipsoid, the axis of rotation of which would not coincide with the axis of the figure.

The adoption for each continent of the special ellipsoid which most closely corresponds to the measurements taken thereon, has been proposed also.

However, it is necessary that geodetic calculations be referred to an analytically simple surface ; an ellipsoid having 3 unequal axes would introduce complication into the calculations without at the same time avoiding having to take into account and to examine the difference between the geoid and such ellipsoid. It is particularly important to use a simple surface of reference which is not subject to continual modifications and, from this point of view, the use of the ellipsoid as adopted by the International Conference at Madrid, the elements of which were given in the preceding numbers of the *Hydrographic Review*, cannot be too highly recommended.

A very interesting article by Mr. Arthur R. HINKS on a graphical method of comparison of results arrived at by the use of various ellipsoids, is to be found in the *Geographical Journal* for June 1927.

Taking as a basis CLARKE'S (1880) ellipsoid, for every other ellipsoid, a curve is drawn the abscissae of which are the latitudes, and the ordinates the differences of the distances of the parallels from the equator, as measured on this ellipsoid and on CLARKE'S (1880).

The drawing of these curves is greatly facilitated by the use of the tables published by the Royal Geographical Society of London, which gives the variation of distances from the equator for a change of 1 % of $\frac{1}{\alpha}$. As they also give the variations of *log R* and *log N* for the same amount of change in $\frac{1}{\alpha}$, these tables may be employed for calculations relating to any ellipsoid.

If a triangulation has been calculated for a certain ellipsoid, by marking off the values : geodetic latitude-astronomical latitude, expressed in metres, starting from the curve relating to this ellipsoid, a series of points is obtained the mean curve through which will be parallel to that of the ellipsoid which best suits this triangulation.

One of the important conclusions reached in Mr. HINK'S article is the advantage which exists in determining as many astronomical positions as possible.

TABLES OF MERIDIONAL PARTS.

Before MERCATOR'S invention, navigators used the projection called the "*plane chart*" on which the meridians and the parallels were equidistant straight lines.

Gerhard MERCATOR realised that, in order to show a loxodrome as a straight line, it was necessary that the spacing of latitude should be on a scale which increased with the distance from the equator, ("gradus latitudinorum versus utrumque polum auximus pro encremento parallelorum supra rationem, quam habent ad aequinoctialem"). It is thus that he drew up, in 1569, the first chart of this kind, a copy of which is at the Bibliothèque Nationale in Paris. It appears that he never knew the actual theory of *meridional parts*; on his chart the distance between the parallels -60° and $+70^{\circ}$ is too short by about one 40th.

The law for the accurate spacing of the parallels is due to Edward WRIGHT, who discovered it in 1590, and explained it to his friend BLUNDEVILLE to whom he sent a small table of meridional parts calculated for every degree.

BLUNDEVILLE published this first table of meridional parts in his "*Exercises*", in 1594. In 1599, WRIGHT explained his method in his work: "*The correction of certain errors in navigation*"; in it he gives a table of meridional parts for every 10', and in a second edition, in 1610, a table for every minute (*).

For these calculations, WRIGHT adopted as the length of the minute of meridian between φ and $\varphi + 1'$, the value : secant $(\varphi + 1')$, and he added up the lengths thus obtained. The error is only 0.8' at 70° and 2.1' at 80°.

The true theory was enunciated in 1645 by Henry BOND. again in 1668 by James GREGORY of Aberdeen, and lastly by HALLEY in 1695; but by all the earth was still presumed to be spherical.

It is known that tables of meridional parts for a sphere are in general use for the calculations for loxodromic navigation and they can be used also, as was demonstrated by GUYOU, for certain calculations in nautical astronomy. They are therefore to be found, to 1 or 2 places of decimals, in nearly all volumes of nautical tables.

1) LAMBERT (1728-1777) appears to have been the first to consider the elliptical shape of the earth from the point of view of navigational problems. He has given a table of meridional parts for every 5° taking $\frac{1}{\alpha} = 230$

(*) In this second edition WRIGHT, a real pioneer, recommends the basing of our units of length on the dimension of the degree at the surface of the earth, in order that they may no longer depend upon the uncertain length of a barley-corn.

(See Volume III, of *Land und Himmelscharten* in *Beiträge zum Gebrauche der Mathematik und deren Anvendung* Berlin, 1772)

2) In his treaties on *Navigation on the Elliptical Spheroid, and in his Loxodromes and its shortest track. Memoirs of the Academy of Turin 1788-89,* VALPERGA DI CALUSO gives a table of meridional parts for every degree for a compression of $\frac{1}{231}$.

3) The *Connaissance des Temps for 1793* gives a table of meridional parts by MENDOZA, which was reproduced later in *Tables para los usos de la navegavion i astronomica nautica,* Paris 1808, then in the *Traité de Navigation* by DU BOURGET, Paris, 1808, and in the *Traité de Géodésie à l'usage des Marins,* by BÉGAT, Paris 1839. Therein the meridional parts were calculated to 2 places of decimals for every 30' between latitudes $0°$ and $27°$ and for every 10' between $27°$ and $90°$ for a compression of $\frac{1}{321}$

4) In the *Connaissance des Temps for 1805,* DELAMBRE gives a method of calculation for obtaining the meridional parts of the ellipsoid. This method consists in substituting in the formula for the meridional parts of a sphere, the geocentric latitude, for the latitude. Therefore it is but necessary to enter the table of meridional parts, calculated for the sphere, using the geocentric latitude. In most collections of nautical tables a table will be found giving, for any particular ellipsoid, the convertion of the latitude into geocentric latitude. Since 1915, the *Connaissance des Temps* has giver this table for every degree to an approximation of one-hundredth of a second, for HAYFORD's ellipsoid, the compression of which is $\frac{1}{297}$.

The DELAMBRE method, however, considers the term e^4, of the development of the expression of meridional parts, as negligible; it cannot be used if an accuracy of more than 1 place of decimals is required.

5) RÜMKER, in his *Handbuch der Schiffahrtskunde,* Hamburg, 1844, gives a table of meridional parts to 1 place of decimals for every 1'. This has been reproduced in the *Tables* by GAILLET, Paris, 1867, in the *Lehrbuch der Kartenprojection* by GRETSCHEL, Weimar, 1873 and in the *Traité des projections* by GERMAIN (for every 30' between $0°$ and $28°$, and every 10' from $28°$ to $90°$). It is calculated with the value $\frac{1}{\alpha} = 303$.

It appears that the tables of MENDOZA and RÜMKER are no longer in use. This is not the case for the following tables which, however, are not very numerous.

It is noted that the various Hydrographic Offices do not always bind themselves, in the construction of the MERCATOR projection, to the use of the same compression of the ellipsoid as that employed for the geodetic calculations. The geographical positions of points are retained, nevertheless; the result is that errors occur only in lengths, measured in minutes from the equator, and in angles errors which are very small from the graphical point of view and nearly always negligible. The length of the equatorial radius appears only in the determination of the metric length of a minute at the equator (or, if preferred, of the length of 1' of longitude of the basic parallel) in order to ascertain the scale of the chart. However, it is certainly better to employ the same ellipsoid in both cases.

6) Several tables of meridional parts have been drawn up based on the the system of

$$BESSEL \frac{1}{\alpha} = 299.1528$$

Projection tables for the use of the U. S. Navy comprising a new table of meridional parts for the Mercator projection, with reference to the terrestrial spheroid, Washington, 1869, Government Printing Office.

In these tables the meridional parts are given to 2 places of decimals. They are no longer used in the UNITED STATES but are used in ITALY for the construction of charts.

Formeln und Tafeln zur Berechnung von Mercatornetzen für die Deutschen Admiraltätskarten, Berlin, 1904.

In these tables the meridional parts are given to 2 places of decimals for every 1' between 0° and 86° in Table II. ; they reproduce the UNITED STATES tables mentioned above. Table III, gives the lengths of meridional parts in meres of every 1' of latitude and their sum for every 1' to 3 places of decimals, from 53°5' to 60°, taking the value 1116.661 m. as the length of 1' of longitude at 53°5'. It is the parallel 53°5' which is adopted as the basic parallel for the charts of GERMAN waters, the scales of which lie between $\frac{1}{50,000}$ and $\frac{1}{150,000}$. (For other charts the basic parallel adopted is, in principle, that of the mean latitude of the chart).

The *Tablas Nauticas*, Cadiz, 1921 contain a table of meridional parts to 1 place of decimals for every 10' between 0° and 70° with proportional parts.

The nautical tables published in *Athens* in 1898 by Admiral MATHEOPOULOS give the meridional parts to 3 places of decimals for every 1' between 0° and 80°, and every 10' between 80° and 90° to 2 places of decimals.

The NORWEGIAN HYDROGRAPHIC OFFICE has calculated tables which give the meridional parts, to 3 places of decimals, for every 1' between 57° and 78° (1)

7) Based on *CLARKE'S* system (1866) $\frac{1}{\alpha} = 294.98$, the JAPANESE HYDROGRAPHIC OFFICE published in 1922: *Tables used in Hydrographic Surveying* which give the meridional parts to 3 places of decimals every 5' between 0° and 75°. (Since 1922, JAPAN has used BESSEL's ellipsoid for geodetic calculations, but has kept to that of CLARKE (1866) for the determination of MERCATOR's projection).

PORTUGAL uses tables drawn up by taking the length 1435.3258 m. for the length of the minute of longitude along the basic parallel 39°25'. They give, to 3 places of decimals, the lengths of meridional parts of 1' of latitude

(1) Certain differences will be noted in the last decimal between these tables and the preceding ones.

between 36°25' and 42°25'. It is the above basic parallel which is adopted for all charts of the coasts of PORTUGAL.

8) *CLARKE'S (1880)* system $\frac{1}{\alpha} = 293.465$

The UNITED STATES HYDROGRAPHIC OFFICE uses the *Table of Meridional Parts for the Terrestrial Spheroid* by G. W. LITTLEHALES and J. S. SIEBERT, Washington, 1889, which gives the meridional parts for every 1' between 0° and 79° to 3 places of decimals. The BRITISH ADMIRALTY uses a *Table* of *Meridional Parts for the Terrestrial Spheroid* by John W. ATHERTON, London 1910, which gives the meridional parts to 2 places of decimals for every 1' between 0° and 90°.

Tables drawn up on this system are to be found also in the *American Practical Navigator,* by Nathaniel BOWDITCH, Washington 1925. They are calculated to 1 place of decimals for every 1' between 0° and 80° (1).

9) The *Tables du Traité d'Hydrographie* by GERMAIN, Paris, 1882 give the meridional parts for the compression $\frac{1}{294.30}$, to 3 places of decimals and for every 1' between 0° and 80°, then to 2 places of decimals for every 10' between 80° and 90°

These tables are used in FRANCE for the charts of Tunis and Madagascar; they are used also by the UNITED STATES COAST AND GEODETIC SURVEY which has reproduced them in its Special Publication N° 68 : "*Elements of Map Projection,* Washington, 1921 ". They are employed also by the Hydrographic Office of CHILE which has reproduced the part between 18° and 57° in the *Tablas Hidrograficas* by Pedro G. COOPS, Valparaiso, 1907.

10) The DANISH HYDROGRAPHIC OFFICE uses, for the construction of Danish charts on Mercator's projection, the Danish ellipsoid in which

$$\log a = 6{,}804\,617\,729$$

$$\alpha = \frac{1}{300}$$

$$\log e^2 = 7{,}823\,184\,31$$

Tables by Commander H. O. RAVN, R. D. N., give the lengths, in metres and decimetres, of the meridional parts for minutes, and their sums, reckoned from the parallels of 55°, 62°, and 65° on the cylinders which cut the earth's surface at the parallels 56°, 62° and 65° respectively.

The tables extend from 54° to 58°, from 61°-30' to 62°-30' and from 63° to 67°.

(1) The Hydrographic Office of SIAM has published :—*The Table of Meridian Parts for the use of Mercator construction in Siamese waters, by Lieutenant-Commander Luang Samrauch, R. S. N.* which gives meridional parts to 3 places of decimals for every 1' between 6° and 14°. It appears to have been calculated by means of the formula for the meridional parts for the sphere, in which the latitude has been reduced to its geocentric value by means of Inman's Table of Reduction. Inman's formula is only true when the term e^4 of the development of the expression of meridional parts can be neglected ; therefore the accuracy of the last decimal of the numbers in this table cannot be relied upon.

The charts are constructed by adopting for the minute of longitude on the basic parallel of 10°, the length 1815,6 m.

The few charts issued by this Office which cover areas outside the limits are constructed by means of the tables by G. W. LITTLEHALES and J. S. SIEBERT, of the U. S. Hydrographic Office, in which the compression $\frac{1}{293,465}$, mentioned above, is used.

SWEDEN uses tables calculated by V. af. KLINT in 1851 to 4 places of decimals for every 1' between 0° and 90°, compression $\frac{1}{300}$. These are probably the only tables calculated to 4 places of decimals which exist; they are not printed. They closely resemble those which are drawn up on BESSEL's compression.

It would be most useful, however, to have tables of meridional parts to 4 places of decimals, especially since many countries publish very large scale charts on MERCATOR's projection. The Directing Committee of the International Hydrographic Bureau has decided therefore to publish Tables of Meridional Parts calculated to 5 places of decimals for the sphere and for the ellipsoid having a compression of $\frac{1}{297}$ as adopted at Madrid in 1924. The calculation is now finished.

BIBLIOGRAPHY

1. — **THE MILITARY ENGINEER.**
January-February, 1927, page 19, Importance of Isostasy to Mapping (William Bowie).

2. — **GEOGRAPHICAL JOURNAL.**
August, 1924. The proposed adoption of a standard figure of the earth. (G. T. McCaw).

3. — **GEOGRAPHICAL JOURNAL.**
June, 1927. A graphical discussion of the figure of the earth. (Arthur R. Hinks).

3. — **CLARKE.**
Account of the comparisons of standards of length made at the Ordnance Survey Office, London, 1866.

4. — **HELMERT.**
Die Mathematischen und Physikalischen Theorieen der höheren Geodäsie, mit Untersuchungen über die mathematische Erdgestalt auf Grund der Beobachtungen — Leipsig, 1884.

5. — **HEISKANEN.**
Untersuchungen über Schwerkraft und Isostasie, veroffentlichungen der Finnischen geodätischen Institutes N° 4. — Helsinki, 1924.

ditto.
Schwerkraft und Isostatische Kompensation in Norwegen. — Helsinki, 1926.

ditto.
Die Erddimensionen nach den Europäischen gradmessungen.
(Veroff. Finn. Geod. Inst. N° 6, Helsinki, 1926).

6. — **HAYFORD.**
The figure of the earth and Isostasy — 1909.
Supplementary Investigation in 1909 of the figure of the Earth and Isostasy — 1910.

7. — **F. MARGUET.**
Une histoire de la navigation (1550-1750), translated from the « Elements of Navigation » by Robertson, 1780 — Paris, 1918.

8. — **Dr HEINRICH GRETSCHEL.**
Lehrbuch der Karten-Projection — Weimar, 1873.

9. — **A. GERMAIN.**
Traité des Projections des Cartes Géographiques — Paris.

10. — **GUYOU.**
Tables de poche — Paris — Berget — Levrault — 1884.
Annales Hydrographiques — Paris, 1887-1888-1895.