ON THE CURVATURE OF THE LEAD LINE AND THE CORRECTION FOR ITS INCLINATION. (*)

by

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1. The analytical determination of the curve which the lead line takes up when the ship is under way depends essentially on the hypotheses put forward on the subject of the resistance which the water exerts on the wire.

It does not follow from this that the problem is definitely solved by accepting the hypothesis which is now propounded and which, in the absence of positive experimental data, appears at least to be founded on a reasoning which, to the author's mind, is worthy of special attention. (**)

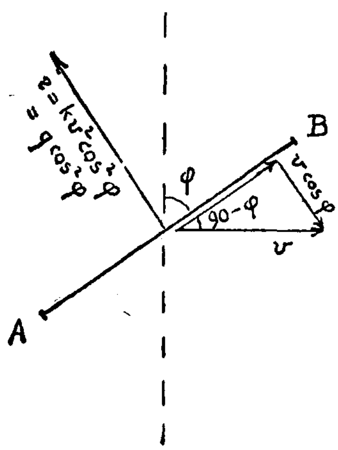

FIG. 1

2. Let AB (Fig. 1) be a rectilinear part of the wire which, for the sake of simplicity, is assumed to be of the unit of length. It has a horizontal speed v which forms an angle $90 - \varphi$ with AB, φ being consequently the inclination of this part of the wire to the vertical.

(*) The idea of this article was suggested by the interesting study by Mr. COURTIER in the *Annales Hydrographiques* 1925-26 (*See* note below).

(**) This hypothesis is accepted in particular for the solution of an analogous problem in the "Cours de Mécanique de l'Ecole Polytechnique" by RÉSAL (1889-90) in which, in the discussion of the problem of the profile assumed by a rectangular sail, considered as being composed of independent bands in a series of vertical planes, this hypothesis is adopted as concerns the pressure exerted by the wind.

The component of v perpendicular to AB, *i. e.* $v \cos \varphi$, gives a hydraulic resistance r perpendicular to the wire and proportional to the square of this same component, *i. e.* to $v^2 \cos^2 \varphi$.

The component of v parallel to AB, $v \sin \varphi$, gives a frictional resistance which it appears legitimate to consider as negligible.

There thus remains the perpendicular resistance only and, consequently, it is inferred that the pressure r exerted by the water on AB *is perpendicular to AB and proportional to the square* of the perpendicular component of the speed v,

$$r = K v^2 \cos^2 \varphi$$

K being a constant.

Seeing that the quantity Kv^2, hereafter referred to as q, is the measure of the resistance of the water on the unit of length of the wire, assumed to be vertical and having a horizontal speed v, then, if $q = K v^2$.

$$(a) \qquad r = q \cos^2 \varphi$$

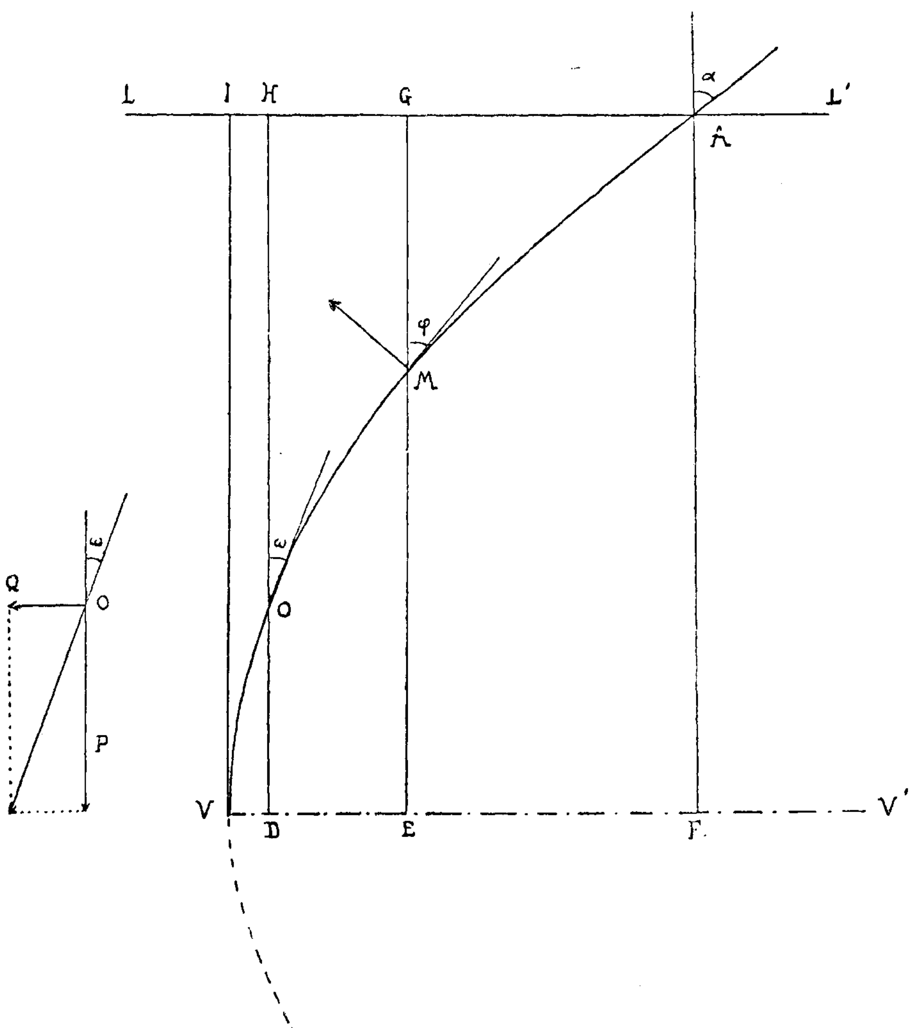

FIG. 2

3. *Form of Equilibrium of the Lead Line.* The curve formed by a lead line towed (without paying out the line) by a ship under way, (*) will now be sought.

An infinitely small linear part ds of the wire at any point M of the lead line (Fig. 2) is in equilibrium under the action of the following forces :

(1) The weight pds, p being the weight of a unit of length of the wire;

(2) The hydraulic resistance rds, perpendicular to the curve and proportional to the square of the perpendicular component of the speed v :

$$r\,ds = Kv^2 \cos^2 \varphi .\ ds.$$
$$= q \cos^2 \varphi .\ ds.$$

φ being the angle between the vertical and the tangent to the curve at the point M, *i. e.* the *inclination of the wire* at M ;

(3) the tensions T and $T + dT$ applied at the two extremities of the part of the wire under consideration (the " element ").

In order to simplify the problem, it will be assumed that the weight of the wire is negligible and, therefore, that the forces which act on the " element " are reduced to the hydraulic resistance and to the difference of the tensions applied to the extremities.

Taking the two straight lines AL (horizontal) and AF (vertical) as axes of the co-ordinates, and stating the equations of equilibrium by projecting these forces on to the axes, then :

$$r\,ds \cos \varphi - d\,(T \sin \varphi) = O$$
$$- r\,ds \sin \varphi - d\,(T \cos \varphi) = O$$

from which :

$$(a) \qquad d\,T = o$$
$$(b) \qquad r\,d\,s - T\,d\,\varphi = o.$$

From (a) may be deduced :

$$(c) \qquad T = \text{constant } T_o$$

The equation (b) may be written :

$$q\ ds\ \cos^2 \varphi = T_o\ d\ \varphi \qquad \text{whence, by taking} \qquad a = \frac{T_o}{q}$$

(*) It will be seen later on that in certain conditions, the results thus obtained can be extended, as may be verified in normal practice of sounding, to the case where " the lead is hove from a ship under way ".

$$(d) \qquad ds = a \frac{d\varphi}{\cos^2 \varphi}$$

On the other hand, by calling the vertical ordinate y and seeing that $dy = ds \cos \varphi$, then from the equation (c):

$$(e) \qquad dy = a \frac{d\varphi}{\cos \varphi}$$

is obtained.

The formulae (d) and (c) solve the problem.

4. The measurement of the arc comprised between the point M and the point V is deduced from the formula (d); the latter being the point at which the tangent to the curve is vertical.

$$(f) \qquad s = \text{arc } AV = a \tan\varphi.$$

This relation shows that the curve, properly speaking, is a catenary the apex of which is V, the axis of symmetry being VV' which is parallel to the direction of the speed v, *i. e.* horizontal. The concavity of the curve is in the direction of movement.

From formula (e) it may be deduced that the difference

$$y - y_m = Y$$

between the ordinate $y = IV$ of the apex V and the ordinate $y_m = GM$ at any point M, *i. e.* the distance of the point M to the axis of the curve, is expressed, as a function of the angle φ between the tangent at M and the vertical, by the relation:

$$(g) \qquad Y = EM = a \log_e \operatorname{tang} \left(45^\circ + \frac{\varphi}{2}\right)$$

in which $\log_e$ is the symbol of the Naperian logarithms.

5. The inclination ω of the wire at its lower extremity O, *i. e.* at the position of the lead, will now be determined.

The tension at θ is directed along the tangent to the curve at this point and its value is To (constant all along the wire) as shown in formula (c).

Let the resultant of the horizontal resistance Q which is offered by the water to the advance of the lead, and of the weight P (in water) of the lead, be considered. Then:

$$(h) \qquad \tan \omega = \frac{Q}{P}$$

Noting also that, as $T_0 = \dfrac{Q}{\sin \omega}$

the parameter $a = \dfrac{T_0}{q}$, which appears in the formulae (*f*) and (*g*), can be expressed in the following form:

$$(i) \qquad a = \frac{Q}{q} \frac{1}{\sin \omega}$$

6. The ratio $\dfrac{Q}{q}$ which appears in the expression of the parameter a is a constant quantity for any given type of wire and for any given lead, *i. e.* for any single given " sounding equipment ".

In fact, both Q, the resistance offered by the water to the advance of the lead, and q, the resistance offered by the water to the unit of length of wire assumed to be vertical, are proportional to the square of the speed v. Therefore, when v varies, the ratio of the two forces remains constant. The ratio $\dfrac{Q}{q}$ is consequently a characteristic quantity for each sounding equipment. It will be indicated in future by the symbol C_o:

$$\frac{Q}{q} = C_o \text{ and therefore :}$$

$$(i') \qquad a = \frac{C_o}{\sin \omega}$$

The inclination ω at the position of the lead is, on the contrary, variable according to the speed v.

In fact, the formula (*h*) shows that, seeing that Q is proportional to v^2 and that P is contant, $\tan \omega$ varies proportionally to the square of the speed.

7. From this may be concluded:

(*a*) *The form of equilibrium of the towed lead line is an arc of a catenary the symetrical axis of which is horizontal and the concavity of which is in the direction of movement. The lead is situated above the axis of the catenary.*

(*b*) Let C_o = the constant of the sounding equipment;
ω = the inclination of the wire to the vertical at the position of the lead;
φ = the inclination of the wire at any point M on the line.

The arc s of the curve lying between any point M on the line and the apex of the curve is given by the formula:

$$(A) \qquad s = \frac{C_o}{\sin \omega} \tan \varphi$$

The vertical distance Y of any point M on the line from the axis of the curve is given by the relation :

$$(B) \qquad Y = \frac{C_o}{\sin \omega} \log_e \tan \left(45^o + \frac{\varphi}{2} \right)$$

8. Then, by formulae (A) and (B) :

$$\frac{Y}{s} = \frac{\log_e \tan \left(45^o + \frac{\varphi}{2} \right)}{\tan \varphi}$$

That particular function of φ which constitutes the second term of this equation, being designated by $H(\varphi)$, *i. e.* by making :

$$H(\varphi) = \frac{\log_e \tan \left(45^o + \frac{\varphi}{2} \right)}{\tan \varphi}$$

$$\frac{Y}{s} = H(\varphi)$$

then for the extremities of the line, *i. e.* for the point A situated on the surface of the water and for the point O, position of the lead :

$$\frac{Y_a}{s_a} = H(\alpha)$$

$$\frac{Y_o}{s_o} = H(\omega)$$

are obtained.

$Y_a = AF$, vertical distance of the point A from the axis of the curve ;
$Y_o = OD$, vertical distance of the point O from the axis of the curve ;
s_a = arc AV, V being the apex of the curve ;

s_o = arc OV ;
α = inclination (to the vertical) of the wire at the point A ;
ω = inclination of the wire at the point O.

If the length of the line paid out be designated by l, then:

$$l + s_o = s_a$$

and consequently:

$$Y_a = s_a H(\alpha) = l H(\alpha) + s_o H(\alpha)$$
$$Y_o = s_o H(\omega)$$

The depth $h = HO$, at which the lead O is situated, is given by the formula:

$$h = Y_a - Y_o = l H(\alpha) - s_o \left[H(\omega) - H(\alpha)\right]$$

But

$$s_o = \frac{C_o}{\sin \omega} \tan \omega = C_o \sec \omega$$

and hence:

$$h = l H(\alpha) - C_o \sec \omega \left[H(\omega) - H(\alpha)\right]$$

The negative *correction* which must be applied to the length l of line paid out in order to obtain the depth h is therefore:

$$(C) \quad \text{Correction} = l - h = C\, l [1 - H(\alpha)] + C_0 \sec \omega [H(\omega) - H(\alpha)]$$

The first term $l\,[1 - H(\alpha)]$ is a function of the two observable elements, l, length of line paid-out, and α, inclination of the wire to the surface of the water.

Taking the constant C_o of the sounding equipment as being known, the second term:

$$C_o \sec \omega\, [H(\omega) - H(\alpha)]$$

is a function of the observable inclination α and of the inclination ω which is not observable and which depends on the effective towing speed.

It will be seen shortly, however, that ω is a function of l and of α and that, therefore, the second part of the correction is also reduced to a function of these two observable elements.

9. According to the formula - (A):

$$\frac{s_a}{s_o} = \frac{tan\ \alpha}{tan\ \omega}$$

$$s_a = l + s_o \qquad s_o = C_o\ sec\ \omega$$

and consequently :

$$(D) \qquad \frac{l}{C_o}\ sin\ \omega + tan\ \omega = tan\ \alpha$$

and also :

$$tan\ \omega = \frac{1}{1 + \frac{l}{C_o}\ sin\ \omega}\ tan\ \alpha$$

This equation gives ω a function of l and of α. Thus it is demonstrated that the two terms of the correction $l - h$ can be determined as a function of the observable elements l and α.

It is necessary, however, to point out that the calculation of ω by the formula (D) would be very long and that, consequently, for practical purposes, it is necessary to seek an easier and shorter method.

10. If the small degree of accuracy with which the inclination α can be observed be taken into account, and if the errors be considered which may be introduced into the calculation of the correction when α attains high values, the conclusion is reached that it is necessary to sound with inclinations which do not exceed a given limit which, for the present purpose, will be taken as 30^o.

In any case $\omega \leqslant \alpha$

($\omega = \alpha$ in the extreme case for which $l = o$, *i. e.* when the lead is towed awash).
and consequently, according to the convention adopted :

$$\omega \leqslant 30^o$$

Now, let it be noted that, as $s_o = C_o$ sin ω
then for $\omega \leqslant 30^o$

$$s - C_{oO} \leqslant C_o\ (sec\ 30^o - 1) \qquad \text{or :}$$

$$s_o - C_o \leqslant 0.15\ C_o, \qquad \text{approximately.}$$

In other terms, admitting that the soundings are made at inclinations α which are not greater than 30^o, it may be said that, with a relative error below $\frac{15}{100}$:

$$s_o = C_o$$

From this it may be seen that the characteristic quantity of each sounding equipment (the expression of which is given in paragraph 6, namely :

$C_o = \frac{Q}{q}$) *approximately* measures the arc of the curve lying between the lead O and the apex of the curve.

If it be accepted, in the first approximation, that $s_o = C_o$ then :

$$s_a = l + C_o$$

is obtained and the formula $\frac{s_o}{s_a} = \frac{\tan \omega}{\tan \alpha}$ will give :

and also :

$$\tan \omega = \frac{C_o}{l + C_o} \tan \alpha$$

$$(D') \qquad \tan \omega = \frac{1}{1 + \frac{l}{C_o}} \tan \alpha$$

This *approximate* formula permits a much more simple calculation of the angle ω as a function of l and of α if the value of C_o which corresponds to the given sounding equipment be known.

The values of ω being known, a table of values of the second term of the correction can be drawn up, *i. e.* of

$$C_o \sec \omega \; [H(\omega) - H(\alpha)]$$

as a function of the observable elements l and α.

11. The maximum limit of error which may be made in the calculation of ω by the approximate formula (D') will now be determined.

Taking : $\frac{l}{C_o} = x$

The exact value of ω is given by the formula :

$$\tan \omega = \frac{1}{1 + x \cos \omega} \tan \alpha$$

On the contrary, the approximate value, which will be referred to as ω', is given by the formula :

$$\tan \omega' = \frac{1}{1 + x} \tan \alpha$$

Consequently :

$$\tan\omega - \tan\omega' = \Delta \tan\omega = \tan\alpha \left(\frac{1}{1 + x\cos\omega} - \frac{1}{1 + x}\right)$$

and, as ω is always less than α :

$$\Delta \tan\omega < \tan\alpha \left(\frac{1}{1 + x\cos\omega} - \frac{1}{1 + x}\right)$$

The second term of this inequality may be put in the following form :

$$(1) \qquad \frac{at}{b + cx + \frac{1}{x}}$$

if $a = 1 - \cos\alpha$ $\quad b = 1 + \cos\alpha$

$c = \cos\alpha$ $\quad t = \tan\alpha$

The maximum of the trinomial which appears in the denominator is obtained for

$$cx = \frac{1}{x} \qquad \text{i. e.} \qquad x = \frac{1}{\sqrt{c}}$$

And consequently the maximum of the expression will be :

$$\frac{at}{b + 2\sqrt{c}} = \frac{\text{tang} - \sin\alpha}{1 + \cos\alpha + 2\sqrt{\cos\alpha}}$$

For $\alpha \leqslant 30^{\circ}$

$$\Delta \tan\omega < 0.021 \text{ approximately.}$$

A variation of two hundredths in the value of the tangent corresponds, at the most, to a variation slightly greater than 1° in the angle. (This can be verified with ω near zero).

Thus it is demonstrated that the use of the approximate formula (D') allows ω to be determined to an approximation of about 1°, even in the most unfaourable case.

12. It now remains to see how the value of $C_{o} = \frac{Q}{q}$, the constant for each given sounding equipment, is determined.

An approximate value of C can be determined theoretically and roughly from the dimensions of the lead and of the wire.

In fact, if the surface swept by the horizontal movement of the lead be designated by s and the surface swept during the same movement by the unit of length of vertical wire by σ, then:

$$\frac{Q}{q} = \frac{s}{\sigma}$$

But it is more convenient to make an experimental determination. With the ship under way at a constant speed v_o the lead is towed successively with two lengths of line l_1 and l_2, and the corresponding inclinations α_1, and α_2 are observed. If the arcs comprised between the surface of the sea and the apex of the curve be designated by s_1 and s_2 then in both cases:

$$l_1 = s_1 - s_o \qquad l_2 = s_2 - s_o$$

$$l_2 - l_1 = s_2 - s_1$$

According to formula (A):

$$l_2 - l_1 = \frac{C_o}{\sin\omega}(\tan\alpha_2 - \tan\alpha_1)$$

and hence:

$$\frac{l_2 - l_1}{\tan\alpha_2 - \tan\alpha_1} = \frac{C_o}{\sin\omega}$$

Therefore, if the value of the inclination ω which corresponds to this speed v_o be known also, the quantity C_o will be determined by the relation:

$$(E) \qquad C_o = \frac{l_2 - l_1}{\tan\alpha_2 - \tan\alpha_1}\sin\omega$$

The determination of ω is made by direct measurement by observing the inclination of the wire when the lead is towed awash at the speed v_o.

Example /

$$l_1 - l_2 = 45 \text{ metres.}$$

$$\text{Speed 3 knots} \left\{ \begin{array}{lll} \alpha_1 = 7^\circ & \tan\alpha_1 = 0.123 \\ \alpha_2 = 26^\circ & \tan\alpha_2 = 0.488 \\ \omega = 5^\circ & \sin\omega = 0.087 \end{array} \right.$$

$$C_o = \frac{45}{0.355} \times 0.087 = 11.03 \text{ metres.}$$

It is needless to add that if l_1 can be reduced to zero, then $\alpha_1 = \omega$ and:

$$C_o = \frac{l_2}{\tan \alpha_2 - \tan \omega} \sin \omega$$

13. The problem for the case where the lead is towed without the line being paid out has been solved.

The same results will be obtained in the case of a lead hove from a ship under way, on condition that the fall takes place without acceleration, which can be verified in practice when the wire runs out at a uniform speed.

It is known that this condition can be satisfied without difficulty and that, particularly for deep-sea soundings, it is necessary, for the accuracy of the operation, that this really be the case.

Instead of the weight (in water) P of the lead, the same weight diminished by the resistance ρ offered by the water to the fall of the lead will have to be considered, *i. e.*

$$P' = P - \rho$$

It is interesting to observe that at the usual speeds of fall (from 1 to 2 metres per second) the resistance ρ reaches considerable values.

Such diminution of weight causes a reduction in the tension on the wire and it is easy to realise that, from this very fact, a larger inclination will be observed at equal speeds, when the lead is hove, than when it is simply towed.

The inclination must always be read at the moment when the lead touches bottom (*)

14. Thus it is demonstrated that, in the practice of sounding, the formula for correction which has been established can be used either for a towed lead or for a falling lead; in other words, that in all cases the formula for reducing the soundings to the vertical can be used.

In deep-sea sounding, the second term of the correction is very small with reference to the first term and, consequently, it can be neglected and the correction is reduced to the first term only, *i. e.* by applying a correction equal to

$$l\,[1 - H\,(\alpha)]$$

15. In order to facilitate the calculation of the correction, a Table of the values of the function $H\,(\varphi)$ is given below.

In addition, as an example, an abridged specimen calculation of a table of corrections for the value $C_o = 10$ is given.

(*) As also in the case of sounding with the "fish lead"

Values of $H(\varphi) = \dfrac{\log_e \tan\left(45^\circ + \frac{\varphi}{2}\right)}{\tan \varphi}$

φ	$H(\varphi)$	φ	$H(\varphi)$	φ	$H(\varphi)$
1°	0,99996	16°	0,98678	31°	0,94791
2	.99980	17	.98504	32	.94425
3	.99954	18	.98319	33	.94044
4	.99919	19	.98122	34	.93647
5	.99873	20	.97914	35	.93235
6	.99817	21	.97694	36	.92806
7	.99751	22	.97462	37	.92361
8	.99674	23	.97217	38	.91898
9	.99588	24	.96960	39	.91418
10	.99490	25	.96691	40	.90920
11	.99381	26	.96408	41	.90403
12	.99262	27	.96112	42	.89867
13	.99132	28	.95803	43	.89311
14	.98992	29	.95480	44	.88735
15	.98841	30	.95143	45	.88138

ABRIDGED SPECIMEN OF A TABLE OF CORRECTIONS FOR INCLINATION OF WHEN LINE $C_o = 10$.

l	INCLINATION α				
	0°	10°	20°	30°	
metres	metres	metres	metres	metres	
0	0	0	0	0	
10	0	0.05 0.04	0.21 0.16	0.49 0.37	1st term of the correction. 2nd » »
20	0	0.10 0.05	0.42 0.19	0.97 0.43	1st term of the correction. 2nd » »
30	0	0.15 0.05	0.63 0.20	1.46 0.46	1st term of the correction. 2nd » »
40	0	0.20 0.05	0.83 0.20	1.94 0.47	1st term of the correction. 2nd » »
50		0.25 0.05	1.04 0.20	2.43 0,47	1st term of the correction. 2nd » »

The two terms of the correction are given separately in this table:

1st term $= l\,[1 - H(\alpha)]$

2nd term $= C_o \sec\omega\,[H(\omega) - H(\alpha)]$.

ω having been calculated, as a function of l and α, by the (approximate) formula:

$$\tan\omega = \frac{1}{1 + \frac{l}{C_o}} \tan\alpha$$

The sum of the two terms gives the total correction.

TABLE

GIVING THE VALUES OF THE ANGLE ω, WHEN $C_o = 10$, AS A FUNCTION OF l AND α. USED FOR THE CALCULATION OF THE 2nd TERM OF THE CORRECTION AS A FUNCTION OF THE SAME ELEMENTS.

l	INCLINATION α				
	0°	10°	20°	30°	
metres					
0	0	10°	20°	30°	
10	0	5°02'	10°19'	16°06'	
20	0	3°22'	6°55'	10°54'	
30	0	2°31'	5°12'	8°13'	
40	0	2°01'	4°10'	6°35'	
50	0	1°41'	3°28'	5°30'	

NOTE ON OTHER FORMULAE FOR CORRECTION BASED ON OTHER HYPOTHESES. (*)

by

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1· Monsieur A. COURTIER, in his " Note sur la Courbure de la ligne et la Correction d'inclinaison, *etc.*" (*Annales Hydrographiques* 1925-1926) assumes that the hydraulic resistance r (*see* paragraph 2) is perpendicular to the wire and equal in magnitude to the pressure which would be exerted on the vertical projection of the " element ".

$$r = q \cos \varphi$$

Consequently, the form of equilibrium of the lead line would be, according to COURTIER'S expression, " a sort of catenary ", with a horizontal axis.

The same symbols and notations as those which have been used in the proceding article will be adopted, and further, for the sake of simplicity, the function $\log_e \left(45^\circ + \frac{\varphi}{2}\right)$ of an angle φ will be referred to as $\lambda(\varphi)$.

The arc of the curve comprised between any point on the line, where the inclination of the wire to the vertical is φ, and the apex of the curve is given by the formula :

$$(A_1) \qquad s = \frac{Co}{\sin \omega} \lambda(\varphi)$$

The vertical distance from this point to the axis of the curve is given by formula :

$$(B_1) \qquad Y = \frac{Co}{\sin \omega} \text{ arc } \varphi \; (^{**})$$

The quantity $C_o = \frac{Q}{q}$ is constant for a given sounding equipment ; in this case it is also the measurement, sufficiently approximate, of the arc of the curve comprised between the lead and the apex of the curve.

In fact, according to the formula (A_1)

$$s_o = C_o \frac{\lambda(\omega)}{\sin \omega}$$

(*) This Note is intended to establish a formula for correction, analogous to that described by the author, (*see* preceeding article, paragr. 8) in cases where the curve assumed by the lead line is determined on the basis of other hypotheses (COUTIER hypothesis, DE MARCHI hypothesis).

(**) The expression *arc* is used to indicate that the value of the angle is expressed in radians.

when $\omega \leqslant 30^{\circ}$, $\frac{\lambda(\omega)}{s\,n\,\omega} \leqslant 1.10$ approximately.

So that, with a relative error of less than about 1/10, it may be assumed that $C_o = s_o$.

2. As the ratio $\frac{Y}{s}$ is solely a function of the inclination of the wire at the point under consideration,

$$\frac{Y}{s} = \frac{\text{arc } \varphi}{\lambda(\varphi)}$$

and consequently, by stating generally :

$$\frac{\text{arc } \varphi}{\lambda(\varphi)} = F(\varphi)$$

the following formula for correction is obtained, by a method identical to that which was followed in the preceding article :

$$(C_1) \quad \text{Correction} = l\left[1 - F(\alpha)\right] + C_o \frac{\lambda(\omega)}{\sin \omega}\left[F(\omega) - F(\alpha)\right]$$

which is in every respect analogous to formula C of paragraph 8.

Let ω be eliminated in the second term, expressing it as a function of the observable elements l and α and observing that ·

$$\frac{s_o}{s_a} = \frac{s_o}{l + s_o} = \frac{\lambda(\omega)}{\lambda(\alpha)}$$

an

$$s_o = C_o \frac{\lambda(\omega)}{\sin \omega}$$

therefore ;

$$(D_1) \quad \frac{l}{C_o} \sin \omega + \lambda(\omega) = \lambda(\alpha)$$

The calculation of this formula, however, is not convenient. Assuming, as a first approximation, that $C_o = s_o$ (as was done in paragraph 10), then :

$$\frac{s_0}{l + s_0} = \frac{C_0}{l + C_0} = \frac{\lambda(\omega)}{\lambda(\alpha)}$$

and hence :

$$(D'_1) \qquad \lambda(\omega) = \frac{1}{1 + \frac{l}{C_0}} \lambda(\alpha)$$

This *approximate* formula enables the values of the second term of the correction to be calculated as a function of l and α much more simply.

In order to determine the constant C_0 experimentally, let the same means be used as that described in paragraph 12, then make the calculation by mean of the formula :

$$C_0 = \sin \omega \quad \frac{l_2 - l_1}{\lambda(\alpha_2) - \lambda(\alpha_1)}$$

3. Professor DE MARCHI in his " Teoria degli scandagli di alto mare " (Memoria XXI del R. Comitato Talassografico, Venezia, 1913) assumes that the hydraulic resistance r is HORIZONTAL and equal in magnitude to the pressure which would be exercised on the vertical projection of the " element " :

$$r = q \cos \varphi$$

The form of equilibrium of a wire, each " element " of which is subjected to a *horizontal* force proportional to the vertical projection of the " element ", is a parabola with a horizontal axis — a result which may be expected from the well-known formula for suspension bridges. The value of the parameter of the parabola is :

$$\frac{P}{q} = \frac{Q}{q} \cot \omega = \frac{C_0}{\tan \omega}$$

And as :

$$(A_2) \qquad s = \frac{1}{2} \frac{C_0}{\tan \omega} \gamma(\varphi)$$

taking, for the sake of simplicity :

$$\gamma(\varphi) = \tan \varphi \sec \varphi + \lambda(\varphi) ;$$

$$(B_2) \qquad Y = \frac{C_0}{\tan \omega} \tan \varphi$$

It is interesting to note that, in this case, the constant $C_o = \frac{Q}{q}$ exactly measures the vertical distance of the lead to the axis of the curve.

In fact, according to the formula ($B2$) :

$$Y_o = \frac{C_o}{tan\ \omega}\ tan\ \omega = C_o$$

4. As in the preceding case, the ratio $\frac{Y}{s}$ is solely a function of the inclination of the wire at the point considered :

$$\frac{Y}{s} = \frac{tan\ \varphi}{\gamma\ (\varphi)}$$

Therefore, by taking :

$$\frac{tan\ \varphi}{\gamma\ (\varphi)} = K\ (\varphi)$$

the following formula for correction is obtained by the usual procedure :

$$(C_2) \quad \text{Correction} = l \left[1 - K\ (\alpha) \right] + \frac{C_o}{K\ (\omega)} \left[K\ (\omega) - K\ (\alpha) \right]$$

which is in every way analogous to the formula previously given.

Eliminating ω in the second term of the correction and expressing it as a function of the observable elements l and α, then :

$$\frac{s_o}{s_a} = \frac{s_o}{l + s_o} = \frac{\gamma\ (\omega)}{\gamma\ (\alpha)}$$

But

$$s_o = \frac{Y_o}{K\ (\omega)} = \frac{C_o}{K(\omega)}$$

and consequently, with the appropriate reductions :

$$\frac{l}{C_o}\ tan\ \omega + \gamma\ (\omega) = \gamma\ (\alpha)$$

is obtained.

The use of this accurate formula for the calculation of ω not being convenient, assuming as a first approximation that:

$s_o = Y_o$ and consequently $s_o = C_o$ then:

$$\frac{s_o}{l + s_o} = \frac{C_o}{l + C_o} = \frac{\gamma(\omega)}{\gamma(\alpha)}$$

and

$$(D'_2) \qquad \gamma(\omega) = \frac{1}{1 + \frac{l}{C_o}} \gamma(\alpha)$$

This approximate formula gives the values of the second term of the correction as a function of l and α, in a much simpler manner.

In order to determine the constant C_o experimentally, the following formula would be used:

$$(E_2) \qquad C_o = 2 \frac{l_2 - l_1}{\gamma(\alpha_2) - \gamma(\alpha_1)} \tan \omega$$

5. *Conclusion.* Whatever hypothesis be admitted as to the hydraulic resistance exerted on the wire, the formula for correction may be expressed in the form:

$$\text{Correction} = l A + C_o B$$

in which l is the length of line paid-out, C_o is a constant peculiar to each sounding equipment, and the coefficients A and B are functions of the observable quantities l and α (inclination of the wire to the surface of the sea).

In order that easy comparison may be made of the numerical results which are obtained from the three formulae for correction (C), (C_1) and (C_2), which correspond to each of the three different hypotheses, the values of the functions $H(\varphi)$, $F(\varphi)$ and $K(\varphi)$ for several values of the angle φ are given in the following table:

φ	5°	10°	15°	20°	25°	30°
$H(\varphi)$	0.9987	0.9949	0.9884	0.9791	0.9669	0.9514
$F(\varphi)$	0.9987	0.9949	0.9885	0.9795	0.9677	0.9532
$K(\varphi)$	0.9987	0.9949	0.9883	0.9788	0.9661	0.9496

Thus it is seen that (for inclinations $< 30^{o}$) the three formulae give results which differ but very slightly from one another.

It may therefore be concluded that, for *practical purposes* of hydrography, it makes no difference which of the hypotheses be accepted.

From the theoretical point of view the difference is greater and the author believes that it would probably be of interest to raise an objective discussion, based, if possible, on experimental data, as to the relative soundness of each of the three hypotheses put forward.