

Latitude d'arrivée
(Objet observé)
Latitude of Arrival
(Object observed on)

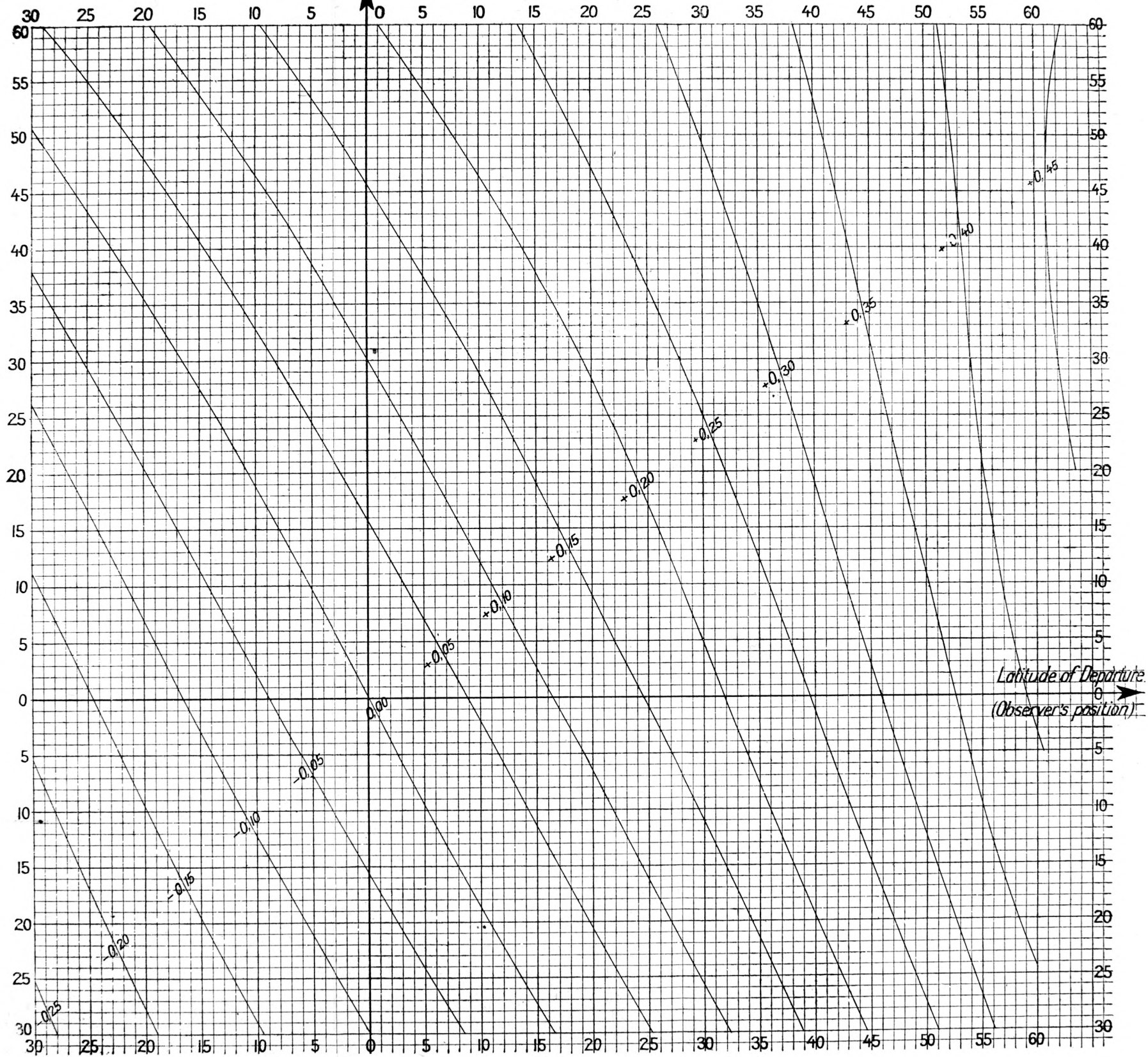
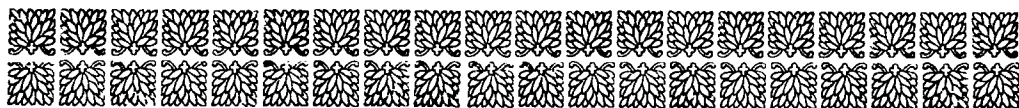


TABLE 1.



GRAPHICAL COMPUTATION OF GREAT CIRCLE COURSE AND RADIO BEARINGS.

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The following graphical table is an attempt to facilitate the laying out of Great Circles courses and Radio bearings on a Mercators chart.

The table gives a factor, which multiplied by the difference of longitude of two positions, represents the correction to be applied to the charts true course (the loxodrome) in order to find the Great Circle course (the orthodrome).

The latitudes of departure and arrival are set off respectively along the horizontal and vertical axes of a system of rectangular coordinates. The curves represent different values of the factor mentioned above.

With reference to radio bearings, the position of the observer, whether he is ashore or afloat, is taken as the departure point, and the position of the object which is observed is used as the point of arrival.

Referring to Table 1.

The procedure is as follows :

The curve passing through the point that corresponds to the latitudes of the two positions, represents the factor. For points between two curves the values are interpolated.

The difference of longitude between the two positions is multiplied by this factor, thus giving the angle by which the true course differs from the Great Circle course at the point of departure.

A positive factor indicates that the Great Circle course lies between the North Pole and the chart course, a negative factor that the Great Circle lies on the other side. (This rule holds good also when crossing the Equator, where the old rule that the Great Circle curves towards the nearest pole may fail).

It is quite accurate enough to take the latitudes and longitudes to the nearest degree.

EXAMPLE :

Departure — The English Channel, Latitude North : 51° . Point of arrival— The Newfoundland Banks, Latitude North : 43° . Difference of longitude : 50° . The table gives the factor as $+0,39$. The correction to be applied to the chart course is therefore : $50 \times 0,39 = +19^{\circ} 1/2$.

Whenever it is found necessary to change the ship's course, in order to follow the great circle, a new correction is computed by entering the table

with the ship's new latitude, and using the new difference in longitude for multiplication.

The table reproduced above was constructed with a small number of computed points only, and it should therefore be regarded as provisional.

This article is published in order to ascertain to what extent the matter is of practical interest.

The table will eventually be extended and could be issued ready for use in a comparatively short time.

All the usual advantages obtained from graphical tables, as compared with numerical tables, are obvious in this case also.

The size of the table is reduced, and no difficulty is encountered when interpolating. It would probably be a good idea to insert, on charts, an extract of the table, corresponding to the limiting latitudes of each chart.

The following table shows an example for a chart between 20° and 50° North. In this table intermediate curves have been added, thus eliminating interpolation. At the same time the scale for the latitude of arrival has been decreased, so as to give the table a more convenient form.

Referring to Table 2.

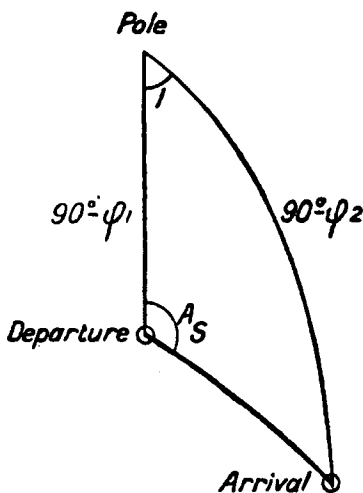
As equal values all lie on one curve, a very clear image is formed, to the eye, of how the function changes with the variables. From numerical tables a corresponding general view is obtained only by experience, based upon computation of a series of function values.

The following is a short account of the mathematical basis of the table.

True Azimuth (A_s) on the earth may be computed from the formula :

$$\text{Cot. } A_s = \frac{\cos \varphi_1 \tan \varphi_2}{\sin l} \cdot \sin \varphi_1 \text{ cot. } l.$$

where φ_1 and φ_2 designate the latitudes of the two positions, and l the difference of longitude.



Azimuth (A_m) obtained from the Mercators chart (the loxodrome course) corresponds to a computed value :

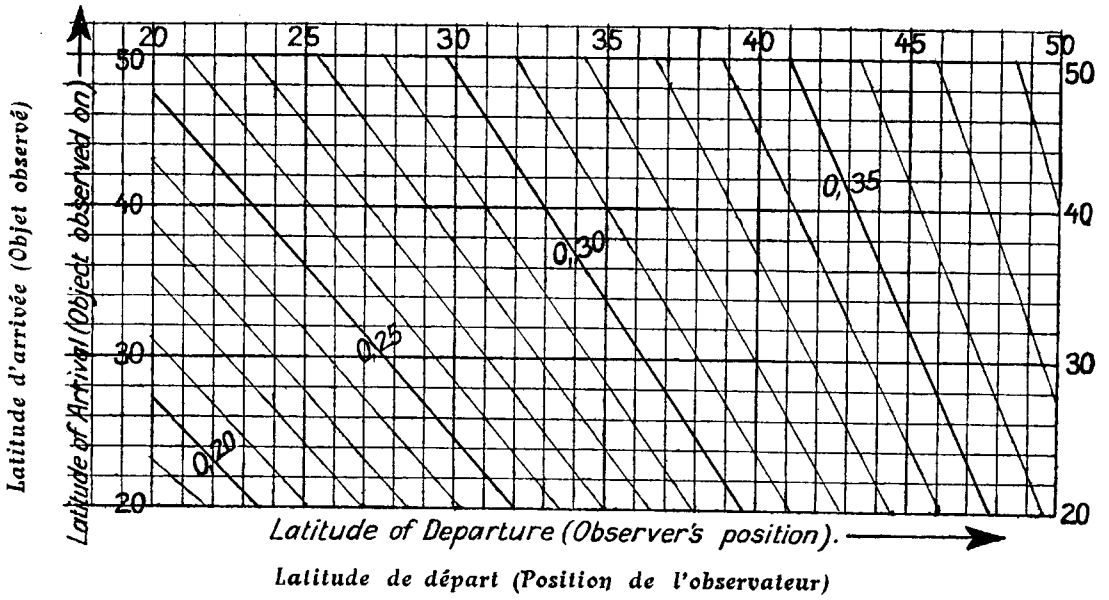


TABLE 2.

$$\cot A_m = \frac{\log \tan \left(\frac{\pi}{4} + \frac{\varphi_2}{2} \right) \div \log \tan \left(\frac{\pi}{4} + \frac{\varphi_1}{2} \right)}{l}$$

The quantity sought is the *correction*, the difference between the chart's azimuth and true azimuth, and this correction is accordingly worked out as the difference between two arc cot. functions.

It is somewhat difficult, taking the above formula only into consideration, to form an image of how the correction varies with each of the 3 quantities φ_1 , φ_2 and l . The writer's aim, however, has been to accomplish this by means of the computation of a series of examples.

As a table with 3 entry columns is very unhandy to use, the writer sought to express the correction as a product, one factor of which was a *function* of one of the *variables*, while the other factor was a *function* of the remaining *two variables*.

In accordance with the already known approximate formula, the longitude was considered separately.

To illustrate the accuracy with which this may be done, an extract of the computations is given below, taking the latitude of departure to be 30° and that of arrival to be 50°.

The following table will show how the correction varies with the difference in longitude, when the values of latitude are kept constant.

<i>Difference in longitude</i>	<i>Exact value of the correction</i>	<i>Approximate value correction = 0.3017</i>
0°	0°0	0°0
10	3,0	3,0
20	5,9	6,0
30	8,9	9,0
40	11,9	12,1
50	15,1	15,1
60	18,4	18,1
70	21,9	21,1
80	25,6	24,1
90	29,6	27,1
100	33,9	30,2
110	38,5	33,2
120	43,6	36,2
130	49,0	39,2
140	54,9	42,2
150	61,2	45,2

The *correction* = 0,3017 l . is the value derived from the graphical table. For every set of latitudes there will be one certain factor that gives the closest approximation, and it was possible to obtain the value of this factor by arranging the work as an adjustment computation.

In all the cases that have been examined however, a factor, based on a difference of longitude of 50° , has given a very good approximation, and in order to effect the highest possible regularity the computation of factors has been based on a fixed difference of longitude of 50° .

As is obvious from the above comparison, the simplified mode of calculation may only be used for differences of longitude smaller than 70° . The same limit, 70° , was found in the other cases examined.

It should be borne in mind, however, that use of the table for over 70° difference of longitude, does not cause any error with cumulative effect in Great Circle sailing, as each successive laying off of the course is quite independent of possible previous errors.

With reference to radio bearings, the matter is of no practical interest, such great distances never being required.

Mainly to show what a great field for research there may be, if the matter be considered to be of practical interest, it may be mentioned that if the formula for the correction be written :

$$\text{Correction} = 0,3017 \cdot l + 3,4 \left(\frac{l}{100} \right)^4$$

the deviation from the correct values will not be noticeable for differences in longitude of less than 150° .

Several other cases that have been examined seem to indicate the same proportion to l^4 .

From this it may probably be the fact that it is possible to arrive also at a practical solution for differences of longitude greater than 70° .

Oslo, December 1926.