ERRORS IN RESECTION

by
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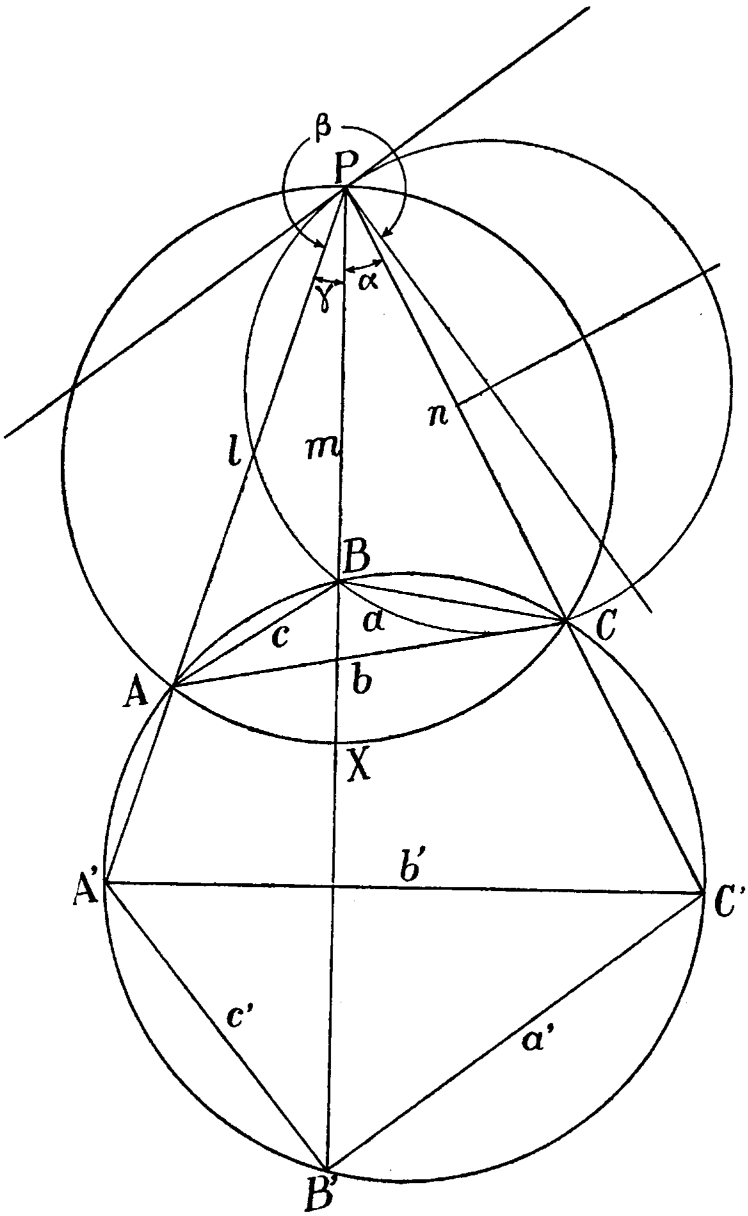
(*Empire Survey Review*, No. 18, Vol. III, Oct. 1935, pp. 218-225).

A very interesting article by Mr. J. E. Jackson, entitled *Errors in Resection* appears in the *Empire Survey Review.*

The subject matter approaches very closely to that which we have ourselves dealt with in *The Hydrographic Review*, Vol. VII, No. 2, Nov. 1930, pp. 7-30; but the author offers new expressions for the displacement undergone by the observation point as a result of errors in the sights or in the angles observed. We think it well to mention them, but without using exactly the same method of explanation and of demonstration.



(a) Let A, B, C be the three datum points seen from the observation point P, which is at distances l, m and n from them (*). Let a, b, c be the sides of the triangle ABC, and A, B, C its angles. Let also α, β, γ be the angles at which the sides of the triangle are seen from P; the signs of these angles being reckoned so that

$$\alpha + \beta + \gamma = 2\pi$$

Consider the circle passing through the three points A, B, C, and the triangle $A'B'C'$ formed by the second points of intersection of the straight lines PA, PB, PC with this circle. Let us denote its sides by a', b', c', the distances of the points A', B', C', from the point P by l', m', n', the distance from the centre of the circle to the point P by d, and its radius by R.

We have the relationships :

$$ll' = mm' = nn' = d^2 - R^2.$$

The triangles such as PAC and $PA'C'$ are thus similar and we shall have :

$$\frac{a'}{a} = \frac{m'}{n} = \frac{mm'}{mn} = \frac{d^2 - R^2}{mn} = \frac{(d^2 - R^2)^2}{lmn}\,\frac{1}{l'},$$

whence :

$$a' = al\,\frac{d^2 - R^2}{lmn} = \frac{a}{l'}\,\frac{(d^2 - R^2)^2}{lmn},$$

and similarly :

$$b' = bm\,\frac{d^2 - R^2}{lmn} = \frac{b}{m'}\,\frac{(d^2 - R^2)^2}{lmn},$$

$$c' = cn\,\frac{d^2 - R^2}{lmn} = \frac{c}{n'}\,\frac{(d^2 - R^2)^2}{lmn}.$$

The tangent to the segment containing the angle α makes an angle at P with the straight line PB equal to PCB. The straight line $B'C'$ making the same angle with the straight line PBB' is thus parallel to this tangent. It will be seen that the sides of the triangle $A'B'C'$ are parallel to the tangents to the segments containing the angles which can be observed at P between the points A, B, C.

The size of the angle A' is half the arc $B'C'$; the size of the angle α is half the same arc, less half the arc BC which is itself the measurement of the arc A. We thus have :

$$A' = \alpha + A, \qquad B' = \beta + B, \qquad C' = \gamma + C.$$

(b) Suppose we have measured the angles β and α, the first correctly, the second with an error of ε_b which may be attributed to a faulty sight of the point B. Instead of obtaining the position P, we shall obtain the position P' on the circle ACP containing the angle β. Neglecting the infinitely small quantities of greater order than the first, we find for the length PP' (now called Mm', page 24), which can then be measured along the tangent, the expression :

$$(1) \qquad PP' = \frac{mn}{\alpha \sin(\gamma + C)}\,\varepsilon_b$$

(*) *In our figure we have assumed the point* P *to be outside the triangle* ABC *because this position corresponds to the usual case which arises in observations at sea. With theodolite stations the point* P *is generally inside the triangle* ABC; *but this does not in any way modify the formulae obtained, which are general.*

Let X be the point of intersection of the straight line PB with the circle PAC; the length of the three chords PX, PA and PC can be expressed as functions of the diameter D of the circle and of the angle which they make with the diameter passing through P. Let φ be the angle made by PX with this diameter. We shall have:

$$PX = D \cos \varphi, \qquad l = D \cos(\varphi + \gamma), \qquad n = D \cos(\alpha - \varphi).$$

Let us eliminate D and φ among these equations. To do this, let us multiply the two latter by $\sin \alpha$ and $\sin \gamma$ respectively, and add them member by member.

$$l \sin \alpha + n \sin \gamma = D \cos \varphi \sin(\alpha + \gamma) = PX \sin \beta = - m \sin \beta - BX \sin \beta.$$

whence:

$$- BX \sin \beta = l \sin \alpha + m \sin \beta + n \sin \gamma,$$

an expression in which we shall denote the second member, which is homogeneous with respect to the three points A, B, C, by $\varkappa$. It should be noted that $\varkappa$ cancels out if the four points A, B, C, P are on the same circle. Further, the angle $\gamma + C$ is equal to BCX, having the same measurement on the circle PAC. Consequently, considering the triangles BCX and PAC in which the angles at X and A are equal, we shall have:

$$\frac{BX}{\sin(\gamma + C)} = \frac{a}{\sin PAC} \quad \text{and} \quad \frac{n}{\sin PAC} = \frac{b}{- \sin \beta}.$$

Equation (1) then becomes:

$$PP' = \frac{mb}{- BX \sin \beta} \varepsilon_b.$$

Introducing the value $\varkappa$, we shall have the expression:

$$(2) \qquad PP' = \frac{bm}{\varkappa} \varepsilon_b.$$

A faulty sight of the points A or C would have similarly given vectors $\frac{al}{\varkappa} \varepsilon_a$ or $\frac{cn}{\varkappa} \varepsilon_c$. If both the angles measured are faulty, the component of two vectors tangent to the two segments containing these angles will furnish the resulting displacement of the point P. The vector corresponding to an error ε_a will be parallel to $B'C'$, that corresponding to an error ε_b will be parallel to $A'C'$, etc. We have furthermore seen in (a) that the lengths a', b', c' are proportional to al, bm, cn; the vector $\frac{al}{\varkappa} \varepsilon_a$ will thus be parallel and proportional to $B'C'$ and so on. The vector resulting from the errors in the two angles measured will be parallel and proportional to one of the diagonals of the parallelogram constructed on the corresponding sides of the triangle $A'B'C'$.

(c) If instead of measuring two angles with a sextant or a hydrographic circle we take readings with a theodolite, the errors in the sights on each of these three points must be taken into consideration. An error ε_a in the sight on the point A does not influence the angle α and only falsifies the angles β and γ. It will therefore produce a displacement of P by a quantity $\frac{al}{\varkappa} \varepsilon_a$ along a vector tangent to the circle PBC. The errors in the sights of the points B and C will give similar displacements.

If the three errors were equal and of the same sign, the displacement of P would obviously be nil, and the same would apply to the resultant of the three vectors which would consequently form a closed triangle.

If one and the same maximum error $\pm E$ is to be feared in the three sights, it must be concluded from this that the end of the resultant will be inside a hexagon of which the apices are at distances of $\frac{2al}{\varkappa} E$, $\frac{2bm}{\varkappa} E$, $\frac{2cn}{\varkappa} E$ from P, in directions parallel to the sides of the triangle $A'B'C'$. The maximum displacement to be feared for P is equal to twice the greatest vector.

We must thus try to obtain for each of the points A, B, C an accuracy of sights proportional to the quantities al, bm, cn, or, which comes to the same thing, to a', b', c'

(d) Since the arc $A'B'$, as we have seen in (a), measures $2(\gamma + C)$, the angle formed by $B'A'$ with the diameter passing through B' will measure $\frac{\pi}{2} - (\gamma + C)$, and consequently :

$$c' = 2R \sin(\gamma + C).$$

The expression (1) for the vector PP' may thus be written :

$$PP' = \frac{2\,Rmn}{ac'}\varepsilon_b = \frac{2\,Rlmn}{abc}\;\frac{bc}{lc'}\varepsilon_b\,;$$

and as

$$\frac{c}{c'} = \frac{l}{m'},$$

we shall have :

$$(3) \qquad PP' = \frac{2\;Rlmn}{abc}\frac{b}{m'}\varepsilon_b\,,$$

and similarly the other vectors :

$$\frac{2\,Rlmn}{abc}\;\frac{a}{l'}\varepsilon_a\,, \qquad \frac{2\,Rlmn}{abc}\;\frac{c}{n'}\varepsilon_c\,.$$

These expressions lend themselves more easily than formulae (2) to calculation by logarithms after measuring the lengths by a graphical construction. They show also that the vector corresponding to ε_a can be considered as proportional to $\frac{a}{l'}$ just as to al or a' : a result obtained also from the values of a' given in (a).

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LOXODROME, ORTHODROME, STEREODROME

by

PROFESSOR W. IMMLER.

(*Annalen der Hydrographie*, Heft VII, 15th July 1935, pp. 275-281).

Professor W. IMMLER shows that the use of Mercator's projection by seamen has been of value to them so long as they found it no disadvantage to follow the straight or "rhumb" line which in this projection joins their point of departure to their point of arrival. But nowadays fast ships and particularly aircraft wish to follow the shortest route, the arc of a great circle; the convenience experienced in following a rhumb line by steering a course on a constant azimuth is rather illusory since changes of variation impose changes of compass course which become more frequent as the speed increases. The stereographic projection, with its great simplicity and its meridians almost rectilinear near the centre, seems to him a suitable one with which to replace Mercator's projection to advantage in such cases, and he proposes the name of "stereodrome" for the straight line joining the points of departure and arrival on the former projection.

This straight line represents an arc of a small circle of the sphere, differing according to the central point adopted for the projection. But the same generally holds good with the other systems of projection. If, for example, one steers by keeping the bearing of a radio-beacon at a constant angle from the course, one describes a rhumb line of a