SURVEY BY SHORT BASE METHOD

(Extract from an article entitled "*Some methods and procedure developed during recent Expeditionary Surveys in South-East Greenland*" by Professor N. E. Nörlund and Michael Spender in Vol. LXXXVI, N° 4, of *The Geographical Journal,* London, October 1935.

By means of a modern rapid-reading precision theodolite the length of a base-line between two instrument tripods can be measured optically with accuracy and speed. Direct measurement of the angle subtended at one end of the base by a subtense-bar set up at the other suffices for bases up to about 250 m. in length; for longer bases a shorter optically determined auxiliary base is desirable.

The light, wind, and conditions of visibility will decide how nearly this length of 250 m. may be approached. In measuring the angle subtended by the bar from the farther end of the base at least four sets of readings should be taken, the lower circle of the theodolite being shifted slightly between each pair. At 250 m. the angle subtended by the bar is about 27 ½'; an error of 2" gives an error of about 0.3 m. in the length of the base, or a scale error of about 1/833, i. e. at a distance of 12.5 km. the error in the range of a point otherwise accurately determined from the base would be about 5 m. The results of a very great number of base measurements indicate that the subtense-bar can be observed at distances up to 250 m., and sometimes greater, with an accuracy approaching the limiting accuracy of observation with the theodolite used (1) i. e. in the case of the Wild "Universal" that of a second of arc.

The base commands an area in which not only the direction but also the range of defined points may be determined. If circumstances are favourable, one end of the base may be a triangulation station and the distances may be calculated from that point; where the principal station is less favourably situated it may be neccessary to lay out an eccentric base. The connection to the main station is however easily made with the subtense-bar; often in such cases the connecting leg may be used as the auxiliary base for a better determination of the length of the main base. At an important station the main base will command points to both sides.

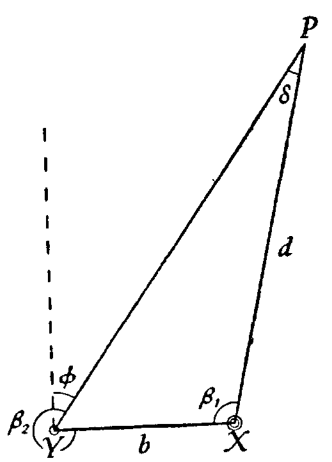

Fig. 1

Any point visible from both end-stations may be defined in relation to the base by three angles, viz. the obliquity φ, the parallax δ, and the vertical angle α. The angle φ is not directly measured in the field and may appear at first sight to be somewhat arbitrarily chosen. It is the divergence of the sighting rays from the normal to the base and its cosine is the degree to which the effective length of the base is reduced. Fig. I. shows φ and δ, and also makes clear the relation $d = b \cos \varphi / \sin \delta$. Note that the obliquity is referred to the secondary end-station. The vertical angle α need only be measured from the primary end-station, and is used in the expression $h = d \tan \alpha$, where h is the difference in height and d the distance of the point from the main station. Where the points are situated at angles less than 45° left and right from the normal to the base, i. e. where the obliquities are less than 45° the parallax δ is comparatively large up to considerable distances. For instance at fifty times the length of the base it amounts to 1°09' for points on the base normal; and 0°48 ½' for points at the extreme obliquity (45° is the limit used in practice). The parallax is obtained as the excess over 180° of the difference between the two angles β_1 and β_2 which are measured in the field.

The accuracy with which these angles may be measured is not so much dependent

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(1) Cf. F. Ackerl. — "*Entfernungsmessungen mit der Wildschen Invar-Basislatte*". Zeitschr. f. Instrumentenkunde, *Vol.* 52, 1932.

on the surveyor or his theodolite as upon the point itself. To meet the criticism that these angles of intersection are ridiculously small we must first have an idea of the error in the determination of the parallactic angle δ which would produce an appreciable error, say ½ mm., in a map at 1:200.000 (1). In a typical case with a base 250 m. long the distance of a point 50 base lengths away is 12.5 km. At this distance the angle of parallax must be known within 35" to give a position of $\pm$ 100 m. Now the base end-stations are carefully set up tripods which remain in position throughout the working of the station. The theodolite is automatically accurately centred at each end and is in each case oriented to an equally accurately centred sighting-pin at the opposite station. Instrumentally speaking therefore the accuracy desired should be very easily attained. It is the point itself which has to be carefully chosen. For certainty there is nothing better than a stone in the middle of a snow-patch. Occasionally a peak will have a well-enough defined pinnacle or outstanding feature; but there is always a risk that even an experienced surveyor can take a knife-edged ridge seen end-on to be a pinnacle, only to discover the mistake at the second station. Before the surveyor has experience in the choice of objects to sight to, he must be prepared for about one in ten of his points to be erroneously determined.

Survey by Short Base Method

23 . 9 . 1933 Station 7770 Base 291.3 m. Height 100 m.

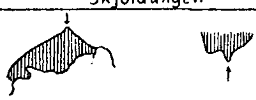
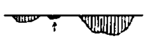
1	2	3	4	5
 High mountain on Skjoldungen	47° 29′ 42″	228° 27′ 21″ 0 57 39	91° 43′ 46″ +3 27 41 11 784	12980 ← 1302 ? 895
	64 30 57	246 25 25 1 54 28	92 21 42 +4 43 33 4 803	8020 907
7756 Foot of cairn	102 25 05	285 22 36 2 57 31	92 29 36 +4 59 21 2 475	5440 577
Stone breaking the surface 	314 30 37 3 29 00	131 01 37	89 12 04 −1 35 43 1 100	3610 0

FIG. 2

The successful application of the method is a matter of routine, rapid reading, and good booking. Not only has every point sighted to be sighted twice, but the base must be reconnoitred and laid out, all of which takes valuable time on a difficult station. Figure 2. shows the method of booking which was finally evolved. It is seen to comprise five columns. To explain the way these are occupied it is necessary to refer to the actual operations at the station. First the tripods are erected at the base end-station; the theodolite is set up at the one and sighted to the centred pin on the other. The lower circle is then so set that the reading is 0°0'0", or only a few seconds from it. The points chosen as suitable for observation are now sketched or otherwise described in the first column. It is best to make a little sketch of the mountain on which the point lies as seen with the naked eye, indicating with an arrow the point's approximate position. A drawing is then made of the details seen in the telescope, so that precisely the same object may be picked up again from the other base end-station. The second sketch is of course drawn inverted, exactly as seen in the telescope. The next two columns contain the horizontal angles to the point, in column 2 the angle from the primary station, in column 3 that from the secondary station. (The primary station is that

(1) *A more analytic treatment of the fine intersection will be found in R. Graf, Flums : "Fehlertheorie des Wildschen Autographen."* Schweiz, Zeitschr. für Vermessungswesen und Kulturtechnik, 1928, *Heft II. See also Richard Finsterwalder,* "Grenzen und Möglichkeiten der terrestrischen Photogrammetrie." *Sonderdruck Jahrg.* 1930, Allgem. Verm. Nachrichten, *Liebenwerda, Saxony.*

selected as pole from which the distances will be calculated). Either end of the base may be the pole and there is no restriction as to which should first be occupied. But naturally enough every attempt will be made to use the central triangulation station as centre for the computation of distances. (1)

The second column contains the horizontal angle at X between Y and the object; the third the horizontal angle at Y between X and the object; below one of which the observer writes in the second minus the first minus 180°, which is the angle δ subtended by the base at the distant object. (See Appendix I). The fourth column contains the half vertical angle observed on one face, the deduced vertical angle corrected for the zero error (in this case 11"), the correction for refraction and curvature, and the quantity $h = d$ tan (vertical angle) calculated by slide rule. The fifth column contains the distance d calculated from $d = b \cos \varphi / \sin \delta$; and the sum of H, $E - R$, and h, which is the height above sea. In the present example the height H of the station has been found from a depression angle in the fourth observation. (See also Appendix I).

APPENDIX I.

The Calculation of Distance and Height.

In fig. 2, showing the method of booking at a "base-method" station, there are two styles of figures. The italics are those entered during the working out of the observations. The parallactic angle δ, the excess of the difference between the two angles in columns 2 and 3 above 180°, will be set in one or the other column, according as to which of the two angles exceeds 180°. The obliquity φ is the arithmetical difference between β_2 entered in column 3 and 270° or 90°, according as β_2 is greater or less than 180°. If the base-length be already worked out from the angle to the subtense-bar, d can now be calculated from the formula $d = b \cos \varphi / \sin \delta$.

This is best done on a 50-cm. slide-rule divided along the fixed scales for distance and along the sliding scale for the angular functions, sine on the face and tangent on the reverse. Such slide-rules have been specially made for the Geodetic Institute in Copenhagen by the firm DENNERT & PAPE, of Altona. They give also the correction $E - R$ for curvature and refraction, which may be read off as soon as d is determined. The whole operation takes no more than a few instants and d can be entered as the upper of the two figures in column 5.

The circle readings of the angles of elevation and depression in column 4 have first to be converted into true angles. In the example given the angles are measured with a WILD "Universal" theodolite in the first, or face-right position. The second figure in column 4 is the true angle; below it is the correction in metres for curvature and refraction. The fourth term, obtained from the slide-rule after reversing the scale, is $d \tan \alpha$, the difference of height h between the observer and the point observed. If this point is at sea-level, it will give the value of H, the height of the station. For all other points the algebraic sum of h and H will give the height of the point concerned, which height is then entered as the lower of the two figures in column 5.

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(1) *For the determination of the range of the object, see Appendix I.*