ANALYSIS OF BUBBLE ACCELERATION ERRORS

by

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So little use has been made of the bubble sextant in the past that few navigators have a practical grasp of its limitations and advantages as compared to a marine sextant. With the regular use of this instrument by Pan American Airways System and operators of other trans-oceanic planes, facts about the bubble errors become of interest to the mariner as well as the aviator.

In an effort to determine the accuracy to be expected of the bubble sextant, the writer has taken thousands of observations in the air, aboard ship, in small boats and in submarines. Strange to say the errors in the air are very similar to those for observations taken at sea.

The errors are less, of course, in large steady ships than for small ones in similar weather. The general conclusions reached are: (*a*) a single bubble sextant observation is not dependable, (*b*) an average of as many as five or ten sights might be in error more than can be allowed, (*c*) an average of a large number of observations, say 50 or more, gives surprisingly close results, say within five miles, which is less than half the conceded error of position in the British Fleet at the Battle of Jutland.

In order to make clear the technique used for checking the bubble sextant errors on which this paper is based, the observed data with graphical analysis are included.

The instrumental error of a good bubble sextant is less than two nautical miles. On the other hand, the acceleration error of the bubble might easily be more than 2 degrees of arc, that is it might exceed 120 miles. This condition would apparently make it impossible to determine the position of a plane in flight close enough for practical purposes.

Fortunately, the law of average completely changes the picture. If there is no acceleration, the bubble is within 2' of the true horizon at all times. Furthermore, if the plane is following a steady course, the acceleration errors will *on the average* give observed altitudes with approximately as many plus errors as minus errors. It remains to be determined how many sights must be averaged to give the desired accuracy. In an effort to determine what accuracy might be expected under average conditions, the writer observed and analysed the data described below.

Through the courtesy of the officers at Maxwell Field, Alabama, the writer was permitted to fly in an open cockpit (O - 2 U) plane from Maxwell Field, Ala., to Langley Field, Va. The flight was made September 30, 1935. The day was generally clear, with clouds forming toward the end of the flight. The plane had a strong tail wind most of the way. The air was fairly smooth, though there were some occasional severe bumps.

In order to get as accurate data as possible, ten sextant observations were taken and the plane's position was then immediately determined as accurately as possible by air pilotage. Then ten more observations of the sun would be taken and the plane's position again determined. This procedure was kept up till the sun set. One hundred and ten observations were taken. No observation was discarded.

The observed data of sextant altitude, the Greenwich Civil Time, and position, were used to compute the true altitude of the sun for the various times. Actually, the altitudes were computed for twenty minute intervals and these altitudes were plotted against Greenwich Civil Time to get a curve of true sextant altitudes. The observed sextant altitudes were then referred to the curve to get the errors.

Errors for single shots ran as high as 122 minutes of altitude. Errors for the average of ten shots ran as high as 23 minutes. The average of successive errors for the 11 series of observations was 14 minutes of altitude, but as some of these errors were plus and some minus, the average mean error for the 110 sun observations was only (—) 3.1 minutes of altitude.

The very low algebraic average error of 3.1 minutes of altitude or 3.1 nautical miles shows the importance of averaging numerous observations. It also shows what accuracy may be expected under average conditions. Some recommend throwing out wild shots.

OBSERVED DATA September 30, 1935.

	I		II		III	
N°	GCT	Hs	GCT	Hs	GCT	Hs
1	18-23-30	53-07	18-45-15	50-49	19-00-05	47-53
2	24-15	52-06	46-00	51-12	1-15	48-06
3	24-50	52-32	46-40	50-50	2-00	48-19
4	25-35	53-07	47-35	48-07	2-40	47-14
5	26-10	52-26	48-20	49-37	3-15	48-53
6	26-40	52-57	49-00	50-22	3-40	48-15
7	27-10	52-53	49-40	49-33	4-05	48-47
8	27-40	51-42	50-20	48-47	4-30	47-32
9	28-20	51-42	51-00	48-35	5-00	48-22
10	28-40	52-01	51-35	49-54	5-45	48-03
Ave.	18-26-17	52-25.3	18-48-32.5	49-46.6	19-03-13.5	48-08.4

	IV		V		VI	
N°	GCT	Hs	GCT	Hs	GCT	Hs
1	19-21-25	45-19	19-45-00	41-25	20-16-35	33-48
2	22-25	43-43	46-25	39-27	17-20	33-33
3	22-50	43-18	47-05	39-33	17-50	33-48
4	23-20	45-00	47-35	40-02	18-15	33-05
5	23-50	44-34	48-25	39-33	19-00	32-58
6	24-10	45-07	48-50	39-04	19-20	34-52
7	24-30	44-56	49-20	39-43	19-55	33-27
8	24-50	44-15	49-50	39-39	20-35	34-34
9	25-20	44-32	50-40	40-25	21-10	32-43
10	25-40	43-45	51-05	39-17	21-35	33-10
Ave.	19-23-50	44-26.9	19-48-25.5	39-48.8	20-19-09.5	33-35.6

	VII		VIII		IX	
N°	GCT	Hs	GCT	Hs	GCT	Hs
1	21-13-40	22-04	21-37-05	16-49	21-59-00	12-51
2	14-40	23-05	37-45	17-30	59-55	11-34
3	16-25	23-54	38-30	18-49	22-00-30	11-41
4	17-00	22-31	39-15	17-47	1-45	12-39
5	17-35	21-50	39-45	17-44	2-30	12-40
6	18-00	21-35	39-45	17-44	3-05	13-45
7	18-40	22-11	41-00	17-15	3-40	10-21
8	19-00	20-53	42-00	16-10	4-00	10-34
9	19-20	21-14	42-30	16-54	4-20	10-39
10	19-40	21-23	43-05	16-37	4-40	10-28
Ave.	21-17-24	22-04	21-40-08.5	17-16.4	22-02-20.5	11-43.2

	X		XI		GROUP	ERRORS +	ERRORS −
N°	GCT	Hs	GCT	Hs	I		23
1	22-28-50	6-10	22-43-35	1-25	II		15
2	29-30	6-04	44-05	2-32	III	13	
3	30-15	5-52	44-30	3-22	IV		9
4	30-45	5-52	44-55	2-23	V		8
5	31-20	5-56	45-20	2-23	VI		14
6	31-40	5-20	45-40	2-18	VII		4
7	32-15	5-50	46-10	1-08	VIII	20	
8	33-10	6-14	46-30	1-27	IX		10
9	33-30	5-00	46-55	3-20	X	27	
10	34-00	4-55	47-20	1-38	XI		11
						60	94
Ave.	22-31-31.5	5-43.3	22-45-30	2-05			34
					Mean ave. (−)		3.1

FIG.1. Observed data for 11 groups of ten observations each with the errors for the each groups Fig.4 shows the same data graphically.

FIG.2. SUN ALTITUDES - SEPTEMBER 30,1935

COMPUTATIONS FOR POINTS ON THE ALTITUDE CURVES.

GCT	18-20-00	18-40-00	19-00-00	19-20-00	19-40-00	20-00-00	20-20-00
GHA	97-28 W	102-28	107-28	112-28	117-28	122-27.9	127-28
LONG	86-23 W	85-48	85-10	84-32	83-50	83-06	82-25
LHA	11-05 W	16-40	22-18	27-56	33-38	39-22	45-03
DEC	2-38.6 S	2-38.9	2-39.2	2-39.5	2-39.8	2-40.2	2-40.5
LHAA	71616	54242	42084	32934	25659	19772	15014
DECB	46	46	47	47	47	47	47
R A	71662	54288	42131	32981	25706	19819	15061
DECA	133680	133503	133503	133367	133350	133100	133096
R B	816	1860	3368	5366	7935	11145	15047
K A	132824	131643	130135	128001	125415	121955	118049
K	2-38 S	2-46	2-52	3-00.5	3-11.5	3-27.5	3-47
LAT	32-23 N	32-45	33-09	33-35	34-00	34-21	34-40
K L	35-01	35-31	36-01	36-35.5	37-11.5	37-48.5	38-27
K LB	8672	8940	9213	9534	9875	10234	10615
R B	816	1860	3368	5366	7935	11145	15047
Ho A	9488	10800	12581	14900	17810	21379	25659
Ho	53-29.5	51-14.5	48-27.5	46-12	41-34.5	37-41	33-38
Corr	.5	.5	.5	1.0	1.0	1.0	1.0
Hs	53-30	51-15	48-28	45-13	41-35.5	37-42	33-39

GCT	20-40-00	21-00-00	21-20-00	21-40-00	22-00-00	22-20-00	22-40-00
GHA	132-28	137-28.2	142-28	147-28	152-28.3	157-28.4	162-28.5
LONG	81-56	81-56	81-18	80-35	79-51	79-08	78-33
LHA	50-32	55-32	61-10	66-53	72-37	78-20	83-55.5
DEC	2-40.8	2-41.1	2-41.4	2-41.7	2-42.1	2-42.4	2-42.7
LHAA	11239	8383	5748	3635	2030	907	245
DECB	48	48	48	48	48	48	49
R A	11287	8431	5796	3683	2078	955	294
DECA	132980	132961	132860	132800	132692	132600	132500
R B	19608	24623	31646	40354	51986	68301	93573
K A	113372	108338	101214	92446	80706	64299	38927
K	4-13	4-44	5-35	6-50	8-58	13-09	24-05
L	34-56	34-56	35-13	35-35	36-04	36-32	36-57
K L	39-09	39-40	40-48	42-25	45-02	49-41	61-02
K LB	11042	11364	12091	13179	15077	18909	30147
R B	19608	24623	31646	40354	51986	68301	93573
Ho A	30650	35987	43737	53533	67063	87210	123720
Ho	29-35	25-53	21-25.5	16-57	12-19.5	7-43	3-19
Corr	1.5	2	2.5	3	4.5	6.5	14
Hs	29-36.5	25-55	21-28	17-00	12-24	7-49.5	3-33

MAXWELL F. to LANGLEY F.

				Distance		Ground Speed		
Check Points	GCT	LAT	LONG	Miles	Kts	Interval	Leg.	Ave.
Maxwell F.	18-20	32-23N	86-23W			m s		
Stroud,Ala.	18-55.3	33-03	85-20	76	66	35-15	128	128
Stone Mountain	19-31	33-50	84-10	86 1/2		35-45	145	137
Anderson,SC	20-12	34-33	82-41	98	85	41-00	143	140
Spartanburg	20-35	34-53	81-56	49 1/2	43	23-00	129 1/2	138
left	21-02.5							
Lexington,NC	21-47	35-45	80-18	108	94	44-30	145	139
So.Boston	22-26	36-41	78-54	101	88	39-00	155	142
Jetersville	22-57.5	37-16 1/2	78-06	60	52	31-30	114	138

This procedure is questioned for the reason that the observer cannot tell which are the wild ones, he only thinks he can by the "feel" of the plane, and apparently the proper altitude of the plane can no more be determined by the navigator than by the pilot who admits that he cannot "fly by instinct".

However, to test the theory of discarding wild shots, the two observations with largest errors in each of the eleven series of ten shots each were discarded. The error of each of the discarded shots was 40 miles or more. The average of the individual error of each series was 13.8 miles as compared with 14 miles without discarding any shots. However, the mean of the average errors with two shots discarded was (—) 4.1 miles as compared with (—) 3.1 miles with no observations discarded. This comes about by the fact that although the largest errors in each series were discarded, the gain in accuracy by discarding the largest errors was more than off-set by the loss in accuracy when averaging 8 instead of 10 sights.

A further study of the discarded shots discloses that 11 of them were plus errors and 11 of them were minus errors. The 11 plus errors discarded totalled 750 miles or an average of 68 miles, as compared with 11 minus errors totalling 713 miles with an average error of minus 65 miles.

The average error of (—) 3.1 for 110 sights includes errors of solution, personal errors, sextant and watch errors and errors of position as determined by pilotage. Since there is only one combination which is right, while one error might off-set another, probability and chance make it almost certain the bubble errors alone would be even less with all other errors eliminated.

For example, when the altitudes were computed for the positions taken as carefully as possible for the maps available, the algebraic mean error was (—) 4.4 miles. Later the positions were carefully taken from Geological Survey maps and the computations re-worked with the resulting average mean error of (—) 3.1 miles. In other words this refinement alone eliminated 30 % of the error which might otherwise have been attributed to the bubble.

This series of tests, supported by the evidence obtained by thousands of other similar observations, clearly prove that the bubble sextant errors can be reduced to an extremely low figure when sufficient observations are averaged. The indications are that it does not pay to throw away "wild" shots for the reason that the observer cannot always tell which shots are wild. The law of averages may be depended upon to reduce the errors to practicable limits, say below 5 miles, when enough sights are taken and averaged.

Since the operation of adding up a series of observations and dividing, even by 10, is laborious, some means for avoiding this operation is most important. Either a suitable averaging device, or else an easy solution, is urgently needed. The writer has found that pre-computed sextant altitudes plotted as a curve offers the most promising means for avoiding a lot of work.

Assume a plane in flight and that the navigator desires to determine his position by celestial observations. A time about fifteen minutes in advance is chosen to start observations. The sextant altitude of the sun or moon or both is computed in advance for the position in which the plane is due to be 15 minutes and 25 minutes later. The altitudes thus computed for an interval of ten minutes are plotted as a curve of altitudes using as coordinates Greenwich Civil Time and sextant altitudes.

At the instant for which the first computation is made, observation may be started and as many as desired may be taken during the ten minutes' interval and plotted directly against time and altitude. If the D. R. position of the plane was correct and the observations accurate, the plotted positions of the observations would be along the pre-computed altitude curve. If, however, the plane should be off its dead reckoning course, this fact would be indicated by the average sextant altitude.

Each observation when referred to the curve gives at once the "altitude intercept" found by laying down a line of position. If a curve can be plotted for the moon and for the sun simultaneously, a series of observations give a definite fix, while if only one body is observed, the same information is obtained as is found by plotting a line of position.

The second setting watch set to Greenwich Civil Time speeds up the operation of plotting observations on the curve. For scheduled flights, the curves may be plotted before taking off. A Bureau of Standards Type Aircraft Sextant was used in this test,

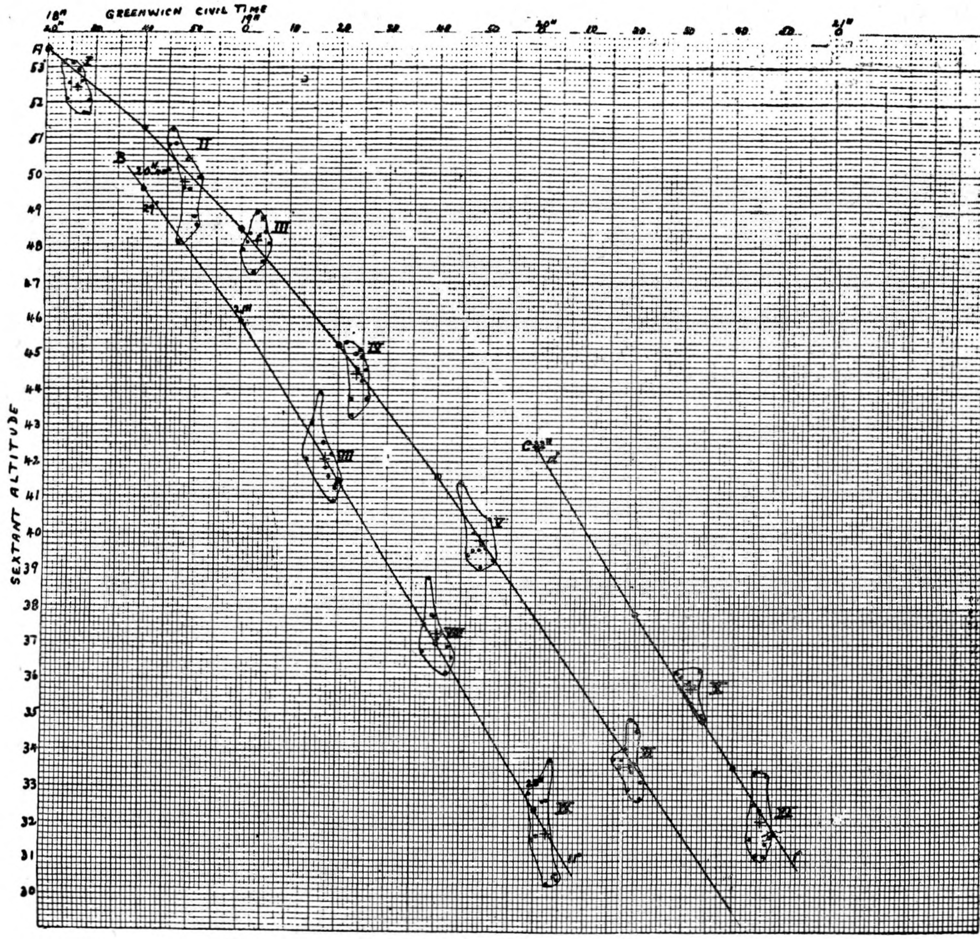

FIG. 4. Observed altitudes plotted against a curve of computed altitudes showing the individual "altitude differences" for each observation. The algebraic error for 110 observations is 3.1 miles minus.

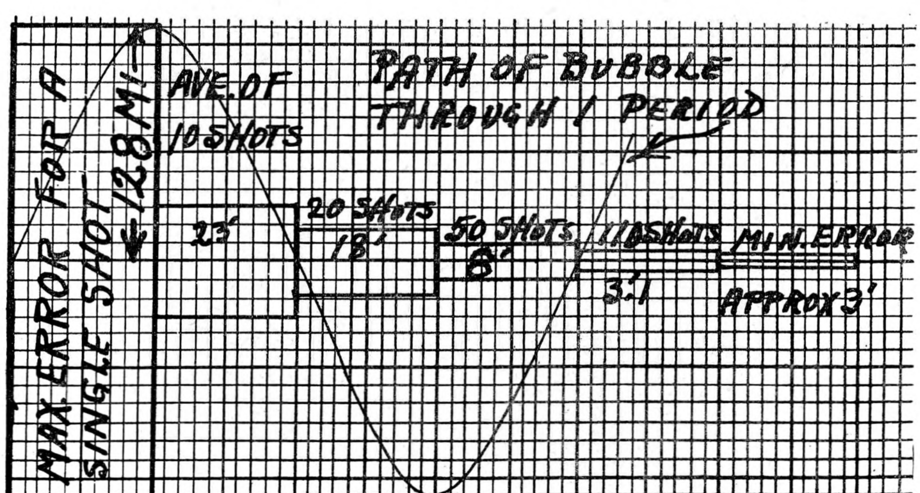

FIGURE 5. Graphic analysis of bubble acceleration errors showing the effect of averaging a large number of observations. The maximum error for a single shot was 128 miles, while the average of 110 shots was only 3.1 miles.

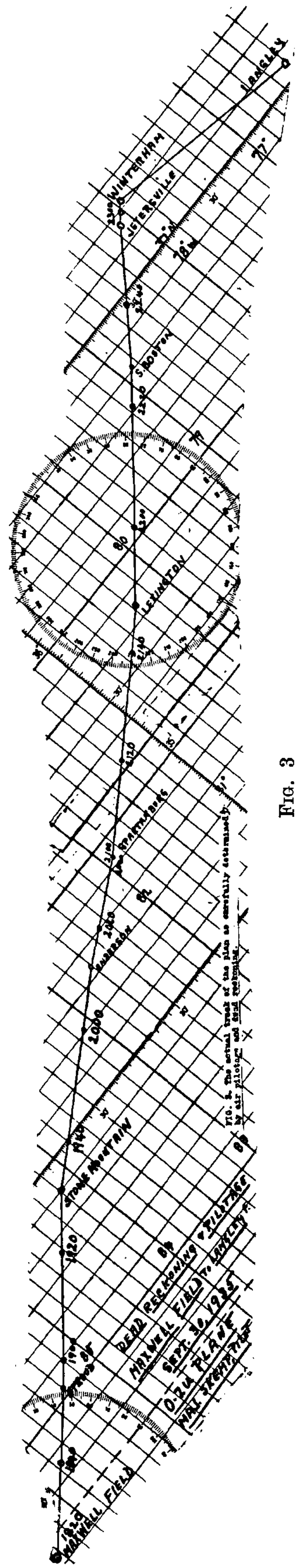

FIG. 3

but any good instrument should give similar results as the nature of the bubble is the same for most sextants.

For night observations of stars, the Star Altitude Curves are recommended since they give a definitie fix in about a minute with only one subtraction of time as the sole computation necessary. Unfortunately, the rapid motion of the sun and the moon in declination as well as irregular motion in hour angle makes it impracticable to use these bodies with fixed altitude curves similar to Star Altitude Curves.

Fig. 1 is the sextant data observed in a flight from Maxwell Field, Alabama, to Langley Field, Virginia.

Fig. 2 shows the computations for points on the curve of altitudes shown in Fig. 4. The computations in Fig. 2 were made with U. S. Hydrographic Office Publication N° 211 (Ageton's Tables). The positions for which the altitudes were computed were taken from the dead reckoning track shown in Fig. 3. Slight refinements were made subsequent to the calculations shown, but the difference is scarcely noticeable.

Fig. 4 shows the individual observations plotted against the curve of altitudes. The distance from the plotted position of any observation to the curve gives at once the "altitude difference" which would be found by working a Sumner line of position in the usual manner.

Fig. 5 is a study of the acceleration errors based on the data from the 110 observations under discussion. While this graph would not conform to the errors resulting from another series of observations, it does show the nature of the errors, and how these errors may be reduced by taking the average of a large number of observations.

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