



**A RAPID METHOD OF CALCULATION  
OF THE HARMONIC TIDAL CONSTANTS  
BY A SYSTEM OF CARDS AND MACHINES.**

BY

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A rough outline of the rapid method in question has already been given on pages 43 and 44 of Special Publication N<sup>o</sup> 12 "Investigation of Harmonic Constants. *etc.*"

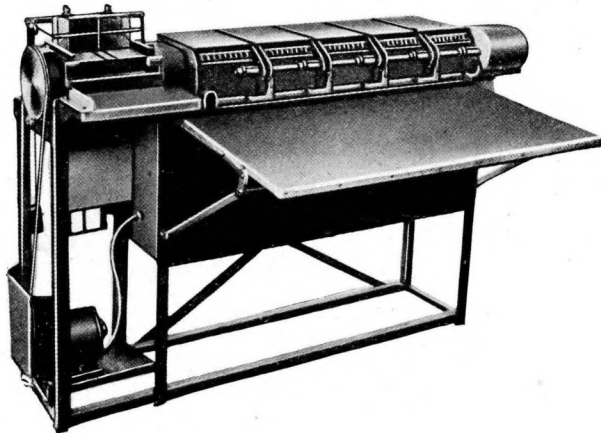
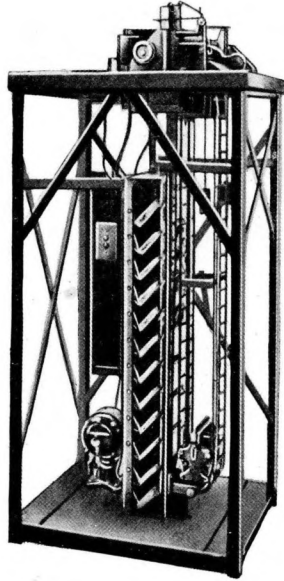
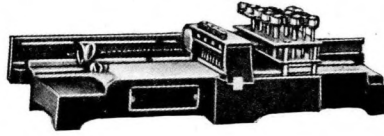
P. M. VAN REIL, the Assistant Director of the Koninklijk Nederlandsch Meteorologisch Institut of DE BILT, gives the detail of this method in the weekly paper "de Ingénieur" N<sup>o</sup> 13 of 1926.

He informs us that, by using the hourly observations taken at a mare-graph during one year, the investigation of constants by a method in which no machines are used requires about 300 hours, while the rapid method in question requires only about one hundred. Besides, the latter method is checked automatically, whereas for the former a particular control is necessary.

The tide-book proposed by the Institute also includes the partial waves K and P, so that the complete type for the first four days is as follows :

DATE OF COMMENCEMENT

Day & Time of S (1)	Observation (2)	Day & Time of					etc.	Day & Time of S (1)	Observation (2)	Day & Time of					etc.									
		M (3)	N (4)	O (5)	K (6)	P (7)				M (3)	N (4)	O (5)	K (6)	P (7)										
1-0	—	1-0	1-0	1-0	1-0	1-0	—	2-8	—	2-7	2-6	2-6	2-8	2-8	2-8	—	3-16	—	3-14	3-13	3-11	3-16	3-16	—
1	—	1	1	1	1	1	—	9	—	8	7	7	9	9	9	—	17	—	15	14	12	17	17	—
2	—	2	2	2	2	2	—	10	—	9	8	8	10	10	10	—	18	—	16	15	13	18	18	—
3	—	3	3	3	3	3	—	11	—	10	9	8	11	11	11	—	19	—	17	16	14	19	19	—
4	—	4	4	4	4	4	—	12	—	11	10	9	12	12	12	—	20	—	18	17	15	20	20	—
5	—	5	5	5	5	5	—	13	—	12	11	11	13	13	13	—	21	—	19	18	16	21	21	—
6	—	6	6	6	6	6	—	14	—	13	12	11	14	14	14	—	22	—	20	19	17	22	22	—
7	—	7	7	7	7	7	—	15	—	14	13	12	15	15	15	—	23	—	21	20	18	23	23	—
8	—	8	8	8	8	8	—	16	—	15	14	13	16	16	16	—	4-0	—	22	21	19	4-0	4-0	—
9	—	9	9	9	9	9	—	17	—	16	15	14	17	17	17	—	1	—	23	22	20	1	1	—
10	—	10	10	10	10	10	—	18	—	17	16	15	18	18	18	—	2	—	4-0	4-0	21	2	2	—
11	—	11	11	11	11	11	—	19	—	18	17	16	19	19	19	—	3	—	4-0	4-0	22	3	3	—
12	—	12	12	12	12	12	—	20	—	19	18	17	20	20	20	—	4	—	1	1	23	4	4	—
13	—	13	13	13	13	13	—	21	—	20	19	18	21	21	21	—	5	—	2	2	4-0	5	5	—
14	—	14	14	14	14	14	—	22	—	21	20	19	22	22	22	—	6	—	3	3	2	6	6	—
15	—	15	15	15	15	15	—	23	—	22	21	20	23	23	23	—	7	—	4	4	3	7	7	—
16	—	16	16	16	16	16	—	3-0	—	23	22	21	3-0	3-0	3-0	—	8	—	5	5	4	8	8	—
17	—	17	17	17	17	17	—	1	—	3-0	23	22	1	1	1	—	9	—	6	6	5	9	9	—
18	—	18	18	18	18	18	—	2	—	3-0	23	22	2	2	2	—	10	—	7	7	6	10	10	—
19	—	19	19	19	19	19	—	3	—	1	3-0	23	3	3	3	—	11	—	8	8	7	11	11	—
20	—	20	20	20	20	20	—	4	—	2	1	3-0	4	4	4	—	12	—	9	9	8	12	12	—
21	—	21	21	21	21	21	—	5	—	3	2	1	5	5	5	—	13	—	10	10	9	13	13	—
22	—	22	22	22	22	22	—	6	—	4	3	2	6	6	6	—	14	—	11	11	10	14	14	—
23	—	23	23	23	23	23	—	7	—	5	4	3	7	7	7	—	15	—	12	12	11	15	15	—
2-0	—	2-0	2-0	2-0	2-0	2-0	—	8	—	6	5	4	8	8	8	—	16	—	13	13	12	16	16	—
1	—	1	1	1	1	1	—	9	—	7	6	5	9	9	9	—	17	168	14	14	13	17	17	—
2	—	2	2	2	2	2	—	10	—	8	7	6	10	10	10	—	18	—	15	15	14	18	18	—
3	—	3	3	3	3	3	—	11	—	9	8	7	11	11	11	—	19	—	16	16	15	19	19	—
4	—	4	4	4	4	4	—	12	—	10	9	8	12	12	12	—	20	—	17	17	16	20	20	—
5	—	5	5	5	5	5	—	13	—	11	10	9	13	13	13	—	21	—	18	18	17	21	21	—
6	—	6	6	6	6	6	—	14	—	12	11	10	14	14	14	—	22	—	19	19	18	22	22	—
7	—	7	7	7	7	7	—	15	—	13	12	11	15	15	15	—	23	—	20	20	19	23	23	—
8	—	8	8	8	8	8	—	16	—	14	13	12	16	16	16	—	4-0	—	21	21	20	4-0	4-0	—
9	—	9	9	9	9	9	—	17	—	15	14	13	17	17	17	—	1	—	22	22	21	1	1	—
10	—	10	10	10	10	10	—	18	—	16	15	14	18	18	18	—	2	—	23	23	22	2	2	—
11	—	11	11	11	11	11	—	19	—	17	16	15	19	19	19	—	3	—	4-0	4-0	23	3	3	—
12	—	12	12	12	12	12	—	20	—	18	17	16	20	20	20	—	4	—	1	1	4-0	4	4	—
13	—	13	13	13	13	13	—	21	—	19	18	17	21	21	21	—	5	—	2	2	1	5	5	—
14	—	14	14	14	14	14	—	22	—	20	19	18	22	22	22	—	6	—	3	3	2	6	6	—
15	—	15	15	15	15	15	—	23	—	21	20	19	23	23	23	—	7	—	4	4	3	7	7	—
16	—	16	16	16	16	16	—	3-0	—	22	21	20	3-0	3-0	3-0	—	8	—	5	5	4	8	8	—
17	—	17	17	17	17	17	—	1	—	23	22	21	1	1	1	—	9	—	6	6	5	9	9	—
18	—	18	18	18	18	18	—	2	—	3-0	23	22	2	2	2	—	10	—	7	7	6	10	10	—
19	—	19	19	19	19	19	—	3	—	1	3-0	23	3	3	3	—	11	—	8	8	7	11	11	—
20	—	20	20	20	20	20	—	4	—	2	1	3-0	4	4	4	—	12	—	9	9	8	12	12	—
21	—	21	21	21	21	21	—	5	—	3	2	1	5	5	5	—	13	—	10	10	9	13	13	—
22	—	22	22	22	22	22	—	6	—	4	3	2	6	6	6	—	14	—	11	11	10	14	14	—
23	—	23	23	23	23	23	—	7	—	5	4	3	7	7	7	—	15	—	12	12	11	15	15	—
2-0	—	2-0	2-0	2-0	2-0	2-0	—	8	—	6	5	4	8	8	8	—	16	—	13	13	12	16	16	—
1	—	1	1	1	1	1	—	9	—	7	6	5	9	9	9	—	17	—	14	14	13	17	17	—
2	—	2	2	2	2	2	—	10	—	8	7	6	10	10	10	—	18	—	15	15	14	18	18	—
3	—	3	3	3	3	3	—	11	—	9	8	7	11	11	11	—	19	—	16	16	15	19	19	—
4	—	4	4	4	4	4	—	12	—	10	9	8	12	12	12	—	20	—	17	17	16	20	20	—
5	—	5	5	5	5	5	—	13	—	11	10	9	13	13	13	—	21	—	18	18	17	21	21	—
6	—	6	6	6	6	6	—	14	—	12	11	10	14	14	14	—	22	—	19	19	18	22	22	—
7	—	7	7	7	7	7	—	15	—	13	12	11	15	15	15	—	23	—	20	20	19	23	23	—



By means of the formulae given on pages 18 and 19 of Special Publication N<sup>o</sup> 12. columns may be added to the proposed tide-book for all the other constituents which might be required.

It should be observed that every single observation, without exception, must be positive; they may be started on any date, but it is easier to start them on the first day of a month. Consequently, the perforated card for observation 168 at 5 p.m. on the fourth day of the fourth month of 1912 will be as follows :

TABLE.

Year	Month	Day	Time of S	Observation	TIME OF						Number
					M	N	K	O	P	etc.	
0 0	0	0 0 0	0 0	0 0 0	0 0	0 0	0 0	0 0	0 0		0
0 1	1	1 1 1	0 1	0 1 1	0 1	0 1	0 1	0 0	0 1		0
2 0	2	2 2 2	2 2	2 2 2	2 2	2 0	2 2	2 2	2 2		2
3 3	3	3 3 3	3 3	3 3 3	3 3	3 3	3 3	3 3	3 3		3
4 4	0	4 4 0	4 4	4 4 4	4 0	4 4	4 4	4 4	4 4		4
5 5	5	5 5 5	5 5	5 5 5	5 5	5 5	5 5	5 5	5 5		5
6 6	6	6 6 6	6 6	6 0 6	6 6	6 6	6 6	6 6	6 6		6
7 7	7	7 7 7	7 0	7 7 7	7 7	7 7	7 0	7 7	7 0		7
8 8	8	8 8 8	8 8	8 8 0	8 8	8 8	8 8	8 8	8 8		8
9 9	9	9 9 9	9 9	9 9 9	9 9	9 9	9 9	9 9	9 9		9
1 2	3	4 5 6	7 8	9 10 11	12 13	14 15	16 17	18 19	20 21		

Therefore the number of cards required is 8800 for 370 days' observations (for S, M, K and P; for O only 355 days' observations are required, and for N 265).

In the example given for Kokas, in Dutch New Guinea, Lat. 2°7' S. and Long. 132°4' E., only 8856 observations were used, because one day is missing in January 1913 out of the observations which were started on 1st April 1912.

The cards are first sorted into 24 packets by the sorting-machine, according to the hours of the constituent S, from 0 to 23 inclusive; the observations of each packet are then added by the tabulator, so that the constants of S can be deduced thereby. The same mode of procedure is also applied to the constituents M, N, etc.

Instead, however, of working out the summations of the 370 cards together, they are taken for each month separately in order to collect the data which will serve for calculating Sa and Ssa. Thus the number of cards for July is 24 × 31, and the Table which gives the constituents S and M for that month is drawn up as follows :

TABLE I.

JULY

Day (1)	OBSERVATIONS (2)	Number of cards (3)	Hour (4)	OBSERVATIONS (5)	Number of cards (6)	Hour (7)	OBSERVATIONS (8)	Number of cards (9)
92	7659	24	00	10520	31	00	9789	31
93	7569	24	01	10245	31	01	10303	31
94	7603	24	02	10030	31	02	10662	31
95	7685	24	03	9860	31	03	11181	32
96	7682	24	04	9650	31	04	10871	31
97	7729	24	05	9415	31	05	10232	30
98	7712	24	06	9255	31	06	10207	31
99	7722	24	07	9101	31	07	9451	31
100	7791	24	08	8905	31	08	8810	31
101	7729	24	09	8811	31	09	8650	31
102	7771	24	10	8650	31	10	8922	31
103	7694	24	11	8660	31	11	8899	30
104	7661	24	12	8700	31	12	10073	32
105	7713	24	13	8909	31	13	10269	31
106	7499	24	14	9145	31	14	10591	31
107	7650	24	15	9733	31	15	10771	31
108	7678	24	16	10280	31	16	10887	31
109	7743	24	17	10743	31	17	10746	31
110	7817	24	18	11105	31	18	10644	32
111	7678	24	19	11325	31	19	9516	31
112	7723	24	20	11350	31	20	8689	30
113	7609	24	21	11245	31	21	8772	31
114	7649	24	22	11010	31	22	9031	31
115	7693	24	23	10745	31	23	9426	31
116	7656	24						
117	7675	24						
118	7659	24						
119	7533	24						
120	7453	24						
121	7449	24						
122	7508	24						
	237392							

COMPONENT S.

ASSEMBLY OF TABLES BY HOURS AND MEANS

TABLE 2.

KOKAS 1912-1913 ( $\phi = 2.7^\circ$  Lat. S.,  $\lambda = 132.4^\circ$  Long. E.)

Month hour	NUMBER OF OBSERVATIONS												HOURLY TOTALS FOR EACH MONTH												TOTAL	Means																		
	Apr	May	Jun	Jul	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	April	April	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.			Mar.	April																
	30	31	30	31	30	31	30	31	30	31	30	28	31	5	369	389	10450	10300	10520	9495	8605	8380	7760	8235			8296	8200	10030															
0	30	31	30	31	30	31	30	31	30	31	30	28	31	5	369	9770	10450	10300	10520	9495	8605	8380	7760	8235	8296	8200	10030	11171E	302.8															
1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9960	10520	10145	10245	9290	8530	8250	7850	8490	8565	8465	10280	112320	304.4															
2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10255	10565	10015	10030	9155	8520	8380	8170	8875	9100	8900	10685	114470	310.2															
3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10480	—	—	—	—	—	—	—	—	—	—	—	117190	317.6															
4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10600	—	—	—	—	—	—	—	—	—	—	—	120005	325.2															
5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10548	—	—	—	—	—	—	—	—	—	—	—	122048	330.8															
6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10370	—	—	—	—	—	—	—	—	—	—	—	123105	333.6															
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10019	—	—	—	—	—	—	—	—	—	—	—	122776	332.7															
8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9640	—	—	—	—	—	—	—	—	—	—	—	121515	329.3															
9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9156	—	—	—	—	—	—	—	—	—	—	—	119514	324.0															
10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8715	—	—	—	—	—	—	—	—	—	—	—	116865	316.7															
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8454	—	—	—	—	—	—	—	—	—	—	—	115033	311.8															
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8200	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	113417	307.4															
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8369	—	—	—	—	—	—	—	—	—	—	—	114417	310.1															
14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8560	—	—	—	—	—	—	—	—	—	—	—	115500	313.0															
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8857	—	—	—	—	—	—	—	—	—	—	—	117628	318.8															
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9155	—	—	—	—	—	—	—	—	—	—	—	119515	323.9															
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9439	—	—	—	—	—	—	—	—	—	—	—	120743	327.2															
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9670	—	—	—	—	—	—	—	—	—	—	—	121185	328.4															
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9840	—	—	—	—	—	—	—	—	—	—	—	120405	326.3															
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9880	—	—	—	—	—	—	—	—	—	—	—	118965	322.4															
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9787	—	—	—	—	—	—	—	—	—	—	—	116625	316.1															
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9660	—	—	—	—	—	—	—	—	—	—	—	114285	309.7															
23	30	31	30	31	30	31	30	31	30	31	30	28	31	5	369	9670	—	—	—	—	—	—	—	—	—	—	—	112255	304.2															
MONTHLY TOTALS															MONTHLY TOTALS												General Means																	
Total															720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	720	744	2821496	318.6

(\*) In January one day's observations are lacking.

COMPONENT M.

ASSEMBLY OF TABLES. BY HOURS AND MEANS.

TABLES 3.

KOKAS 1912 — 1913 ( $\phi = 2.7^\circ$  Lat. S.,  $\lambda = 132.4^\circ$  Long. E.)

Month hour	NUMBER OF OBSERVATIONS												HOURLY TOTALS FOR EACH MONTH												TOTAL	Means					
	April		May		Jun.		Jul.		Aug.		Sept.		Oct.		Nov.		Dec.		Jan.		Feb.		Mar.				April				
	29	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30			31	30	31	30	31
0	29	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31		
1	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	
2	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	
3	30	32	30	32	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	
4	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	
5	30	30	31	30	32	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31	30	31
6	29	32	29	31	31	30	32	30	30	31	27	32	5	369	9502	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	30	31	30	31	31	30	31	30	32	30	28	31	5	370	9138	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	31	31	30	31	31	30	31	29	32	29	29	31	5	370	8916	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	29	31	31	31	31	30	31	30	31	30	31	27	31	5	369	8200	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10	30	31	31	31	31	30	31	30	31	30	31	28	31	5	369	8727	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
11	30	31	30	30	32	29	32	30	32	31	28	31	4	370	9056	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	30	32	29	32	30	31	30	31	30	31	28	32	4	369	9436	etc.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13	30	31	30	31	31	30	31	30	31	30	28	31	4	368	9824	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14	31	30	31	31	31	30	31	30	31	30	29	30	4	368	10510	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
15	29	32	29	31	32	30	31	31	30	31	28	30	6	369	10010	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
16	30	31	30	31	31	30	31	30	31	30	28	28	5	368	10409	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	31	31	30	31	31	29	32	29	31	30	28	32	5	370	10532	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	31	30	31	32	30	31	29	31	30	30	28	31	5	368	9747	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	30	31	30	31	32	28	31	30	31	30	28	31	5	368	8996	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20	30	31	30	30	31	30	31	30	31	30	28	30	6	368	8571	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
21	30	32	29	31	30	30	32	30	31	31	27	32	5	370	8456	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
22	30	30	30	31	31	30	31	30	31	30	28	31	5	368	8679	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23	31	29	31	31	31	31	30	31	29	32	29	31	5	369	9392	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
TOTAL		744	720	744	720	744	720	744	720	744	720	744	120	8856	22908	23397	22762	23739	23721	22928	23671	23015	21770	24376	4049	12821	1496	318.6			
		MONTHLY TOTALS												MONTHLY TOTALS												General Means					

(\*) In January one day's observations are lacking.



The three left hand columns in Table 1 are used for testing. Columns 5 and 6, 8 and 9 (and so on for the other constituents) are inserted in Tables 2 and 3, which are drawn up for the constituents  $S$  and  $M$  only, and the results of which are as follows: on the lower left hand side the total number of observations for each month, and in the last column the total number for each hour during the year, as well as the grand total of these. On the lower right hand side is the sum of the observations made in each month, and in the last column but one are the totals, for the whole year, of the observations made at the same hour and the general total. The last column shows the mean values for each hour, *i. e.* for the zero hour,  $\frac{111715}{369} = 302.8$ , *etc.*, and finally the general mean 318.6.

In order to test the accuracy with which the cards have been perforated according to the days and observations made during one month, the cards are counted (column 3, Table 1) for each day (column 1, Table 1), and the observations made during that month are added (column 2, Table 1). The total for that month must be equivalent to the monthly total of Table 2, and the total of the sums in column 2, Table 1, for the twelve months must be equivalent to the monthly total of the last column but one, at the bottom of Table 2. Lastly the total of the monthly totals on the right hand side, 2821496, divided by the total of the monthly totals on the left hand, 8856, must come very near to the general mean 318.6.

Sometimes double insertions or blanks (for constituents other than  $S$ ) give rise to differences in number which can only be revealed by the test.

In Table 3 (Constituent  $M$ ) the observations made at the lunar hour 16 in the months of December and January are shown as amounting to 32 and 28 in number, while the sorting of the cards gives 31 and 29. The cause of this discrepancy is that the first observation in January is at the same time the double insertion at the 16th lunar hour in December. In the addition, therefore, this card should be added to those of December.

From the total number of monthly observations it will be seen that the divergence affects both months, because this number should be equal to 24 times the number of days which have elapsed since the observations were begun, that is, for December and January, 744 and 720, since in January no observations were made during one day. On the contrary Table 3 shows 745 and 719 observations.

Should the key-punch allow several cards to be punched at a time, a certain number of these might be prepared beforehand, except for the year and the observations, which are punched only when about to be used.

