THE TRACK OF THE ORTHODROMIC CURVE ON MERCATOR CHARTS.

by

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(Translated from the French).

INTRODUCTORY.

Although in both aerial and marine navigation, the tendency is towards greater utilization of the orthodromes (or the arcs of the great circles) for plotting the course over great distances, the compass still remains the instrument in common use which permits the successive courses to be steered correctly. These courses are the loxodromes into which the orthodrome in question is subdivided in practice.

It is important therefore that in aerial navigation, as in maritime navigation, use should be made of charts on which the loxodromes are represented by straight lines, that is, Mercator charts, on condition that the arcs of the great circles may be plotted easily and rapidly regardless of the chart scale.

We shall first say a few words about the principal solutions which have been proposed for the problem of orthodromic navigation.

I

THE KAHN CHARTS.

One of the most modern and at first sight most fascinating, is that of the French naval engineer KAHN. The solution consists in taking the great circle course desired as the equator of a chart constructed on the Mercator system. We know in fact that in the equatorial regions the orthodrome coincides very closely with the loxodrome and in practice differs from the latter by a negligible quantity only. M. KAHN claims that in the zone comprised between $\pm$ 15° of the equator this error becomes practically nil.

But the parallels and the meridians being represented by curves, and no longer by straight lines as on the Mercator charts, the carrying forward of the lines of position resulting from the astronomical observations becomes rather impracticable.

Another objection results from the utilization of radiogoniometric bearings. As long as one remains in the pseudo-equatorial zone, these bearings may well be represented by straight lines. But we must anticipate that in the future more and more use will be made of bearings of this kind taken on stations lying outside of this zone, in which case one can no longer consider the corresponding spherical arcs as straight lines.

HILLERET CHARTS.

Another and simpler solution was furnished some time ago by the charts of *Lieutenant de Vaisseau* HILLERET, which have been much neglected or forgotten in these days.

These charts, which comprise part of the regular service issue of the French Hydrographic Service, are constructed on a system of projection (gnomonic projection on the plane tangent at the equator) where all the arcs of the great circles of the sphere are represented by straight lines. Since they also represent the parallels and geographic meridians as curves and as straight lines respectively, it suffices to plot the straight line between two points given by their geographic positions to obtain the true orthodrome joining these two points on the terrestrial sphere.

The latitudes of the points where the straight line thus obtained intersects the successive meridians, are measured on the chart. These are then transferred to the Mercator chart and by fairing a smooth curve through these successive points one obtains the track of the orthodrome sought.

The accuracy thus obtained appears adequate for the purposes of aerial navigation at least.

It is only necessary that the Hilleret charts, which have been constructed solely for purposes of maritime navigation and on which the seaports only are charted, should be completed by indications for the airports and the principal cities in the interior comprising the aeronautical centres — a task which could easily be carried out. (1)

THE FAVÉ TRACINGS.

This is a solution analogous to that of the Hilleret charts and is furnished by the tracings invented and prepared by Ingénieur hydrographe en chef FAVÉ, of the French Hydrographic Service. The solution is based on the principle that the orthodrome is defined by the angle at which it intersects the equator and the corresponding longitude.

The tracings, constructed for the Mercator system to scales corresponding to those of the planispheres or General Charts of the Hydrographic Service, are placed over the chart of the same scale so that the equator of the tracing being maintained on that of the chart one of the orthodromes of the tracing passes through the two points defining the orthodrome sought.

Thereupon one measures the latitudes of the points where the orthodromic curve of the tracing intersects the successive meridians of the chart on which the curve is to be plotted. The points are then joined by a smooth curve which represents the orthodrome sought.

Briefly, in this process as in that of Hilleret, the Favé tracings constitute a simple calculating device.

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(1) For a more detailed description of the charts of Kahn and Hilleret, see the more complete articles of Ingénieur Hydrographe Général P. DE VANSSAY DE BLAVOUS, published in Hydrographic Review, Vol. V, N° 2, Monaco, Nov. 1928, pp. 39 and following.

II

FORMULAE FOR THE DIRECT CONSTRUCTION OF ORTHODROMES ON MERCATOR CHARTS.

It may be asked whether it might not be simpler, more rapid and accurate to have tables calculated in advance giving the intersections with the parallels or the meridians of the orthodromes which cut the equator at given angles.

For this it would doubtless be necessary to calculate the angle under which the orthodrome passing through the two given points intersects the Equator and the longitude of the corresponding point.

We give here the very simple formulae which permit the two quantities to be found with the requisite accuracy.

Let A and B represent the two points which are to be joined by the orthodrome.

L_1 and L_2 are their latitudes, considered as positive in the northern hemisphere and negative in the southern hemisphere.

G is their difference in longitude reckoned positively from W. to E.

ω is the angle made by the orthodrome sought with the plane of the Equator.

γ the longitude of the point C where that orthodrome intersects the Equator. The longitude is reckoned positively in the sense given above. (See Fig. 1).

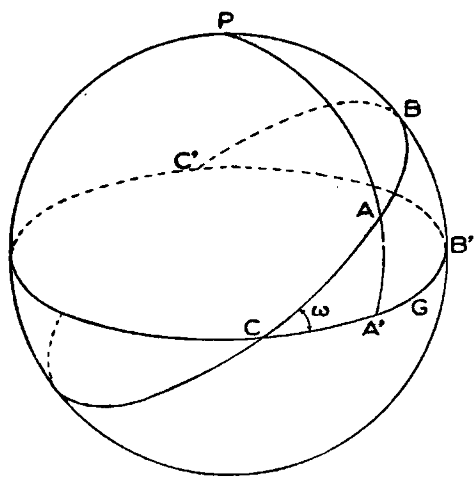

FIG. 1

Let us consider the meridians of the points A and B; they intersect the Equator at the points A' and B'.

The rectangular spherical triangles CAA' and CBB', give respectively:

$$\tan L_1 = \sin \gamma \tan \omega$$
$$\tan L_2 = \sin (\gamma + G) \tan \omega \qquad (1)$$

from which we readily deduce :

$$\text{(2)} \qquad \cot \gamma = \frac{\tan L_2}{\tan L_1 \sin G} - \cot G$$

a formula which can readily be computed by means of the logarithmic tables of addition and subtraction (1); or, where there is no need for such great accuracy, by means of the tables of natural trigonometric functions.

Once γ has been determined by the formula (2) we can calculate ω easily by one of the formulae in (1), or better by both of them as a check.

Finally, knowing γ and ω, we can calculate the arcs CA and CB by the formulae

$$\text{(3)} \qquad \sin CA = \frac{\sin L_1}{\sin \omega} \qquad \sin CB = \frac{\sin L_2}{\sin \omega}$$

The distance AB will be the difference or the sum of the two arcs; the difference if the two points A and B are located on the same side of the Equator and the sum if they are located on opposite sides of the Equator.

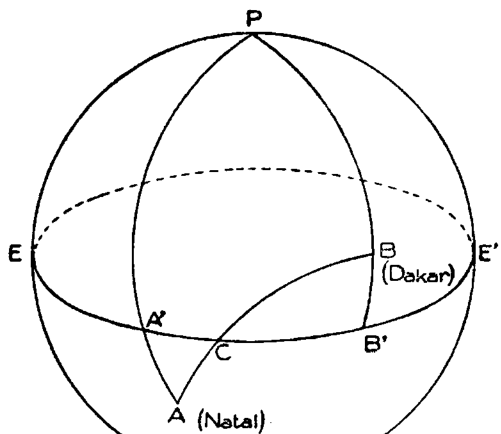

FIG. 2

(1) This formula can easily be made calculable by logarithms, by putting

$$\tan \varphi = \frac{\cos G \tan L_1}{\tan L_2}$$

We then find (2) $\tan \gamma = \dfrac{\tan L_1}{\tan L_2} \times \dfrac{\sin G \cos 45^\circ \cos \varphi}{\sin (45^\circ - \varphi)}$

An example will clarify the above statements :

Let us consider the arc of the great circle which traverses the Atlantic from Dakar to Natal (the flight of Coste and Joseph Le Brix).

We have for Dakar .. $\left\{ \begin{array}{l} L_2 = + 14^\circ 40' \\ G_2 = - 17^\circ 28' \end{array} \right\}$ (Longitude to W. of Greenwich).

Natal .... $\left\{ \begin{array}{l} L_1 = - 5^\circ 40' \\ G_1 = - 35^\circ 20' \end{array} \right\}$ (Longitude to W. of Greenwich).

$$\text{from which } G = G_2 - G_1 = + 17^\circ 52'$$

Calculation of γ

$$\cot \gamma = \frac{\tan L_2}{\tan L_1 \sin G} - \cot G$$

1st term	log. tan L_2	1. 41784 (+)			
	col. tan L_1	1. 00338 (—)			
	col. sin G	0. 51314 (+)			
	log.	0. 93436 (—)		—	8.60
2nd term	cot G	....		—	3.10
	cot γ	....		—	11.70
	γ =	....		—	4°53'

We see by the sign of γ that the point A' is to the Westward of the point C, the point where the arc of the great circle considered cuts the Equator. The longitude of this point C is therefore :

— 35° 20' + 4° 53' = 30° 27'.0 W. of Greenwich.

The formulae (1) then give us for ω $\left\{ \begin{array}{l} \text{by } AA'C \text{ .... } 49^\circ 22' \\ \text{by } AB'C \text{ .... } 49^\circ 21' \end{array} \right.$

The formulae (3) give $\left\{ \begin{array}{l} \text{for the arc } AC \text{ .... } 7^\circ 30' \\ \text{» » » } CB \text{ .... } 19^\circ 30' \end{array} \right.$

from which the arc AB 27° 00'
(or 1620 nautical miles).

Note : In cases where there is no need for such great accuracy, we may determine the position of the point C on the Equator, by means of one of the Hilleret charts or the Favé tracing. The value of ω can then be deduced immediately from the two equations (1); the difference between the two solutions thus obtained gives some idea of the accuracy which can be counted on for the method.

III

DESCRIPTION AND USE OF TABLES.

The tables which are found further on (pp. 28-32) are for the purpose of plotting the orthodromes (or the arcs of the great circles) on the Mercator charts without logarithmic calculation, and frequently by inspection. These orthodromes are defined by the angle ω under which they intersect the Equator and by the position on the Equator of the corresponding point *C*, whether this point be calculated by the formulae given above or, (in cases where less accuracy is needed). it has been obtained from the Hilleret chart or determined by means of the Favé tracings.

Tables I and II give the angles as horizontal arguments and for vertical arguments the longitudes are listed, counted from the point *C* thus determined.

Except for the equatorial region, where it suffices to consider the orthodromes corresponding to the values of ω equal to 5° and 10°, Table I gives the angles ω for every 2°, from 10° to 40°. Table II gives them for each degree from 40° to 70°.

The interpolations for the intermediate values of ω may readily be made by inspection to within 1' to 2'; which will generally suffice for all practical purposes.

For the values of the Longitude *M*, one can dispense with interpolations. It will suffice to plot on the chart the meridians corresponding to a round number of degrees to the Eastward or the Westward of the point where the orthodrome intersects the Equator.

Thus for the track of the orthodrome Dakar-Natal, which we studied above, the point where this orthodrome cuts the Equator being as we have seen in Longitude 30° 27' W. of Greenwich, we shall plot on the chart the meridians 25° 27', 20° 27', 15° 27', 10° 27', and we enter Table II with the corresponding values of *M*; that is, 5°, 10°, 15°, 20°, 25° for the Eastern branch of the orthodrome sought. The value found for ω being 49° 22', the corresponding Latitudes given by Table II will be found by inspection to be 5° 50' N., 11° 30' N., 16° 50' N., 21° 50' N., which permits us to plot the orthodromic curve with all necessary accuracy on the chart of the North Atlantic (N° 5588).

At values greater than 70° the orthodromes cut the meridians at angles which are more and more acute, except in the portions of the curve in the vicinity of the Latitude equal to this same value of ω.

It is therefore advisable to give the intersections of the orthodrome no longer with the meridians, but with the parallels to which, in proportion with the approach of ω to 90°, the orthodrome tends to become more and more perpendicular over an ever increasing portion of the track.

Table III gives the Longitudes of the points where the orthodrome having a value of ω intersects the successive parallels of Latitude L — Longitudes which must be augmented or diminished with respect to the point where the given orthodrome cuts the Equator, depending upon the branch under consideration.

The Latitudes which are given as arguments in Table III are given for each 10° only up to Latitude 40°; the corresponding parts of the orthodromes being almost straight lines approach closer and closer to the initial meridian the more the angles ω approach 90°.

Let us add that Table III stops at Latitude 70°, which is above the limit of the Mercator charts in use in the French Hydrographic Service and which we believe will rarely be exceeded by aerial navigation.

TABLE I

Latitudes of the points where an orthodrome forming the angle ω with the Equator intersects the meridians of longitude $\pm M$ (Longitude reckoned from the point where this orthodrome intersects the Equator).
($\tan L = \sin M \tan \omega$)

TABLE I

Latitudes des points où une orthodromie faisant l'angle ω avec l'équateur coupe les méridiens de longitudes $\pm M$ (longitude comptée à partir du point où cette orthodromie coupe l'équateur).
($\mathrm{tg}\ L = \sin M\ \mathrm{tg}\,\omega$)

ω \ M	5°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	M / ω
5°	0°26'	0°52'	1°03'	1°15'	1°26'	1°38'	1°49'	2°01'	2°13'	2°26'	2°39'	2°52'	3°07'	3°22'	3°38'	3°54'	4°11'	5°
10°	0°52'	1°45'	2°06'	2°28'	2°51'	3°14'	3°37'	4°01'	4°25'	4°50'	5°17'	5°44'	6°12'	6°41'	7°11'	7°44'	8°17'	10°
15°	1°18'	2°37'	3°09'	3°42'	4°15'	4°49'	5°23'	5°58'	6°34'	7°12'	7°51'	8°30'	9°12'	9°54'	10°39'	11°26'	12°14'	15°
20°	1°43'	3°27'	4°09'	4°52'	5°36'	6°21'	7°06'	7°53'	8°39'	9°28'	10°19'	11°10'	12°05'	13°00'	13°57'	14°59'	16°01'	20°
25°	2°08'	4°16'	5°09'	6°00'	6°55'	7°49'	8°44'	9°42'	10°39'	11°39'	12°41'	13°43'	14°49'	15°55'	17°04'	18°16'	19°32'	25°
30°	2°31'	5°02'	6°04'	7°06'	8°10'	9°14'	10°19'	11°26'	12°33'	13°42'	14°54'	16°06'	17°22'	18°38'	19°58'	21°21'	22°45'	30°
35°	2°53'	5°46'	6°57'	8°08'	9°20'	10°34'	11°47'	13°03'	14°19'	15°38'	16°58'	18°19'	19°44'	21°09'	22°37'	24°08'	25°41'	35°
40°	3°13'	6°28'	7°47'	9°06'	10°27'	11°48'	13°10'	14°34'	15°58'	17°24'	18°53'	20°21'	21°54'	23°26'	25°03'	26°41'	28°20'	40°
45°	3°33'	7°07'	8°33'	10°00'	11°28'	12°56'	14°25'	15°57'	17°28'	19°02'	20°37'	22°12'	23°51'	25°30'	27°12'	28°56'	30°41'	45°
50°	3°50'	7°42'	9°15'	10°49'	12°23'	13°59'	15°35'	17°12'	18°50'	20°29'	22°10'	23°52'	25°35'	27°19'	29°06'	30°55'	32°44'	50°
55°	4°06'	8°13'	9°53'	11°33'	13°13'	14°54'	16°36'	18°19'	20°02'	21°47'	23°32'	25°19'	27°07'	28°55'	30°45'	32°37'	34°30'	55°
60°	4°20'	8°41'	10°26'	12°12'	13°57'	15°43'	17°30'	19°17'	21°05'	22°54'	24°44'	26°34'	28°26'	30°17'	32°10'	34°05'	36°00'	60°
65°	4°32'	9°05'	10°55'	12°44'	14°34'	16°25'	18°15'	20°07'	21°58'	23°51'	25°44'	27°37'	29°31'	31°26'	33°22'	35°18'	37°15'	65°
70°	4°42'	9°24'	11°18'	13°11'	15°05'	16°59'	18°53'	20°47'	22°42'	24°37'	26°33'	28°29'	30°25'	32°22'	34°19'	36°17'	38°15'	70°
80°	4°55'	9°52'	11°50'	13°48'	15°46'	17°45'	19°43'	21°42'	23°40'	25°39'	27°38'	29°37'	31°36'	33°35'	35°35'	37°35'	39°34'	80°
90°	5°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	90°
ω / M	5°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°	ω / M

TABLE II

tan $L = \sin M \tan \omega$

TABLE II

tg $L = \sin M$ tg ω

M ω	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	M ω
5°	4°11'	4°20'	4°29'	4°39'	4°49'	4°59'	5°10'	5°20'	5°31'	5°43'	5°56'	6°09'	6°22'	6°36'	6°51'	7°06'	5°
10°	8°17'	8°35'	8°53'	9°12'	9°31'	9°51'	10°12'	10°33'	10°55'	11°18'	11°41'	12°06'	12°32'	12°59'	13°27'	13°56'	10°
15°	12°14'	12°41'	13°07'	13°34'	14°02'	14°31'	15°01'	15°31'	16°02'	16°35'	17°08'	17°43'	18°20'	18°57'	19°37'	20°18'	15°
20°	16°01'	16°34'	17°07'	17°41'	18°15'	18°52'	19°30'	20°09'	20°49'	21°30'	22°11'	22°54'	23°39'	24°26'	25°14'	26°03'	20°
25°	19°32'	20°11'	20°50'	21°30'	22°12'	22°55'	23°39'	24°23'	25°09'	25°56'	26°43'	27°33'	28°24'	29°17'	30°11'	31°07'	25°
30°	22°45'	23°30'	24°14'	25°00'	25°46'	26°34'	27°22'	28°12'	29°03'	29°55'	30°47'	31°42'	32°37'	33°34'	34°33'	35°32'	30°
35°	25°41'	26°29'	27°18'	28°09'	28°59'	29°51'	30°42'	31°35'	32°30'	33°25'	34°21'	35°18'	36°17'	37°17'	38°17'	39°19'	35°
40°	28°20'	29°11'	30°03'	30°56'	31°50'	32°45'	33°40'	34°35'	35°31'	36°28'	37°27'	38°27'	39°27'	40°28'	41°30'	42°33'	40°
45°	30°41'	31°35'	32°29'	33°24'	34°20'	35°16'	36°13'	37°11'	38°09'	39°08'	40°08'	41°08'	42°09'	43°11'	44°14'	45°17'	45°
50°	32°44'	33°40'	34°36'	35°32'	36°29'	37°27'	38°25'	39°24'	40°23'	41°23'	42°23'	43°24'	44°26'	45°28'	46°31'	47°34'	50°
55°	34°30'	35°27'	36°25'	37°23'	38°21'	39°20'	40°18'	41°18'	42°18'	43°18'	44°18'	45°19'	46°22'	47°24'	48°26'	49°29'	55°
60°	36°00'	36°59'	37°57'	38°55'	39°54'	40°53'	41°53'	42°53'	43°53'	44°54'	45°54'	46°55'	47°57'	48°58'	50°00'	51°02'	60°
65°	37°15'	38°13'	39°12'	40°11'	41°11'	42°11'	43°11'	44°11'	45°11'	46°11'	47°12'	48°13'	49°15'	50°16'	51°17'	52°18'	65°
70°	38°15'	39°14'	40°14'	41°13'	42°13'	43°13'	44°13'	45°13'	46°13'	47°13'	48°13'	49°15'	50°16'	51°17'	52°18'	53°19'	70°
80°	39°34'	40°34'	41°34'	42°34'	43°34'	44°34'	45°34'	46°34'	47°34'	48°34'	49°34'	50°34'	51°34'	52°34'	53°35'	54°35'	80°
90°	40°00'	41°00'	42°00'	43°00'	44°00'	45°00'	46°00'	47°00'	48°00'	49°00'	50°00'	51°00'	52°00'	53°00'	54°00'	55°00'	90°
ω M	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	ω M

TABLE II (cont^{d}.)

$\tan L = \sin M \tan \omega$

TABLE II (suite)

$\text{tg } L = \sin M \text{ tg } \omega$

ω ↓ M →	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M ← ω ↓
5°	7°06'	7°22'	7°39'	7°57'	8°16'	8°35'	8°56'	9°19'	9°42'	10°08'	10°36'	11°05'	11°37'	12°11'	12°48'	13°28'	5°
10°	13°56'	14°26'	14°58'	15°30'	16°06'	16°44'	17°24'	18°05'	18°49'	19°36'	20°26'	21°18'	22°14'	23°15'	24°20'	25°30'	10°
15°	20°18'	21°00'	21°44'	22°31'	23°18'	24°09'	25°02'	25°57'	26°56'	27°57'	29°02'	30°10'	31°23'	32°40'	34°00'	35°25'	15°
20°	26°03'	26°53'	27°46'	28°41'	29°38'	30°38'	31°40'	32°45'	33°53'	35°03'	36°16'	37°32'	38°51'	40°15'	41°43'	43°13'	20°
25°	31°07'	32°04'	33°03'	34°04'	35°07'	36°12'	37°19'	38°29'	39°41'	40°55'	42°11'	43°30'	44°53'	46°17'	47°45'	49°16'	25°
30°	35°32'	36°33'	37°36'	38°40'	39°46'	40°53'	42°03'	43°15'	44°28'	45°43'	47°00'	48°19'	49°40'	51°03'	52°29'	53°57'	30°
35°	39°19'	40°22'	41°27'	42°33'	43°40'	44°49'	45°59'	47°11'	48°24'	49°38'	50°51'	52°11'	53°30'	54°50'	56°12'	57°36'	35°
40°	42°33'	43°37'	44°42'	45°48'	46°56'	48°04'	49°14'	50°24'	51°36'	52°48'	54°02'	55°17'	56°33'	57°51'	59°09'	60°29'	40°
45°	45°17'	46°21'	47°26'	48°32'	49°39'	50°47'	51°56'	53°05'	54°14'	55°24'	56°36'	57°48'	59°01'	60°15'	61°30'	62°45'	45°
50°	47°34'	48°38'	49°43'	50°48'	51°54'	53°00'	54°07'	55°15'	56°23'	57°31'	58°40'	59°50'	61°00'	62°11'	63°23'	64°35'	50°
55°	49°28'	50°32'	51°36'	52°40'	53°44'	54°49'	55°55'	57°01'	58°07'	59°14'	60°21'	61°28'	62°36'	63°45'	64°53'	66°02'	55°
60°	51°02'	52°05'	53°08'	54°11'	55°15'	56°19'	57°23'	58°28'	59°32'	60°37'	61°42'	62°47'	63°53'	65°00'	66°06'	67°12'	60°
65°	52°18'	53°20'	54°23'	55°25'	56°27'	57°30'	58°33'	59°37'	60°40'	61°43'	62°46'	63°50'	64°54'	65°58'	67°02'	68°07'	65°
70°	53°19'	54°20'	55°21'	56°23'	57°24'	58°26'	59°28'	60°31'	61°32'	62°34'	63°36'	64°39'	65°41'	66°44'	67°46'	68°50'	70°
80°	54°35'	55°35'	56°36'	57°36'	58°36'	59°37'	60°38'	61°39'	62°39'	63°39'	64°40'	65°40'	66°41'	67°41'	68°42'	69°43'	80°
90°	55°00'	56°00'	57°00'	58°00'	59°00'	60°00'	61°00'	62°00'	63°00'	64°00'	65°00'	66°00'	67°00'	68°00'	69°00'	70°00'	90°
ω ↑ M →	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M ← ω ↑

TABLE III

For the determination of longitude of the points where the orthodrome having the angle ω intersects the parallels of latitude.

(Longitudes reckoned from the point where the orthodrome intersects the Equator).

$$\sin M = \tan L \cot \omega$$

TABLE III

Pour la détermination des longitudes des points où l'orthodromie d'angle ω coupe les parallèles de latitude.

(longitudes comptées à partir du point où l'orthodromie coupe l'équateur).

$$\sin M = \text{tg } L \text{ cotg } \omega$$

ω ↓ / L →	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	L → / ω ↓
10°	3°41'	3°29'	3°17'	3°05'	2°54'	2°42'	2°31'	2°20'	2°09'	1°58'	1°47'	1°36'	1°25'	1°14'	1°04'	0°53'	0°42'	0°32'	0°21'	0°10'	0°	10°
20°	7°37'	7°18'	6°48'	6°24'	6°00'	5°36'	5°12'	4°49'	4°26'	4°04'	3°41'	3°18'	2°56'	2°34'	2°12'	1°50'	1°28'	1°06'	0°44'	0°22'	0°	20°
30°	12°08'	11°28'	10°49'	10°10'	9°32'	8°54'	8°17'	7°40'	7°03'	6°26'	5°50'	5°15'	4°39'	4°04'	3°28'	2°53'	2°18'	1°44'	1°09'	0°35'	0°	30°
40°	17°47'	16°48'	15°49'	14°52'	13°55'	13°00'	12°04'	11°10'	10°16'	9°23'	8°31'	7°38'	6°47'	5°55'	5°03'	4°12'	3°21'	2°31'	1°40'	0°50'	0°	40°
45°	21°20'	20°08'	18°58'	17°48'	16°40'	15°32'	14°26'	13°21'	12°16'	11°12'	10°09'	9°07'	8°05'	7°03'	6°02'	5°01'	4°00'	3°00'	2°00'	1°00'	0°	45°
50°	25°42'	24°14'	22°47'	21°22'	20°00'	18°37'	17°17'	15°58'	14°41'	13°24'	12°08'	10°53'	9°38'	8°25'	7°12'	5°59'	4°47'	3°35'	2°23'	1°11'	0°	50°
55°	31°19'	29°27'	27°39'	25°53'	24°10'	22°30'	20°51'	19°15'	17°40'	16°07'	14°35'	13°05'	11°35'	10°06'	8°38'	7°11'	5°44'	4°17'	2°51'	1°26'	0°	55°
60°	39°05'	36°37'	34°15'	31°58'	29°47'	27°39'	25°36'	23°34'	21°37'	19°41'	17°47'	15°55'	14°05'	12°17'	10°29'	8°43'	6°57'	5°12'	3°28'	1°44'	0°	60°
65°	51°19'	47°36'	44°10'	40°58'	37°57'	35°04'	32°19'	29°41'	27°07'	24°38'	22°13'	19°50'	17°32'	15°16'	13°02'	10°49'	8°37'	6°27'	4°17'	2°09'	0°	65°
70°	90°00'	71°06'	63°13'	57°09'	51°59'	47°24'	43°14'	39°22'	35°44'	32°17'	28°59'	25°48'	22°43'	19°43'	16°47'	13°54'	11°04'	8°17'	5°30'	2°45'	0°	70°
ω → / L ↓	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	90°	ω ← / L ↓